国外数学名著系列（续一）

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Vladimir I. Arnold　Valery V. Kozlov　Anatoly I. Neishtadt

Mathematical Aspects of Classical and Celestial Mechanics

Third Edition

经典力学与天体力学中的数学问题

（第三版）

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《国外数学名著系列》(影印版)序

要使我国的数学事业更好地发展起来，需要数学家淡泊名利并付出更艰苦地努力。另一方面，我们也要从客观上为数学家创造更有利的发展数学事业的外部环境，这主要是加强对数学事业的支持与投资力度，使数学家有较好的工作与生活条件，其中也包括改善与加强数学的出版工作。

从出版方面来讲，除了较好较快地出版我们自己的成果外，引进国外的先进出版物无疑也是十分重要与必不可少的。从数学来说，施普林格(Springer)出版社至今仍然是世界上最具权威的出版社。科学出版社影印一批他们出版的好的新书，使我国广大数学家能以较低的价格购买，特别是在边远地区工作的数学家能普遍见到这些书，无疑是对推动我国数学的科研与教学十分有益的事。

这次科学出版社购买了版权，一次影印了 23 本施普林格出版社出版的数学书，就是一件好事，也是值得继续做下去的事情。大体上分一下，这 23 本书中，包括基础数学书 5 本，应用数学书 6 本与计算数学书 12 本，其中有些书也具有交叉性质。 这些书都是很新的，2000 年以后出版的占绝大部分，共计 16 本，其余的也是 1990 年以后出版的。这些书可以使读者较快地了解数学某方面的前沿，例如基础数学中的数论、代数与拓扑三本，都是由该领域大数学家编著的“数学百科全书”的分册。对从事这方面研究的数学家了解该领域的前沿与全貌很有帮助。按照学科的特点，基础数学类的书以“经典”为主，应用和计算数学类的书以“前沿”为主。这些书的作者多数是国际知名的大数学家，例如《拓扑学》一书的作者诺维科夫是俄罗斯科学院的院士，曾获“菲尔兹奖”和“沃尔夫数学奖”。这些大数学家的著作无疑将会对我国的科研人员起到非常好的指导作用。

当然，23 本书只能涵盖数学的一部分，所以，这项工作还应该继续做下去。更进一步，有些读者面较广的好书还应该翻译成中文出版，使之有更大的读者群。

总之，我对科学出版社影印施普林格出版社的部分数学著作这一举措表示热烈的支持，并盼望这一工作取得更大的成绩。

王　元

2005 年 12 月 3 日

Preface

In this book we describe the basic principles, problems, and methods of classical mechanics. Our main attention is devoted to the mathematical side of the subject. Although the physical background of the models considered here and the applied aspects of the phenomena studied in this book are explored to a considerably lesser extent, we have tried to set forth first and foremost the "working" apparatus of classical mechanics. This apparatus is contained mainly in Chapters 1, 3, 5, 6, and 8.

Chapter 1 is devoted to the basic mathematical models of classical mechanics that are usually used for describing the motion of real mechanical systems. Special attention is given to the study of motion with constraints and to the problems of realization of constraints in dynamics.

In Chapter 3 we discuss symmetry groups of mechanical systems and the corresponding conservation laws. We also expound various aspects of order-reduction theory for systems with symmetries, which is often used in applications.

Chapter 4 is devoted to variational principles and methods of classical mechanics. They allow one, in particular, to obtain non-trivial results on the existence of periodic trajectories. Special attention is given to the case where the region of possible motion has a non-empty boundary. Applications of the variational methods to the theory of stability of motion are indicated.

Chapter 5 contains a brief survey of the various approaches to the problem of integrability of the equations of motion and some of the most general and efficient methods of their integration. Diverse examples of integrated problems are given, which form the "golden reserve" of classical dynamics. The material of this chapter is used in Chapter 6, which is devoted to one of the most fruitful parts of mechanics – perturbation theory. The main task of perturbation theory is studying the problems of mechanics that are close to problems admitting exact integration. Elements of this theory (in particular, the well-known and widely used "averaging principle") arose in celestial mechanics in connection with attempts to take into account mutual gravitational perturbations of the planets of the Solar System. Adjoining Chapters 5 and 6 is Chapter 7, where the theoretical possibility of integrating the equations of motion (in a precisely defined sense) is studied. It turns out that integrable systems are a rare exception and this circumstance increases the importance of approximate integration methods expounded in Chapter 6. Chapter 2 is devoted to classical problems of celestial mechanics. It contains a description of the integrable two-body problem, the classification of final motions in the three-body problem, an analysis of collisions and regularization questions in the general problem of n gravitating points, and various limiting variants of this problem. The problems of celestial mechanics are discussed in Chapter 6 from the viewpoint of perturbation theory. Elements of the theory of oscillations of mechanical systems are presented in Chapter 8.

The last Chapter 9 is devoted to the tensor invariants of the equations of dynamics. These are tensor fields in the phase space that are invariant under the phase flow. They play an essential role both in the theory of exact integration of the equations of motion and in their qualitative analysis.

