THE THEORY OF MAGNETISM MADE SIMPLE

磁性理论

An Introduction to Physical Concepts and to Some Useful Mathematical Methods

DANIEL C. MATTIS

World Scientific

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Prologue

Research into magnetism and magnetic media is undergoing a sea change. Consider the following points.

- In the attempt to satisfy an ever-increasing need for higher density recording materials ultra-small *mesoscopic* and even *nanoscopic* magnetic materials (magnetic *dots* or *grains*) are being made in the laboratory while their electrical and magnetic properties are undergoing intense scrutiny. Giant — even *colossal* — magnetoresistance are newly identified physical phenomena already being exploited for the latest in high-density media.
- Quantum tunneling of spins, virtually unknown a decade ago, has suddenly become an important and popular topic in its theory and in its practice; this new field has been named *spintronics*.
- "Quantum computation" based on quantum entanglement of spins vies with its electronic counterparts for a role in the information processing units of the future.
- "Quantum fluctuations" drive *long-range order* in a phenomenon colorfully denoted "order from disorder."
- Artificially engineered metallic alloys with fully spin-polarized conduction electrons have been dubbed "half-metals."
- A purely magnetic phenomenon (i.e., *antiferromagnetism*) is reputedly at the basis of the extraordinary discovery, high-temperature superconductivity — although the facts are either not all yet known or are insufficiently understood at the time of writing.

At the dawn of this new millennium it becomes apparent that the study of magnetism — arguably already dating back several millennia — has acquired a new urgency along with its new vocabulary. In a previous version of this book (*The Theory of Magnetism*, vols. I and II, Springer, Berlin, 1981 and 1985), the statics and dynamics of magnetism and its thermodynamic properties were examined. Insofar as they dealt with basic theory and documented the history of the science, these texts should still serve regardless of their flaws. However in other aspects the earlier works are now as hopelessly out of date as is the original 1965 edition, based on a set of lecture notes for a Summer course.

But even when applications evolve rapidly in time they are still secondary to the immutable basic physical principles that empower them. The purpose of the present text is to expound the basics of quantum magnetism to a new audience, whether undergraduate, graduate, or professional. The presentation is intended to be quickly and easily grasped by the reader — preferably with an instructor's help. The field is vast and not all the important topics could be covered. But most concepts that *are* introduced in these pages are worked out in minute detail. A lot of redundancy should make it easier to master some technical aspects. The mathematics is held at first to the minimum required for the formulation of classical and then quantum phenomena, but is then further developed to whatever extent needed as the *many-body* aspects that are at the heart of the theory are progressively allowed to unfold. There is a deliberate attempt to use similar tools wherever possible so that the reader can hone newly acquired skills on various topics; additionally, references to more advanced methods abound.

Following a historical Chapter 1 (co-authored with Dr. Noémi P. Mattis), Chapter 2 provides a quick overview of the subject matter and touches upon all the topics that follow. The study of the theory of magnetism provides more than just an entry into today's hot topics: it gives insight into all of theoretical physics, of which it is an integral part. A half-century ago when this author was still a graduate student, one of his instructors joked that, "all of mathematics is a special case of quantum mechanics." In the spirit of this witty but profound observation, let us see whether all of theoretical physics might not be "a special case of the theory of magnetism!" And there are several points to buttress this claim.

- The study of spin glasses, first undertaken thirty years ago, was intended as a first step to understanding amorphous SiO_2 (window glass) and random media in general rather than as an end in itself. One of the purposes was to understand whether a *phase transition* from a disordered high-temperature phase into a disordered low-temperature phase was even conceptually possible. The question was asked, does the "glassy" temperature signal a true discontinuity in thermodynamic properties, or merely an enhanced viscosity? Is the spin glass a genuine thermodynamic phase? This question is answered in the affirmative by some exactly solvable examples. Actually, this is just a special case of a larger class of so-called "critical phenomena." The very creation of ferromagnetism at or near the Curie temperature T_c, provides us with the original example of what is now classified as a *second-order phase transition*. This phenomenon is now known to be common to all physical sciences. Historically, the first serious theory purporting to explain how a paramagnet attains a macroscopic magnetic moment and long-range order when the temperature drops below T_c was devised one hundred years ago by Pierre Weiss, inspired by van der Waals' seminal theory of vapor-to-liquid phase transitions.

- All mean-field approximations in current use are outgrowths of Weiss' "molecular field" theory of ferromagnetism. Spin waves (magnons) are the simplest realization of Goldstone's theorem in quantum field theory. The interacting quantum spins in the Heisenberg model are the very prototype of a nonlinear quantum field theory. The first exact solution of *any* quantum "many-body problem" was "Bethe's ansatz," which revealed the eigenstates and eigenvalues of the one-dimensional Heisenberg model. And so it goes ...

Let us now mention up front what was purposely left out of this book for want of space, expertise or time. At the top of this list is the now famous "Quantum Hall Effect," whereby the two-dimensional electron gas became a laboratory for particles of fractional spin and charge. There is also little discussion of latter-day uses of Bethe's ansatz — as in the Lieb–Wu solution of the one-dimensional Hubbard model or the Andrei–Wiegman analysis of the Kondo effect. There is no development of the renormalization group (the numerical technique that first allowed insight into the second-order phase transition) nor of conformal field theory, central to the characterization of phase transitions in 2D, nor of Baxter's IRF transfer matrices and 8-Vertex models. The Replica Method for spin-glasses and the replica-breaking solutions of Parisi are also omitted.

