

Gauge Field Theories

2nd Edition

STEFAN POKORSKI

规范场理论

第2版

CAMBRIDGE MONOGRAPHS
ON MATHEMATICAL PHYSICS

CAMBRIDGE

世界图书出版公司

www.wpcbj.com.cn

Gauge Field Theories

Second Edition

STEFAN POKORSKI

Institute for Theoretical Physics, University of Warsaw



图书在版编目 (C I P) 数据

规范场理论=Gauge Field Theories: 英文 / (波)波考斯基 (Pokorski, S.) 著. —2版.—北京: 世界图书出版公司北京公司, 2008.5

ISBN 978-7-5062-9207-8

I. 规… II. 波… III. 规范场 —英文 IV. 0413.3

中国版本图书馆CIP数据核字 (2008) 第051045号

书 名: Gauge Field Theories 2nd ed.

作 者: Stefan Pokorski

中译名: 规范场理论 第2版

责任编辑: 高蓉 刘慧

出 版 者: 世界图书出版公司北京公司

印 刷 者: 三河国英印务有限公司

发 行: 世界图书出版公司北京公司 (北京朝内大街 137 号 100010)

联系电话: 010-64015659

电子信箱: kjsk@vip.sina.com

开 本: 16 开

印 张: 39.5

版 次: 2008 年 5 月第 1 次印刷

版权登记: 图字:01-2008-1909

书 号: 978-7-5062-9207-8 / O • 598

定 价: 99.00 元

世界图书出版公司北京公司已获得 Cambridge University Press 授权在中国大陆独家重印发行

Gauge Field Theories, Second Edition

Quantum field theory forms the present theoretical framework for our understanding of the fundamental interactions of particle physics. This up-dated and expanded text examines gauge theories and their symmetries with an emphasis on their physical and technical aspects.

Beginning with a new chapter giving a systematic introduction to classical field theories and a short discussion of their canonical quantization and the discrete symmetries C , P and T , the book provides a brief exposition of perturbation theory, the renormalization programme and the use of the renormalization group equation. It then explores topics of current research interest including chiral symmetry and its breaking, anomalies, and low energy effective lagrangians and some basics of supersymmetry. A chapter on the basics of the electroweak theory is now included.

PROFESSOR POKORSKI, a distinguished theoretical physicist, has presented here a self-contained text for graduate courses in physics, the only prerequisite is some grounding in quantum field theory.

Born in 1942, Professor Stefan Pokorski received his PhD in theoretical physics in 1967 from Warsaw University, where he is now holder of the Chair in Theoretical Particle Physics. He has been a member of the Polish Academy of Science since 1991 and was Director of the Institute for Theoretical Physics, University of Warsaw, from 1984 to 1994 and President of the Polish Physical Society from 1992 to 1994. A visiting scientist at CERN and the Max-Planck Institute for Physics, Munich, for periods of time totalling almost ten years, Professor Pokorski has also made regular visits to universities and laboratories around the world. Professor Pokorski has had many scientific articles published in leading international journals and in recent years has concentrated on physics beyond the standard model and supersymmetry.

Erratum for the Book:
S. Pokorski, Gauge Field Theories
 (2nd Edition)
 Cambridge University Press

Last update: February 15, 2008

Place:	Instead of:	Should be:
p. 109, lines below eq. (2.153):	It can be proved, for example, by induction, that ...	It can be proved that ...
line 1 below eq. (2.154):	The effective action ...	An easy way to prove eq. (2.154) is to consider the relation between the functionals $W[J]$ and $W'[J]$ for the fields Φ and $\bar{\Phi}$, respectively, and similarly $Z[J]$ and $Z'[J]$ [†] . The effective action ...
p. 110, 2 lines above eq. (2.156):	Also, we can prove, for example, by induction, that ...	Similarly to eq. (2.154) one can prove that ...
p. 185, line 1 below eq. (5.26):	... reads	... reads (see p. 266 for the discussion of the running $\alpha(q^2)$)
p. 214, line 2 below eq. (6.20):	the 1PI Green's function $\Gamma^{(n)}$ by ...	the 1PI Green's function $\Gamma^{(n)}$ (in a complete discussion one should include 1P reducible Green's functions as well; such an extension is straightforward) by ...
p. 411, line 2 from the bottom:	and process-dependent corrections consisting ...	and corrections, often called process-dependent, consisting ...
p. 412, starting at line 5 under Fig. 12.5:	the splitting into universal and process-dependent parts is gauge-dependent and to get physical ...	the splitting into universal and so-called process-dependent parts is gauge-dependent: the vertex and box corrections contain also contributions which are independent of the external legs and cancel the gauge dependence of the corrections to the gauge boson propagators. To get physical ...
line 13 under Fig. 12.5:	of at least part of the process-dependent corrections ...	of the external leg independent vertex corrections ...
line 1 in the footnote [†] :	is cancelled by the vertex corrections. ...	is cancelled by the external leg independent part of vertex corrections. ...