The book is significantly expanded by comparison with its previous editions (VINITI, 1985; Springer-Verlag, 1988, 1993, 1997). We have added Ch. 4 on variational principles and methods (§ 4.4.5 in it was written by S. V. Bolotin), Ch. 9 on the tensor invariants of equations of dynamics, § 2.7 of Ch. 2 on dynamics in spaces of constant curvature, §§ 6.1.10 and 6.4.7 of Ch. 6 on separatrix crossings, § 6.3.5 of Ch. 6 on diffusion without exponentially small effects (written by D. V. Treshchev), § 6.3.7 of Ch. 6 on KAM theory for lower-dimensional tori (written by M. B. Sevryuk), § 6.4.3 of Ch. 6 on adiabatic phases, § 7.6.3 of Ch. 7 on topological obstructions to integrability in the multidimensional case, § 7.6.4 of Ch. 7 on the ergodic properties of dynamical systems with multivalued Hamiltonians, and § 8.5.3 of Ch. 8 on the effect of gyroscopic forces on stability. We have substantially expanded § 6.1.7 of Ch. 6 on the effect of an isolated resonance, § 6.3.2 of Ch. 6 on invariant tori of the perturbed Hamiltonian system (with the participation of M. B. Sevryuk), § 6.3.4 of Ch. 6 on diffusion of slow variables (with the participation of S. V. Bolotin and D. V. Treshchev), § 7.2.1 on splitting of asymptotic surfaces conditions (with the participation of D. V. Treshchev). There are several other addenda. In this work we were greatly helped by S. V. Bolotin, M. B. Sevryuk, and D. V. Treshchev, to whom the authors are deeply grateful.

This English edition was prepared on the basis of the second Russian edition (Editorial URSS, 2002). The authors are deeply grateful to the translator E. I. Khukhro for fruitful collaboration.

Our text, of course, does not claim to be complete. Nor is it a textbook on theoretical mechanics: there are practically no detailed proofs in it. The main purpose of our work is to acquaint the reader with classical mechanics on the whole, both in its classical and most modern aspects. The reader can find the necessary proofs and more detailed information in the books and original research papers on this subject indicated at the end of this volume.

V. I. Arnold, V. V. Kozlov, A. I. Neishtadt

Contents

1

Basic Principles of Classical Mechanics

For describing the motion of a mechanical system various mathematical models are used based on different "principles" – laws of motion. In this chapter we list the basic objects and principles of classical dynamics. The simplest and most important model of the motion of real bodies is Newtonian mechanics, which describes the motion of a free system of interacting points in three-dimensional Euclidean space. In § 1.6 we discuss the suitability of applying Newtonian mechanics when dealing with complicated models of motion.

1.1 Newtonian Mechanics

1.1.1 Space, Time, Motion

The space where the motion takes place is three-dimensional and Euclidean with a fixed orientation. We shall denote it by E^3. We fix some point $o \in E^3$ called the "origin of reference". Then the position of every point s in E^3 is uniquely determined by its position vector $\vec{os} = \mathbf{r}$ (whose initial point is o and end point is s). The set of all position vectors forms the three-dimensional vector space $\mathbb{R}^3$, which is equipped with the scalar product $\langle\ ,\ \rangle$.

Time is one-dimensional; it is denoted by t throughout. The set $\mathbb{R} = \{t\}$ is called the *time axis*.

A *motion* (or *path*) of the point s is a smooth map $\Delta \to E^3$, where Δ is an interval of the time axis. We say that the motion is defined on the interval Δ. If the origin (point o) is fixed, then every motion is uniquely determined by a smooth vector-function $\mathbf{r}\colon\ \Delta \to \mathbb{R}^3$.

The image of the interval Δ under the map $t \mapsto \mathbf{r}(t)$ is called the *trajectory* or *orbit of the point s*.

The *velocity* $\mathbf{v}$ of the point s at an instant $t \in \Delta$ is by definition the derivative $d\mathbf{r}/dt = \dot{\mathbf{r}}(t) \in \mathbb{R}^3$. Clearly the velocity is independent of the choice of the origin.

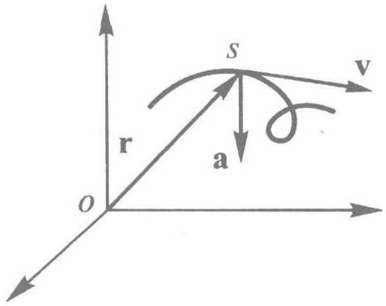

Fig. 1.1.

The *acceleration* of the point is by definition the vector $\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} \in \mathbb{R}^3$. The velocity and acceleration are usually depicted as vectors with initial point at the point s (see Fig. 1.1).

The set E^3 is also called the *configuration space* of the point s. The pair $(s, \mathbf{v})$ is called the state of the point, and the set $E^3 \times \mathbb{R}^3\{\mathbf{v}\}$, the *phase* (or *state*) *space*.

