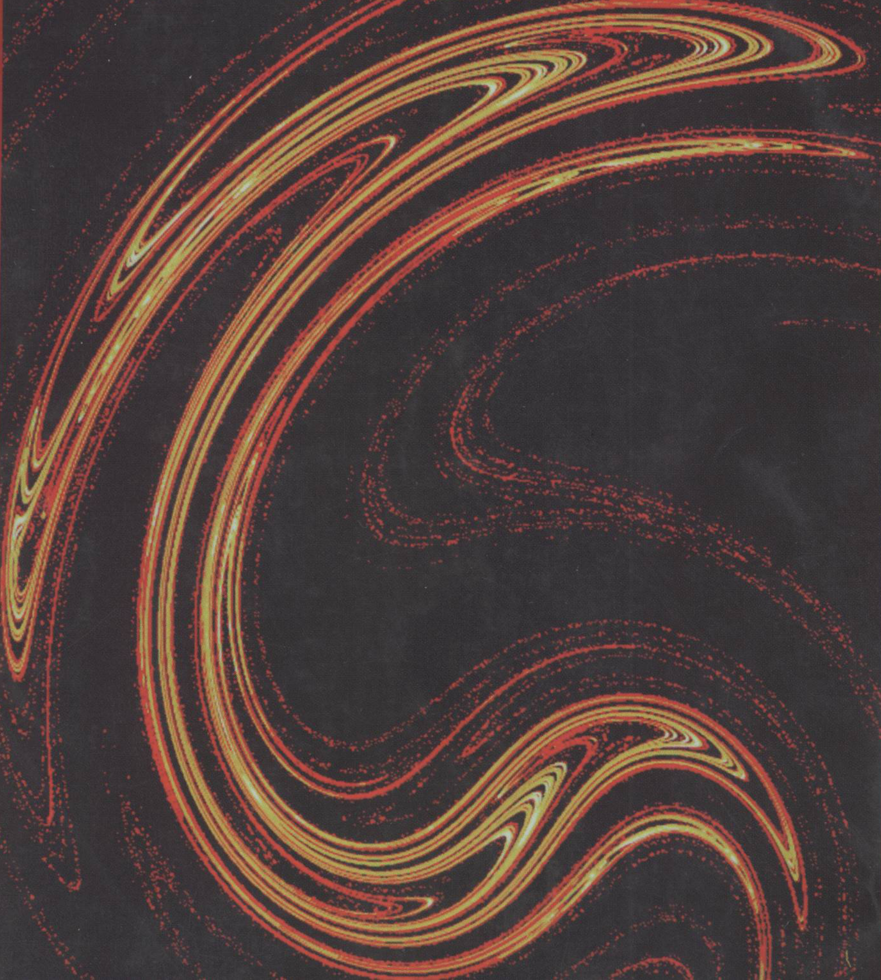
The Impact of Chaos on Science and Society

Edited by Celso Grebogi and James A. Yorke



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The impact of chaos on science and society

Preface

The growing body of knowledge about chaotic behaviour in nature and the identification of such behaviour in an increasing number of scientific disciplines prompted the United Nations University and the University of Tokyo to assemble a body of knowledgeable experts to evaluate the impact that chaos has had on the conduct of science and on our understanding of society.

The resulting symposium on the Impact of Chaos on Science and Society, convened at the University of Tokyo from 15 to 17 April 1991, was the first in the series of United Nations University international seminars on the Frontiers of Science and Technology. The symposium brought together mathematicians, physicists, biological and medical scientists, geoscientists, engineers, economists, and social scientists from nine nations, including many pioneers and world leaders in research on chaos for an interdisciplinary exchange of experiences in dealing with chaotic phenomena.

The proceedings of the symposium comprise the present volume. Its publication, we believe, will help people in different areas assess the general impact of chaos theory developed over the past two decades or so. It will further enhance the interaction among scientists who may find that chaotic dynamics plays an important role in their respective fields. In what follows we briefly comment on each of the contributions.

The contribution by Ding, Grebogi, and Yorke is an overview of the field of chaotic dynamics. Their history-oriented presentation sets the stage on which important questions can be raised and evaluated.

The paper by Campbell and Mayer-Kress addresses the issue of applying nonlinear dynamics to the modelling of socio-political problems. In particular, they investigated models of an international arms race involving two or more nations. An important point stressed by the authors is that even though mathematical modelling of social phenomena will never replace political intuition and wisdom, mathematical analysis may reveal some inescapable consequences about the system that should be considered in the decision-making process.