CAMBRIDGE MONOGRAPHS ON MATHEMATICAL PHYSICS

General editors: P. V. Landshoff, D. R. Nelson, D. W. Sciama, S. Weinberg

- J. Ambjørn, B. Durhuus and T. Jonsson *Quantum Geometry: A Statistical Field Theory Approach*
A. M. Anile *Relativistic Fluids and Magneto-Fluids*
J. A. de Azcárraga and J. M. Izquierdo *Lie Groups, Lie Algebras, Cohomology and Some Applications in Physics*†
J. Bernstein *Kinetic Theory in the Early Universe*
G. F. Bertsch and R. A. Broglia *Oscillations in Finite Quantum Systems*
N. D. Birrell and P. C. W. Davies *Quantum Fields in Curved Space*†
S. Carlip *Quantum Gravity in 2 + 1 Dimensions*
J. C. Collins *Renormalization*†
M. Creutz *Quarks, Gluons and Lattices*†
P. D. D'Eath *Supersymmetric Quantum Cosmology*
F. de Felice and C. J. S. Clarke *Relativity on Curved Manifolds*†
P. G. O. Freund *Introduction to Supersymmetry*†
J. Fuchs *Affine Lie Algebras and Quantum Groups*†
J. Fuchs and C. Schweigert *Symmetries, Lie Algebras and Representations: A Graduate Course for Physicists*
R. Gambini and J. Pullin *Loops, Knots, Gauge Theories and Quantum Gravity*†
M. Göckeler and T. Schücker *Differential Geometry, Gauge Theories and Gravity*†
C. Gómez, M. Ruiz Altaba and G. Sierra *Quantum Groups in Two-dimensional Physics*
M. B. Green, J. H. Schwarz and E. Witten *Superstring Theory, volume 1: Introduction*†
M. B. Green, J. H. Schwarz and E. Witten *Superstring Theory, volume 2: Loop Amplitudes, Anomalies and Phenomenology*†
S. W. Hawking and G. F. R. Ellis *The Large-Scale Structure of Space-Time*†
F. Iachello and A. Aruna *The Interacting Boson Model*
F. Iachello and P. van Isacker *The Interacting Boson-Fermion Model*
C. Itzykson and J.-M. Drouffe *Statistical Field Theory, volume 1: From Brownian Motion to Renormalization and Lattice Gauge Theory*†
C. Itzykson and J.-M. Drouffe *Statistical Field Theory, volume 2: Strong Coupling, Monte Carlo Methods, Conformal Field Theory, and Random Systems*†
J. I. Kapusta *Finite-Temperature Field Theory*†
V. E. Korepin, A. G. Izergin and N. M. Bogoliubov *The Quantum Inverse Scattering Method and Correlation Functions*†
M. Le Bellac *Thermal Field Theory*†
N. H. March *Liquid Metals: Concepts and Theory*
I. M. Montvay and G. Münster *Quantum Fields on a Lattice*†
A. Ozorio de Almeida *Hamiltonian Systems: Chaos and Quantization*†
R. Penrose and W. Rindler *Spinors and Space-time, volume 1: Two-Spinor Calculus and Relativistic Fields*†
R. Penrose and W. Rindler *Spinors and Space-time, volume 2: Spinor and Twistor Methods in Space-Time Geometry*†
S. Pokorski *Gauge Field Theories*, 2nd edition
J. Polchinski *String Theory, volume 1: An Introduction to the Bosonic String*
J. Polchinski *String Theory, volume 2: Superstring Theory and Beyond*
V. N. Popov *Functional Integrals and Collective Excitations*†
R. G. Roberts *The Structure of the Proton*†
J. M. Stewart *Advanced General Relativity*†
A. Vilenkin and E. P. S. Shellard *Cosmic Strings and Other Topological Defects*†
R. S. Ward and R. O. Wells Jr *Twistor Geometry and Field Theories*†

