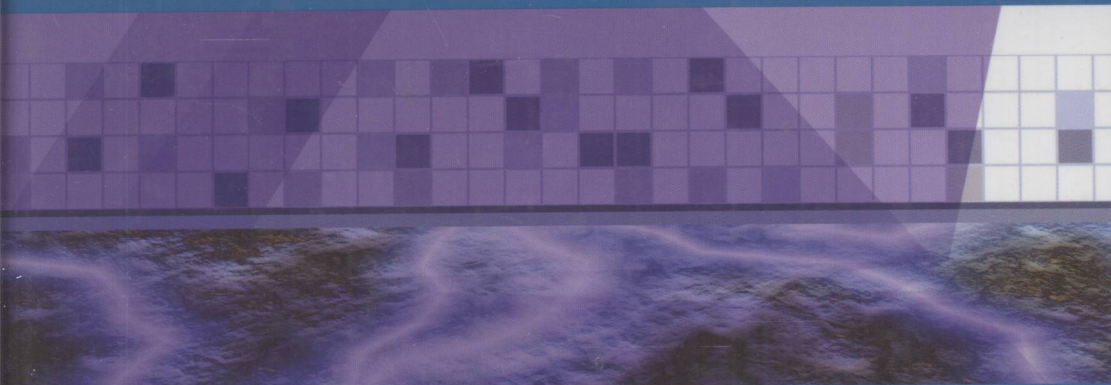


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Two-Dimensional Signal Analysis

Edited by René Garello

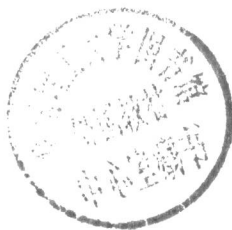
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Two-Dimensional Signal Analysis

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Introduction

The scientist studies nature, not because it is useful; but because he takes pleasure in doing so and he derives pleasure because nature is beautiful.

H. Poincaré, *Science et Methode*, 1905

For a long time, the signal processing (SP) community has limited itself to the field of 1-D signals, very often considering them to be an electronic quantity evolving over time. This approach was fundamentally associated with the type of acquisition which very often converted it in such a way as to display it on the screen of an oscilloscope and processed it with the help of electronic equipment. This occurred whatever the observation and its physical nature. This “engineering science” is strongly linked to a mathematical description and the tools that it uses in order to carry the information from a sender to a receiver. It was then necessary to allow the latter to extract the information with minimum errors or ambiguity. Finally, the field was enlarged to the understanding and modeling of our environment by adopting developed methods in a specific manner. All the initial processing of this type was directed to radar or sonar signals and the application was more military than environmental. It is necessary to note that starting from an electronic – and often mathematical – basis which underlines it, the discipline is strongly associated with computer science for the digital processing of a signal. Nevertheless, the methods continue to be based on the development of mathematical tools and the interpretation of results obtained is found principally in the frequency field. One of the principal components of SP continues to be spectral analysis (the subject of [CAS 06]).

Image processing (IP) has developed over the past two decades and is found, by its very nature, to be strongly linked to signal processing. Indeed, the French (ISIS) and international (IEEE) associations have included it in their signal processing communities. IP methods are essentially developed in the spatial domain and often propose a computer science approach (in the general sense of this discipline). Indeed, it is not rare to see the shape recognition, fuzzy logic, as well as artificial intelligence present in the field of IP in the same way as probabilistic approaches. These topics can be studied in more depth in [COC 01]. From its origin the spatial approach of processing has been guided by the object represented by the image, which in most cases happens to be a digital version of a photograph (or a series of photographs with regard to video). This image (in the visible as well as the infrared domain) is interpreted by the eye and the brain.

What happens then to processing and analysis of 2-D signals which are not images but are no longer 1-D? We will see, as the chapters in this book unfold, that the processing appropriate to the extraction of information favors the spectral approach, but that the 2-D aspect is strongly present. Causality, for example, is not a very intuitive notion in this case. As a result, the position of 2-D processing and analysis is difficult compared to the processing of the 1-D signal (which is often very theoretical and does not allow us to tackle the problem of multi-dimensionality) and of image processing (2-D, but almost exclusively spatial). The study of the m -D case (particularly, 2-D) has generated valuable works, which are often identified in scientific publications in relation to signal processing. In spite of all this, the extension of 1-D techniques to 2-D is not swift, except in simple cases implying a separateness of the signals and the systems. The Fourier transform, which is a good example, illustrates a frequency approach of 2-D processing (and image processing) but quickly shows its limits concerning non-stationarities inherent in the majority of processed data. As a result, the multi-dimensional polynomials in z are not factorable in general and this does not allow an immediate extension of the developed methods in the 1-D cases. “Images” which are most often processed in the framework of 2-D analysis thus arise from the active sensors (sonar, radar, etc.) and are not necessarily directly interpreted by people. A very important part in the extraction of information – for example, in radar imaging – is linked to the traditional domain of image processing (detection of contours, segmentation, classification). Moreover, another work is dedicated to this [MAI 08]. However, the nature of the acquisition and the particularity of certain applications (or imaged scenes) make it one of the preferred supports for the implementation of analysis methods specifically developed within the 2-D framework. Indeed, in a majority of cases, the 2-D signal obtained is strongly non-stationary and, for certain applications, non-linear and non-Gaussian.

At first, an articulation of this work concerning methods called “high resolution spectral analysis” had been envisaged (specifically Chapters 3 and 5). However,

very quickly, it appeared that the characterization of 2-D signals also uses a modeling phase which is based as much on statistics as on the spectral behavior of the entire information. Would it be necessary to construct the analysis on a modeling *a priori* of the 2-D signal or to use the data as support for a processing model? The answer to this question leads to a division of the work into two parts, after this introduction:

- a modeling approach (Chapters 2 to 4) which presents the 2-D extensions of the stochastic spectral analysis methods;
- an approach for which the described methods relate more to the 2-D structure and are thus less sensitive to their random character (Chapters 5 and 6).

In the first approach the model is stochastic, which allows us to reduce the uncertainty created by the finite and random character of the data. The book is representative of a traditional SP approach, which stresses a statistical approach of the concerned phenomenon. The methods appropriate to the processing of random 2-D signals are thus covered (Chapters 1 and 2), and non-stationary (Chapter 3) and non-linear fields (Chapter 4) are also tackled. The strong connection between these chapters based on the random character of 2-D signals is thus favored. In the second approach, data is a simple support of information. The uncertainty character present in the traditional processing methods (the Fourier transform, for example) is compensated for here, in some way, by the complementary dimensions (frequency, scale) introduced in Chapters 5 and 6.

The objective of this work is to show that, at the conjunction of the fields of signal processing and image processing, the processing and analysis of 2-D signals have their own specificity. Thus, the objective consists of presenting all the methods of 2-D signal processing and their complementarities. The stochastic aspect and the structural aspect are not yet intimately linked, but we believe this is one of the keys to future developments in the field of signal and image processing and its applications.

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