There has been much debate over the past few years about whether the brain obeys certain deterministic dynamics. Evidence indicates that in some pathological situations such as epilepsy this may indeed be the case. For the normal brain, positive identification of its dynamic character is still very elusive, but the prevailing wisdom lends support to the notion that the electrical activity as recorded on an EEG of the brain may be chaotic in nature. The article by Mandell and Selz raises the issue again in a different light. They argue that the neurological evidence accumulated thus far may point to the fact that the brain exhibits quasiperiodic dynamics. Their paper is concise in summarizing relevant physiological facts and insightful in mustering the theoretical arguments supporting the conclusion alluded to.

Researchers in the field of chaos theory typically employ a variety of approaches to tackle the problems encountered in nonlinear systems. These approaches can be generally categorized as numerical simulations, physical hand-waving, and rigorous mathematical theorem proving. As Ruelle points out in his contribution, this interweaving of approaches has produced exceptionally fruitful intellectual work in the past decade or two. His paper is succinctly written and should be of interest to people who want to find out about the complex and subtle relationship between chaos and mathematics.

It is a well-established fact that the giant squid axon and other relatively simple biological systems exhibit chaotic dynamics under suitable circumstances. In his contribution, one of the pioneers in this field, Aihara, provides a concise summary of chaos and its occurrence in both biological and artificial networks with an abundance of references. Moreover, the text is well illustrated with both experimental and numerical examples.

If one wants to make a list of the traditional subfields of the natural sciences which have benefited most from the development of chaos theory, physics will undoubtedly top the list. To evaluate the impact of chaos on physics and vice versa, the symposium has elicited the opinion of two experts in the field, Feigenbaum and Hao. In Feigenbaum's contribution, he mentions several specific ways in which the daily investigations of physics have been modified by the advent of chaos. In particular, researchers' thoughts are more organized now in asking questions concerning the onset of turbulence in fluids and chemical reactions. In addition, the arsenal of data analysis techniques has also been dramatically expanded with novel approaches from nonlinear dynamics, thereby revealing as never before possible information about the underlying systems. On the other hand, existing physics techniques also strongly influence the investigations of chaotic dynamics as demonstrated by Feigenbaum's own work on period-doubling bifurcations using the renormalization group method from the theory of phase transitions in physics.

Hao's article starts with a historical overview of the origin of chaos as a scientific term. He goes on to discuss the duality of two disruptions of nature in science, namely determinism and probabilism. He also touches on such issues as quantum chaos and symbolic dynamics. To further illustrate the impact and growth of chaos in the past two decades, he refers to the mammoth collection of literature compiled by his group, which contains more than 200 books and 7,000 articles! In concluding consideration of the impact of chaos on physics, one observes that before the recognition of chaos, researchers turned their backs on the seemingly random phenomena generated by such well-defined systems as nonlinear circuits and such simple mechanical systems as the driven pendulum; once convinced of the existence of chaos, physicists began noticing it throughout the universe.

The term "quantum chaos" is somewhat of a misnomer. The term is actually intended to refer to the quantum mechanical signature of classical chaos, that is, quantum mechanical systems whose classical counterparts exhibit chaotic dynamics. The contribution by Ikeda reviews one direction in this exciting field, namely, quantum chaos as a possible origin of irreversibility and dissipation. His paper should prove to be worthwhile reading for those interested in details of this subject.

The term chaos, if used alone, typically refers to the erratic temporal behaviour of a system. This is clearly not enough when one is confronted with the task of understanding fully developed turbulence, whose hallmark is spatial and temporal complexity. One step in this direction has been taken by Kaneko and his colleagues who investigate systems consisting of a large number of coupled chaotic subsystems known as coupled map lattices (CML). In his contribution to this volume, Kaneko summarizes recent progress in the field and, in particular, elaborates on the possible applications of CML to biological systems with regard to pattern formation, information processing, etc.