The reason for all these omissions is obvious. These are all "advanced" topics beyond the scope of this book, in the sense that each requires a distinct approach and mind-set and grasp of mathematical principles. These are the domains of specialists. What gave this author the impetus to omit or skirt these important topics? By now, each of these topics has become so popular that one or more reviews, conference proceedings and textbooks have been written on it, sometimes even by the greatest experts: the discoverers or inventors, their close collaborators or their students. It is not for an introductory treatise to reinterpret what has been authoritatively stated elsewhere. Instead, we stick to a few of the author's favorite themes and exploit them from various points of view.

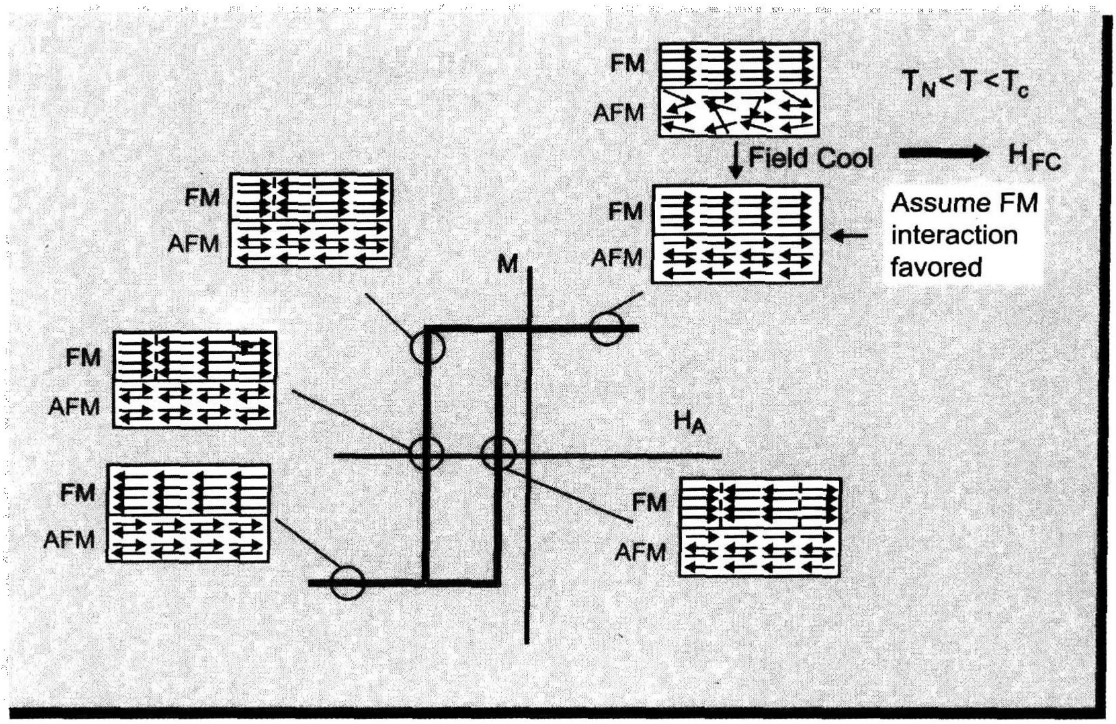
The goal is to prepare the reader, as best as possible, for further adventures into advanced research and for unforeseeable discoveries to come, all surely requiring a good background and a consistent methodology. Where this could not be done conveniently references to the best available tutorials are given, in keeping with another goal: to keep it *simple*, as in the title.

The author is grateful to Prof. E. H. Lieb at Princeton, Prof. J. Hirsch of UCSD and Dr. G. Ortiz of Los Alamos National Laboratory for discussions of material pertinent to their work and other helpful comments. I thank the many others who have altruistically allowed their results to be copied or quoted in this text. Thanks also to my dear wife for the historical research she performed in 1963 *incorporated in Chapter* 1; *forty-two years later this part of the text remains fresh as new*!

I also thank my editors — Dr. K. K. Phua and Kim Tan at World Scientific Publishing for their boundless patience with this procrastinating author.

Salt Lake City, *July 2005*

Exchange bias



This picture illustrates “Exchange Bias”: the effects of an antiferromagnetic substratum on the switching of a ferromagnet. As shown, the hysteresis loop becomes quite asymmetric. The mechanism for this broken symmetry is quite sturdy but poorly understood at the present time. [Figure courtesy Prof. Ivan K. Schuller, UCSD]

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Chapter 1

History of Magnetism

Western scientists and historians generally believe it was the Greeks who first reflected upon the wondrous properties of magnetite, the magnetic iron ore $FeO–Fe_2O_3$ and famed lodestone (leading stone, or compass). This mineral, which even in the natural state often has a powerful attraction for iron and steel, was mined in the province of Magnesia.

