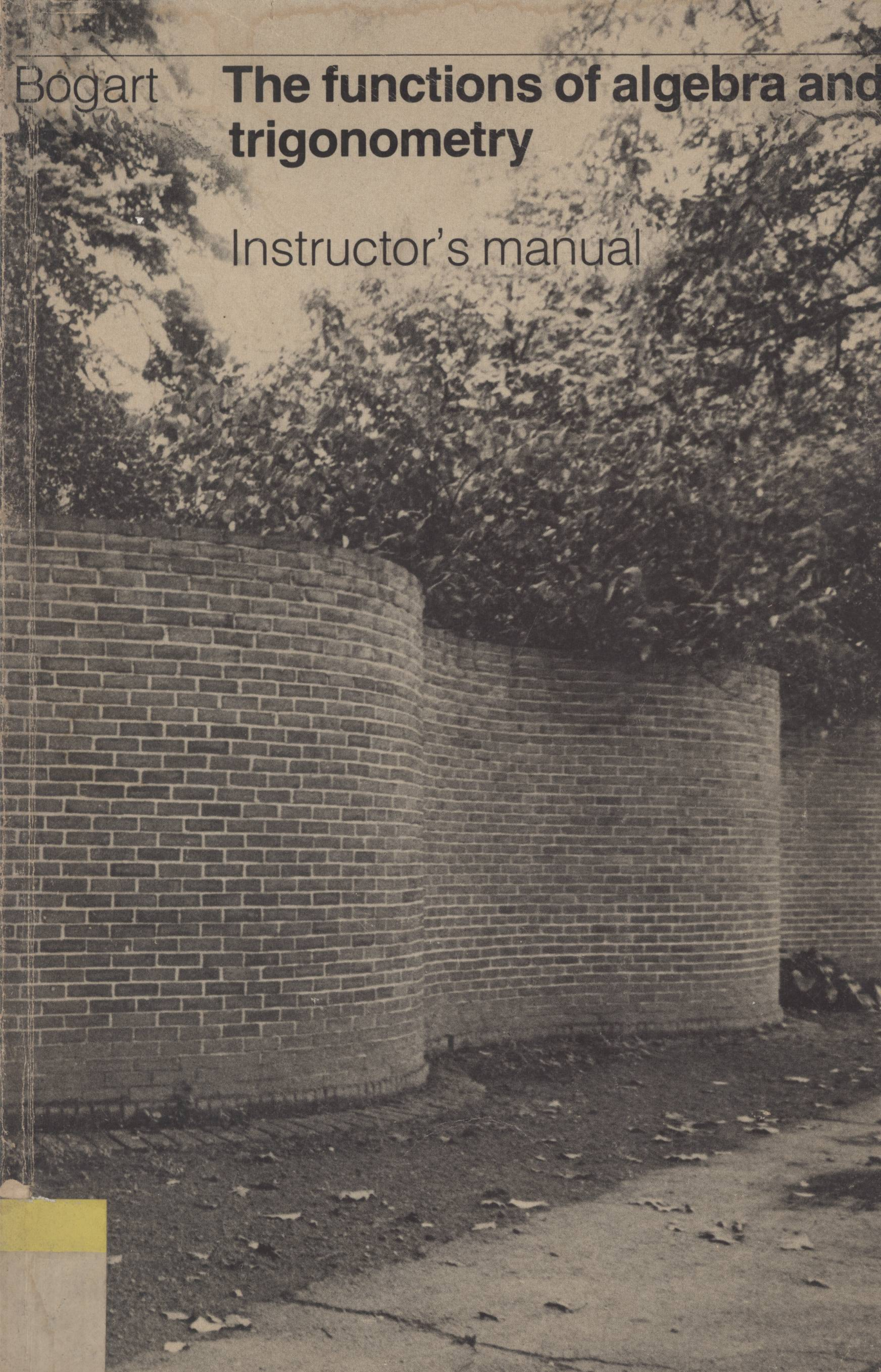
Bogart

The functions of algebra and trigonometry

Instructor's manual



Instructor's Manual

THE FUNCTIONS OF ALGEBRA AND TRIGONOMETRY

Kenneth P. Bogart
Dartmouth College

HOUGHTON MIFFLIN COMPANY BOSTON

Atlanta Dallas Geneva, Illinois
Hopewell, New Jersey Palo Alto London

Printed in the U.S.A.