† Issued as a paperback

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK www.cup.cam.ac.uk
40 West 20th Street, New York, NY 10011-4211, USA www.cup.org
10 Stamford Road, Oakleigh, Melbourne 3166, Australia
Ruiz de Alarcón 13, 28014, Madrid, Spain

© Cambridge University Press 1987, 2000

This book is in copyright. Subject to statutory exception
and to the provisions of relevant collective licensing agreements,
no reproduction of any part may take place without
the written permission of Cambridge University Press.

First published 1987
First paperback edition 1989
Reprinted 1990
Second edition 2000

Gauge Field Theories, 2nd ed. (ISBN 978-0-521-47816-8) by Stefan
Pokorski first published by Cambridge University Press 2000.

All rights reserved.

This reprint edition for the People's Republic of China is published by
arrangement with the Press Syndicate of the University of Cambridge,
Cambridge, United Kingdom.

© Cambridge University Press & Beijing World Publishing Corporation
2008

This edition is for sale in the mainland of China only, excluding Hong Kong
SAR, Macao SAR and Taiwan, and may not be bought for export therefrom.

本书由世界图书出版公司北京公司和剑桥大学出版社合作出版。本书任何部分之文字
及图片，未经出版者书面许可，不得用任何方式抄袭、节录或翻印。

此版本仅限中华人民共和国境内销售，不包括香港、澳门特别行政区及中国台湾。不
得出口。

In memory of Osterns – my mother's family

Preface to the First Edition

This book has its origin in a long series of lectures given at the Institute for Theoretical Physics, Warsaw University. It is addressed to graduate students and to young research workers in theoretical physics who have some knowledge of quantum field theory in its canonical formulation, for instance at the level of two volumes by Bjorken & Drell (1964, 1965). The book is intended to be a relatively concise reference to some of the field theoretical tools used in contemporary research in the theory of fundamental interactions. It is a technical book and not easy reading. Physical problems are discussed only as illustrations of certain theoretical ideas and of computational methods. No attempt has been made to review systematically the present status of the theory of fundamental interactions.

I am grateful to Wojciech Królikowski, Maurice Jacob and Peter Landshoff for their interest in this work and strong encouragement. My warm thanks go to Antonio Bassetto, Wilfried Buchmüller, Wojciech Królikowski, Heinrich Leutwyler, Peter Minkowski, Olivier Piguet, Jacek Prentki, Marco Roncadelli, Henri Ruegg and Wojtek Zakrzewski for reading various chapters of this book and for many useful comments, and especially to Peter Landshoff for reading most of the preliminary manuscript.

I am also grateful to several of my younger colleagues at the Institute for Theoretical Physics in Warsaw for their stimulating interactions. My thanks go to Andrzej Czechowski for his collaboration at the early stage of this project and for numerous useful discussions. I am grateful to Wojciech Dębski, Marek Olechowski, Jacek Pawelczyk, Andrzej Turski, Robert Budzynski, Krzysztof Meissner and Michał Spalinski, and particularly to Paweł Krawczyk for checking a large part of the calculations contained in this book.

Finally my thanks go to Zofia Ziółkowska for her contribution to the preparation of the manuscript.

Stefan Pokorski
Warsaw, 1985

Preface to the Second Edition

This new edition offers a substantial extension of topics covered by the book. The main additions are Chapter 1 and Chapter 12 and extended Appendices. Chapter 1 makes the book more self-contained. It gives a systematic introduction to classical field theories and a brief discussion of their canonical quantization, as some intuition based on canonical quantization proves to be very useful even if the main emphasis is on the path integral approach. Also in Chapter 1 the reader can find a thorough discussion of discrete symmetries C , P and T .