As Hao remarks in his paper, the work of ecologist Robert May has, to some extent, awakened physicists from their 300-year-old dream of determinism. Since May's early investigation of a simple ecological model called the logistic map, much progress has been made towards understanding complex ecological models. The question is: how does this theoretical understanding relate to the real world? This is the issue addressed in the paper by Kendall, Schaffer, Tidd, and Olsen. The first interesting result they present shows that, contrary to naive intuition, the presence of chaos in an ecological system may enhance the survivability of species making up the system in the face of regional and local perturbations. Another result described in the paper concerns the dynamic origin of measles epidemics; they propose that the variation in the size of the patient population over a multi-year period can be understood as a crisis-induced intermittent phenomenon. There are also other speculative notes in this intriguing paper.

Yet another contribution to the volume from the perspective of biological and medical sciences was written by a leading expert in the field. Over the past decade or so, Glass and his colleagues have been consistently breaking new ground in applying the dynamics approach to understanding complex physiological phenomena. In this paper, he mainly focuses on the theory and practice of dynamical disease – pathological conditions of the body which are characterized by irregular time series. What we find most striking is his portrait of the intimate relationship between theory and experiment in cardiology. As he puts it, "Such an interweaving of theory and experiment is rare in the biological sciences and is a clear demonstration of the relevance of nonlinear dynamics to medical problems."

Two papers deal with the earth sciences, covering meteorology and seismology. Speranza reviews the long history of chaos in meteorology, one of the very first fields in which chaotic phenomena were identified. He concludes that, despite this early start, the impact of chaos on meteorology still remains confined to a limited theoretical area of the whole discipline. His analysis indicates ways in which the new mathematical approaches to systems of intermediate dimension, when applied to atmospheric dynamics, can be expected to lead to major contributions of chaos theory to meteorology. Keilis-Borok assesses the impact of chaos on earthquake studies and predictions. He concludes that, even at the present stage when the applicability of chaos theory is far from clear, the concepts of nonlinear dynamics and chaos might already have important consequences.

The contribution evaluating the impact of chaos on engineering comes from Bishop. In his paper, he stresses that, in such engineering applications as preventing a ship from capsizing, it is not the chaotic attractor that plays the critical role but rather the chaotic saddle that gives rise to a chaotic transient, which is the most important element underlying global engineering integrity. Drawing from his own experience with nonlinear analysis, he concludes that the new ideas of chaotic dynamics, and the associated global geometric concepts, will undoubtedly have a profound effect on the analysis of engineering problems.

The traditional wisdom of economic theory attributes the ups and downs of economic time series to random external perturbations. Only recently has serious attention been paid to the presence of nonlinearities in economic data and possible deterministic mechanisms in economic systems. Many methods have been proposed by economists as well as others to establish whether a given time series is generated by a deterministic or a random source. Boldrin, in his contribution, briefly summarizes the current status of these developments. He points out that many economists, inspired by the success of chaos theory in the physical sciences, have made contributions to establishing the foundation of dynamic time-series analysis techniques. But the application of such techniques to real data does not always yield satisfactory results. Nevertheless, the discovery of nonlinear dynamics and chaos has provided new angles for reviewing old data and a new perspective for developing theories in economics.

The paper of Mayntz discusses the relationship between theories in the natural sciences and in the social sciences. One of the tendencies in theory transfer, about which she expresses serious doubt, is the borrowing of concepts from the natural sciences and the rephrasing of well-known social phenomena in terms of these concepts. She argues that this process has not led to new understanding.

Finally, the article by Ueda concerns strange attractors and the origin of chaos. It is a personal account of his courageous struggle against the immense barriers of tradition and misunderstanding in carrying out unconventional ideas in which he truly believes. It is wonderful reading for anyone interested in the history of chaos.

Perceptive observations about the impact of chaos on science and society evolved from an interdisciplinary discussion led by a panel under the chairmanship of Feigenbaum. Highlights of this discussion are recorded in the final chapter of these proceedings.

In conclusion, the contributions to this volume represent a wide spectrum of disciplines, which, in turn, underscores the explosive growth of chaotic dynamics research we have witnessed over the past two decades and provides a perspective on the effect chaos is beginning to have on nature and society.

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Contents

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Chaotic dynamics

Mingzhou Ding, Celso Grebogi, and James A. Yorke

Abstract

In this paper we discuss concepts and recent developments in chaotic dynamics. Our goal is to set the platform on which important questions can be raised and discussed in the future. In addition, from the history we review in this presentation it will become apparent that chaotic dynamics, before its vast implications outside of mathematics were generally appreciated over the past two decades, had endured steady progress ever since the end of the last century. This progress is forever emblematized by names such as Poincaré, Birkhoff, and Kolmogorov, whose seminal works laid the foundation for the explosive growth of research and applications of chaotic dynamics we are witnessing today.