Now consider a more general case when there are n points $s_1, \dots, s_n$ moving in the space E^3. The set $E^{3n} = E^3\{s_1\} \times \cdots \times E^3\{s_n\}$ is called the configuration space of this "free" system. If it is necessary to exclude collisions of the points, then E^{3n} must be diminished by removing from it the union of diagonals

$$\Delta = \bigcup_{i<j} \{s_i = s_j\}.$$

Let $(\mathbf{r}_1, \dots, \mathbf{r}_n) = \mathbf{r} \in \mathbb{R}^{3n}$ be the position vectors of the points $s_1, \dots, s_n$. A motion of the free system is given by smooth vector-functions $\mathbf{r}(t) = (\mathbf{r}_1(t), \dots, \mathbf{r}_n(t))$. We define in similar fashion the velocity

$$\mathbf{v} = \dot{\mathbf{r}} = (\dot{\mathbf{r}}_1, \dots, \dot{\mathbf{r}}_n) = (\mathbf{v}_1, \dots, \mathbf{v}_n) \in \mathbb{R}^{3n}$$

and the acceleration

$$\mathbf{a} = \ddot{\mathbf{r}} = (\ddot{\mathbf{r}}_1, \dots, \ddot{\mathbf{r}}_n) = (\mathbf{a}_1, \dots, \mathbf{a}_n) \in \mathbb{R}^{3n}.$$

The set $E^{3n} \times \mathbb{R}^{3n}\{\mathbf{v}\}$ is called the phase (or state) space, and the pair $(s, \mathbf{v})$, the state of the system.

1.1.2 Newton–Laplace Principle of Determinacy

This principle (which is an experimental fact) asserts that the state of the system at any fixed moment of time uniquely determines all of its motion (both in the future and in the past).

Suppose that we know the state of the system $(\mathbf{r}_0, \mathbf{v}_0)$ at an instant t_0. Then, according to the principle of determinacy, we know the motion $\mathbf{r}(t)$,

$t \in \Delta \subset \mathbb{R}$; $\mathbf{r}(t_0) = \mathbf{r}_0$, $\dot{\mathbf{r}}(t_0) = \dot{\mathbf{r}}_0 = \mathbf{v}_0$. In particular, we can calculate the acceleration $\ddot{\mathbf{r}}$ at the instant $t = t_0$.[1] Then $\ddot{\mathbf{r}}(t_0) = \mathbf{f}(t_0, \mathbf{r}_0, \dot{\mathbf{r}}_0)$, where $\mathbf{f}$ is some function whose existence follows from the Newton–Laplace principle. Since the time t_0 can be chosen arbitrarily, we have the equation

$$\ddot{\mathbf{r}} = \mathbf{f}(t, \mathbf{r}, \dot{\mathbf{r}})$$

for all t.

This differential equation is called the *equation of motion* or *Newton's equation*. The existence of Newton's equation (with a smooth vector-function $\mathbf{f}\colon \mathbb{R}\{t\} \times \mathbb{R}^{3n}\{\mathbf{r}\} \times \mathbb{R}^{3n}\{\dot{\mathbf{r}}\} \to \mathbb{R}^{3n}$) is equivalent to the principle of determinacy. This follows from the existence and uniqueness theorem in the theory of differential equations. The function $\mathbf{f}$ in Newton's equations is usually determined in experiments. The definition of a mechanical system includes specifying this function.

We now consider examples of Newton's equations.

a) The equation of a point in free fall in vacuum near the surface of the Earth (obtained experimentally by Galileo) has the form $\ddot{\mathbf{r}} = -g\mathbf{e}_z$, where $g \approx 9.8\ \mathrm{m/s^2}$ (the acceleration of gravity) and $\mathbf{e}_z$ is the vertical unit vector. The trajectory of a falling point is a parabola.

b) Hooke showed that the equation of small oscillations of a body attached to the end of an elastic spring has the form $\ddot{x} = -\alpha x$, $\alpha > 0$. The constant coefficient α depends on the choice of the body and spring. This mechanical system is called a *harmonic oscillator* (see Fig. 1.2).

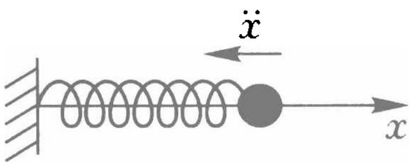

Fig. 1.2. Harmonic oscillator

It turned out that in experiments, rather than finding the acceleration $\mathbf{f}$ on the right-hand side of Newton's equations, it is more convenient to determine the product $m\mathbf{f} = \mathbf{F}$, where m is some positive number called the mass of the point (an instructive discussion of the physical meaning of the notion of mass can be found in [601, 401, 310]). For example, in Hooke's experiments the constant $m\alpha = c$ depends on the properties of the elastic spring, but not on the choice of the body. This constant is called the coefficient of elasticity.

The pair (s, m) (or $(\mathbf{r}, m)$, where $\mathbf{r}$ is the position vector of the point s) is called a *material point* of mass m. In what follows we shall often denote a point s and its mass m by one and the same symbol m. If a system of material

[1] We assume that all the functions occurring in dynamics are smooth.