Chapter 12 gives a concise but systematic and self-contained introduction to the electroweak theory. This is an important completion for a modern book on quantum field theory and fundamental interactions, which was missing in the first edition. Appendices A, C and D are new. In particular, Appendix C contains the complete set of Feynman rules for the Standard Model, including counterterms, which is not easily available in the literature. The new Appendix A is a substantial extension of the previous Appendix C. Several smaller changes and corrections have been made in a number of places in the text. An important addition is Section 7.7, in which the modern approach to effective field theories is presented.

I hope that the additions leave intact the main feature of the book: it is still not easy reading!

I am grateful to many people for their help in the completion of this second edition. Very special thanks go to Piotr Chankowski. His help and collaboration in writing Chapters 1 and 12 and the Appendices was absolutely invaluable. Thanks, Piotr. I also thank Mikołaj Misiak and Janusz Rosiek for their collaboration on Sections 7.7 and 15.6, respectively.

I am deeply indebted to Howie Haber for his careful reading of a large part of the new material. I am grateful to Zygmunt Ajduk, Ratindranath Akhoury, Riccardo Guida, Krzysztof Meissner, Marek Olechowski, Jacek Pawełczyk, Carlos Savoy and Kay Wiese for their numerous comments and corrections. I would also like to

thank Peter Landshoff for his constant encouragement and patience during quite a few years before this edition finally materialized.

Comments and corrections are welcome by e-mail at pokorski@fuw.edu.pl
They will be made accessible at the www page: <http://www.fuw.edu.pl/~pokorski/>

Stefan Pokorski
Warsaw, 1999

Contents

<i>Preface to the First Edition</i>	<i>page</i> xvii
<i>Preface to the Second Edition</i>	xviii
0 Introduction	1
0.1 Gauge invariance	1
0.2 Reasons for gauge theories of strong and electroweak interactions	3
QCD	3
Electroweak theory	5
1 Classical fields, symmetries and their breaking	11
1.1 The action, equations of motion, symmetries and conservation laws	12
Equations of motion	12
Global symmetries	13
Space-time transformations	16
Examples	18
1.2 Classical field equations	20
Scalar field theory and spontaneous breaking of global symmetries	20
Spinor fields	22
1.3 Gauge field theories	29
$U(1)$ gauge symmetry	29
Non-abelian gauge symmetry	31
1.4 From classical to quantum fields (canonical quantization)	35
Scalar fields	36
The Feynman propagator	39
Spinor fields	40
Symmetry transformations for quantum fields	45
1.5 Discrete symmetries	48
Space reflection	48
Time reversal	53
Charge conjugation	56
Summary and the CPT transformation	62
CP violation in the neutral K^0 - \bar{K}^0 -system	64
Problems	68

2	Path integral formulation of quantum field theory	71
2.1	Path integrals in quantum mechanics	71
	Transition matrix elements as path integrals	71
	Matrix elements of position operators	75
2.2	Vacuum-to-vacuum transitions and the imaginary time formalism	76
	General discussion	76
	Harmonic oscillator	78
	Euclidean Green's functions	82
2.3	Path integral formulation of quantum field theory	83
	Green's functions as path integrals	83
	Action quadratic in fields	87
	Gaussian integration	88
2.4	Introduction to perturbation theory	90
	Perturbation theory and the generating functional	90
	Wick's theorem	92
	An example: four-point Green's function in $\lambda\Phi^4$	93
	Momentum space	97
2.5	Path integrals for fermions; Grassmann algebra	100
	Anticommuting c -numbers	100
	Dirac propagator	102
2.6	Generating functionals for Green's functions and proper vertices; effective potential	105
	Classification of Green's functions and generating functionals	105
	Effective action	107
	Spontaneous symmetry breaking and effective action	109
	Effective potential	111
2.7	Green's functions and the scattering operator	113
	<i>Problems</i>	120
3	Feynman rules for Yang–Mills theories	124
3.1	The Faddeev–Popov determinant	124
	Gauge invariance and the path integral	124
	Faddeev–Popov determinant	126
	Examples	129
	Non-covariant gauges	132
3.2	Feynman rules for QCD	133
	Calculation of the Faddeev–Popov determinant	133
	Feynman rules	135
3.3	Unitarity, ghosts, Becchi–Rouet–Stora transformation	140
	Unitarity and ghosts	140
	BRS and anti-BRS symmetry	143
	<i>Problems</i>	147
4	Introduction to the theory of renormalization	148
4.1	Physical sense of renormalization and its arbitrariness	148
	Bare and 'physical' quantities	148
	Counterterms and the renormalization conditions	152