I. Introduction

As early as in the middle of last century it was already known to Maxwell [1] that physical systems could be sensitive to initial data. But systematic studies of chaotic dynamics were nonexistent until the works of Poincaré. In a series of papers written during the period 1881–1886, Poincaré [2] analysed and named many of the qualitative features displayed by dynamical systems, which have since become part of the standard knowledge for people working in this area. The influence of Poincaré's contribution can be further seen in the fact that many questions that intrigued him a century ago are still in the forefront of research today, although large strides of progress have been made since his time.

A typical dynamical system can exhibit a variety of temporal behaviour. To understand what delineates one type of behaviour from the other one may start by considering a simple continuous system on the plane. From the Poincaré–Bendixson theorem we know that, if a solution of such an autonomous system of differential equations is bounded and does not approach an equilibrium point, then the trajectory must spiral asymptotically to a periodic orbit (limit cycle). Hence, such autonomous systems in two dimensions cannot be chaotic. Higher dimensional autonomous differential equations and also autonomous discrete systems in the plane can exhibit much richer types of dynamics including chaos. An example of higher dimensional systems is the three-body problem in celestial mechanics, the study of which has direct implications on important questions such as whether the solar system is stable. Poincaré showed that three-body problems are in general non-integrable and phase space trajectories near some very special points, which he called homoclinic points, are necessarily very complicated. This observation may be regarded as the very first indication of chaotic behaviour in dynamical systems. The understanding of what happens near such a homoclinic point has evolved over many years. An important step was taken by Birkhoff [3] earlier this century whose results provided a more detailed depiction of the dynamics near homoclinic points. Cartwright and Littlewood [4] came across the same phenomenon of homoclinic points in their study of the Van der Pol equation. Levinson [5] simplified some of the work of Cartwright and Littlewood and was able to provide a more precise description of the very complicated behaviour of the simplified Van der Pol equation. Smale [6] explained their results by relating them to his horseshoe map (a rectangle is mapped across itself with the image folded into a horseshoe shape) which exhibits a chaotic invariant set whose dynamics can be thoroughly analysed. He showed that such horseshoes must be found near homoclinic points and furthermore embedded within such a horseshoe are infinitely many periodic points (of different periods) and infinitely many trajectories that remain in the horseshoe but oscillate irregularly without ever settling down to a periodic behaviour. Horseshoes are subsequently shown to exist in numerous nonlinear systems such as the Duffing and Van der Pol equations, and in prototypical maps, such as the Hénon map. At about the same time came the work of Peixoto [7] who made substantial contributions by introducing the concept of topological space into the study of differential equations and by making precise the meaning of two systems being qualitatively equivalent. Parallel to these developments are the works of the Russian school of mathematicians, notably among them Kolmogorov and his students [8–10]. We will comment further on their contributions in subsequent sections.

The existence of horseshoe *per se* does not imply that chaotic behaviour will be observed. The invariant set of the horseshoe itself is unstable. It is also interesting to note that while chaotic attractors seem always to contain homoclinic points, the existence of such points does not ensure that the attractor is chaotic, since there may be periodic attractors within the region. Proof of the existence of chaotic attractors for nonlinear maps which model physical systems remains an outstanding, but difficult, unresolved problem. Progress has been made for some prototypical maps. For example the one-dimensional quadratic map has been shown to have chaotic attractors for a set of parameter values with positive measure [11]. Recently, an analogous result for the Hénon map with sufficiently small Jacobian was proved by Carleson and Benedicks [12].

The presence of chaotic orbits in a system has significant ramifications and can be observed indirectly even when the orbits are not attracting. When a system has multiple attractors, the boundaries between respective basins of attraction can exhibit very complicated patterns. For invertible maps of the plane, these boundaries can be fractal, making final state prediction extremely difficult for initial conditions near the boundary. Such boundaries typically contain horseshoes that, in turn, contain infinitely many unstable periodic orbits. There are still other non-attracting chaotic sets in a dynamical system that may manifest themselves as transient chaos. Such transient chaos and its characterization are important in practice to gain a proper understanding of the system. Thus, in general, studying the behaviour of chaotic sets, both attracting and not attracting, is an important problem for experimental and numerical systems.