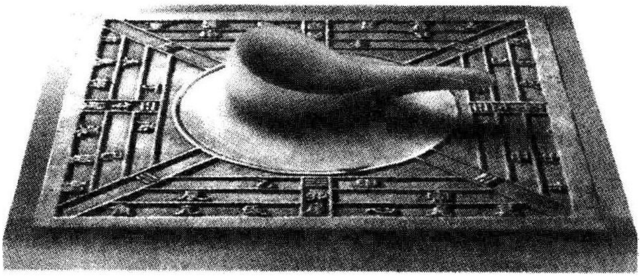
> The magnet's name the observing Grecians drew
> From the magnetick region where it grew.[1]

This origin is not incontrovertible. According to Pliny's account the magnet stone was named after its discoverer, the shepherd Magnes, "the nails of whose shoes and the tip of whose staff stuck fast in a magnetick field while he pastured his flocks."[1]

But more than likely, according to Chinese writings dating back to 4000 B.C. that mention magnetite, the original discoveries were made in China.[2] Meteoric iron was disovered and utilized in the period 3000–2500 B.C. A primitive compass illustrated below dates from that period.

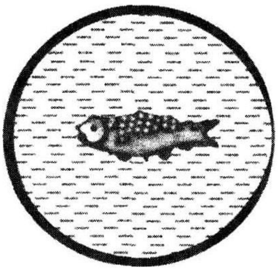
[1] Lucretius Carus, *De Rerum Natura*, 1st century B.C. References are to *vv.* 906 ff., in the translation by Th. Creech, London, 1714. Pliny, quoted in W. Gilbert, *De Magnete*, trans., Gilbert Club, London, 1900, rev. ed., Basic Books, New York, 1958, p. 8.

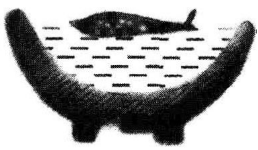
[2] The text and illustrations relating to China are adapted from Yu-qing Yang's paper "*Magnetic Materials in China*," presented in Poland at the 3rd International Conference on *Physics of Magnetic Materials*, (9–14 Sept. 1986), W. Gorzkowski, H. Lachowicz and H. Szymczak, eds., World Scientific, 1987.



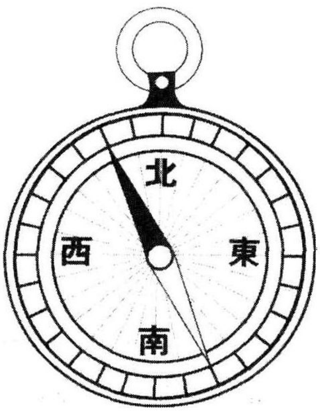
It consists of a spoon-shaped magnetite object with a smooth bottom, set on a polished copper surface. When pushed it rotated freely and usually came to rest with the handle pointing to the South.

The head of a floating "fish" made of magnetized iron, illustrated in a military manual of 1044 A.D., also pointed South.





Dated only a couple of decades later, the book "Notes by the Dream Brook" describes the making of a magnetic steel needle in the remarkably modern needle-and-pivot compass shown here.



1.1. Physics and Metaphysics

The lodestone also appeared in Greek writings by the year 800 B.C., and Greek thought and philosophy dominated all thinking on the subject for some 23 centuries following this. A characteristic of Greek philosophy was that it did not seek so much to explain and predict the wonders of nature as to force them to fit within a preconceived scheme of things. It might be argued that this seems to be precisely the objective of modern physics as well, but the analogy does not bear close scrutiny. To understand the distinction between modern and classical thought on this subject, suffice it to note the separate meanings of the modern word *science* and of its closest Greek equivalent, ἐπιστήμη. We conceive science as a specific activity pursued for its own sake, one which we endeavor to keep free from "alien" metaphysical beliefs. Whereas, ἐπιστήμη meant *knowledge* for the Greeks, with aims and methods undifferentiated from those of philosophy.

The exponents of one important school of philosophy, the *animists*, took cognizance of the extraordinary properties of the lodestone by ascribing to it a divine origin. Thales, then later Anaxagoras and others, believed the lodestone to possess a soul. We shall find this idea echoed into the seventeenth century A.D.

The school of the *mechanistic*, or atomistic, philosophers should not be misconstrued as being more scientific than were the animists, for their theories were similarly deductions from general metaphysical conceptions, with little relation to what we would now consider "the facts." Diogenes of Apollonia (about 460 B.C.), a contemporary of Anaxagoras, says there is humidity in iron which the dryness of the magnet feeds upon. The idea that magnets feed upon iron was also a long lived superstition. Still trying to check on it, John Baptista Porta, in the sixteenth century, reported as follows:

> I took a Loadstone of a certain weight, and I buried it in a heap of Iron-filings, that I knew what they weighed; and when I had left it there many months, I found my stone to be heavier, and the Iron-filings lighter: but the difference was so small, that in one pound I could finde no sensible declination; the stone being great, and the filings many: so that I am doubtful of the truth.[3]

[3] John Baptista Porta, *Natural Magick*, Naples, 1589, reprint of 1st English ed., Basic Books, New York, 1957, p. 212.