	Arbitrariness of renormalization	153
	Final remarks	156
4.2	Classification of the divergent diagrams	157
	Structure of the UV divergences by momentum power counting	157
	Classification of divergent diagrams	159
	Necessary counterterms	161
4.3	$\lambda\Phi^4$: low order renormalization	164
	Feynman rules including counterterms	164
	Calculation of Fig. 4.8(b)	166
	Comments on analytic continuation to $n \neq 4$ dimensions	168
	Lowest order renormalization	170
4.4	Effective field theories	173
	<i>Problems</i>	175
5	Quantum electrodynamics	177
5.1	Ward–Takahashi identities	179
	General derivation by the functional technique	179
	Examples	181
5.2	Lowest order QED radiative corrections by the dimensional regularization technique	184
	General introduction	184
	Vacuum polarization	185
	Electron self-energy correction	187
	Electron self-energy: IR singularities regularized by photon mass	190
	On-shell vertex correction	191
5.3	Massless QED	194
5.4	Dispersion calculation of $O(\alpha)$ virtual corrections in massless QED, in $(4 \mp \varepsilon)$ dimensions	196
	Self-energy calculation	197
	Vertex calculation	198
5.5	Coulomb scattering and the IR problem	200
	Corrections of order α	200
	IR problem to all orders in α	205
	<i>Problems</i>	208
6	Renormalization group	209
6.1	Renormalization group equation (RGE)	209
	Derivation of the RGE	209
	Solving the RGE	212
	Green's functions for rescaled momenta	214
	RGE in QED	215
6.2	Calculation of the renormalization group functions β , γ , γ_m	216
6.3	Fixed points; effective coupling constant	219
	Fixed points	219
	Effective coupling constant	222
6.4	Renormalization scheme and gauge dependence of the RGE parameters	224

	Renormalization scheme dependence	224
	Effective α in QED	226
	Gauge dependence of the β -function	227
	<i>Problems</i>	228
7	Scale invariance and operator product expansion	230
7.1	Scale invariance	230
	Scale transformations	230
	Dilatation current	233
	Conformal transformations	235
7.2	Broken scale invariance	237
	General discussion	237
	Anomalous breaking of scale invariance	238
7.3	Dimensional transmutation	242
7.4	Operator product expansion (OPE)	243
	Short distance expansion	243
	Light-cone expansion	247
7.5	The relevance of the light-cone	249
	Electron–positron annihilation	249
	Deep inelastic hadron leptonproduction	250
	Wilson coefficients and moments of the structure function	254
7.6	Renormalization group and OPE	256
	Renormalization of local composite operators	256
	RGE for Wilson coefficients	259
	OPE beyond perturbation theory	261
7.7	OPE and effective field theories	262
	<i>Problems</i>	269
8	Quantum chromodynamics	272
8.1	General introduction	272
	Renormalization and BRS invariance; counterterms	272
	Asymptotic freedom of QCD	274
	The Slavnov–Taylor identities	277
8.2	The background field method	279
8.3	The structure of the vacuum in non-abelian gauge theories	282
	Homotopy classes and topological vacua	282
	Physical vacuum	284
	Θ -vacuum and the functional integral formalism	287
8.4	Perturbative QCD and hard collisions	290
	Parton picture	290
	Factorization theorem	291
8.5	Deep inelastic electron–nucleon scattering in first order QCD (Feynman gauge)	293
	Structure functions and Born approximation	293
	Deep inelastic quark structure functions in the first order in the strong coupling constant	298
	Final result for the quark structure functions	302

	Hadron structure functions; probabilistic interpretation	304
8.6	Light-cone variables, light-like gauge	306
8.7	Beyond the one-loop approximation	312
	Comments on the IR problem in QCD	314
	<i>Problems</i>	315
9	Chiral symmetry; spontaneous symmetry breaking	317
9.1	Chiral symmetry of the QCD lagrangian	317
9.2	Hypothesis of spontaneous chiral symmetry breaking in strong interactions	320
9.3	Phenomenological chirally symmetric model of the strong interactions (σ -model)	324
9.4	Goldstone bosons as eigenvectors of the mass matrix and poles of Green's functions in theories with elementary scalars	327
	Goldstone bosons as eigenvectors of the mass matrix	327
	General proof of Goldstone's theorem	330
9.5	Patterns of spontaneous symmetry breaking	333
9.6	Goldstone bosons in QCD	337
10	Spontaneous and explicit global symmetry breaking	342
10.1	Internal symmetries and Ward identities	342
	Preliminaries	342
	Ward identities from the path integral	344
	Comparison with the operator language	347
	Ward identities and short-distance singularities of the operator products	348
	Renormalization of currents	351
10.2	Quark masses and chiral perturbation theory	353
	Simple approach	353
	Approach based on use of the Ward identity	354
10.3	Dashen's theorems	356
	Formulation of Dashen's theorems	356
	Dashen's conditions and global symmetry broken by weak gauge interactions	358
10.4	Electromagnetic $\pi^+-\pi^0$ mass difference and spectral function sum rules	362
	Electromagnetic $\pi^+-\pi^0$ mass difference from Dashen's formula	362
	Spectral function sum rules	363
	Results	366
11	Brout-Englert-Higgs mechanism in gauge theories	369
11.1	Brout-Englert-Higgs mechanism	369
11.2	Spontaneous gauge symmetry breaking by radiative corrections	373
11.3	Dynamical breaking of gauge symmetries and vacuum alignment	379
	Dynamical breaking of gauge symmetry	379
	Examples	382
	<i>Problems</i>	388
12	Standard electroweak theory	389
12.1	The lagrangian	391

12.2	Electroweak currents and physical gauge boson fields	394
12.3	Fermion masses and mixing	398
12.4	Phenomenology of the tree level lagrangian	402
	Effective four-fermion interactions	403
	Z^0 couplings	406
12.5	Beyond tree level	407
	Renormalization and counterterms	407
	Corrections to gauge boson propagators	411
	Fermion self-energies	418
	Running $\alpha(\mu)$ in the electroweak theory	419
	Muon decay in the one-loop approximation	422
	Corrections to the Z^0 partial decay widths	430
12.6	Effective low energy theory for electroweak processes	435
	QED as the effective low energy theory	438
12.7	Flavour changing neutral-current processes	441
	QCD corrections to CP violation in the neutral kaon system	445
	<i>Problems</i>	456
13	Chiral anomalies	457
13.1	Triangle diagram and different renormalization conditions	457
	Introduction	457
	Calculation of the triangle amplitude	459
	Different renormalization constraints for the triangle amplitude	464
	Important comments	465
13.2	Some physical consequences of the chiral anomalies	469
	Chiral invariance in spinor electrodynamics	469
	$\pi^0 \rightarrow 2\gamma$	471
	Chiral anomaly for the axial $U(1)$ current in QCD; $U_A(1)$ problem	473
	Anomaly cancellation in the $SU(2) \times U(1)$ electroweak theory	475
	Anomaly-free models	478
13.3	Anomalies and the path integral	478
	Introduction	478
	Abelian anomaly	480
	Non-abelian anomaly and gauge invariance	481
	Consistent and covariant anomaly	484
13.4	Anomalies from the path integral in Euclidean space	486
	Introduction	486
	Abelian anomaly with Dirac fermions	488
	Non-abelian anomaly and chiral fermions	491
	<i>Problems</i>	492
14	Effective lagrangians	495
14.1	Non-linear realization of the symmetry group	495
	Non-linear σ -model	495
	Effective lagrangian in the $\xi_a(x)$ basis	500
	Matrix representation for Goldstone boson fields	502
14.2	Effective lagrangians and anomalies	504