ISBN: 0-395-20469-0

Dependencies of sections in the text. Earlier sections in a group are prerequisite to the later ones, as are prerequisites to the direct prerequisites listed.

Sections	Directly Prerequisite Sections	Optional Background Sections
1.1, 1.2, 1.3	0.1, 0.2	
1.4	1.1, 1.2, 1.3	
2.1, 2.3	1.1, 1.2, 1.3	
2.2, 2.4	1.1, 1.2, 1.3	2.1
3.1, 3.2, 3.3	1.1, 1.2, 1.3, 1.4	
3.4	3.1, 3.2, 3.3	2.2
3.5	3.1, 3.2, 3.3, 3.4	
3.6	3.1, 3.2, 3.3, 3.4	2.2, 3.5
4.1, 4.2	3.1, 3.2, 3.3, 3.4	
4.3	3.5, 4.1, 4.2	
4.4	4.3	8.4
4.5, 4.6	4.1, 4.2, 4.3	2.2
5.1, 5.2	4.1, 4.2	
5.3	3.3	3.4
6.1, 6.2	1.1, 1.2, 1.3, 1.4	8.4
6.3, 6.4, 6.5	6.1, 6.2	2.2, 4.1, 4.2, 5.2
7.1, 7.2, 7.3, 7.4, 7.5	1.1, 1.2, 1.3, 1.4	3.1, 3.2, 3.3, 3.4, 4.1, 4.2
7.6	7.1, 7.2, 7.3 (6.1, 6.2, 6.3 for optional section)	
7.7	7.1, 7.2, 7.3, 7.4	2.2, 3.4, 7.5

Sections	Directly Prerequisite Sections	Optional Background Sections
7.8	3.5, 7.1, 7.2, 7.3, 7.4 (7.7 Arctan only)	8.4
8.1, 8.2, 8.3	1.1, 1.2, 1.3, 1.4	
8.4	8.1	

CONTENTS

CHAPTER 1 INTRODUCTION

PURPOSES AND PREREQUISITES

This book is designed to serve as a launching pad for the study of college mathematics and college subjects outside mathematics that use mathematical concepts. In particular, it is designed to provide solid groundwork in the concepts and computations of algebra and trigonometry, to develop the student's understanding of functions and their graphs, to help the student "see through" word problems to their mathematical statements, and to furnish the student with mathematical reading ability. It assumes that the student has solved simple equations before and has studied enough geometry to know about such things as similar triangles, the Pythagorean theorem for right triangles, and areas and volumes of the simplest geometric figures and solids.

THE WORD PROBLEMS

You'll find that I've tried to include at least one word problem in every homework set when a word problem makes sense. I include at least one word problem in each homework assignment I give on the theory that translating a problem from English to mathematics and then solving it is the most realistic and meaningful experience that a student can have in a mathematics course.

THE PACE OF THE COURSE

I normally cover one section of the book in each lecture, giving a detailed discussion, with examples, of one-half to three-fourths of the ideas in the section, and assign reading the section as part of the homework for the next day. I assign at least half (and often most) of the problems marked with daggers (the answers to which are in the back of the textbook) and some which are not marked. The set of all daggered exercises would make a solid homework assignment. I give quizzes weekly as well as a midterm and a final. By giving quizzes outside regularly scheduled class hours (a Dartmouth tradition) I can cover about 35 sections of the book, meeting 4 times a week in a 9- to 10-week quarter. The result is a very substantial and fast-paced one-quarter course. At the opposite extreme from the course I give is the two-semester algebra and trig course for students without much background and perhaps without much aptitude for math. In such a course, I plan to spend two days on each section and, with time out for exams and quizzes, to cover Chapters 0, 1, 2, most of 3, and most of 4 in the first semester. In the second semester, I plan to cover Chapters 5, 6, most of 7, and 8.

In the preface to the text, I outline what I feel is a core course. Page vi of this manual shows the interdependencies of the sections. I tried to design the book so that the core would fit into a much slower paced one-quarter course than my own. A course meeting 3 hours per week covering the core course plus perhaps a selection of the recommended sections should provide students with a completely adequate background for virtually any calculus course.

HOW CAN WE GET STUDENTS TO READ THE BOOK?

Once I decided that I want to try to develop the students' reading skills, I had to deal with the problem of getting students to read the book. I found, in frustration, that students would do assigned problems without reading the text, unless the lecture did not have an example that showed them how to work a given problem. Then they used the book as a reference. My solution is this. I try to assign one problem that relates directly to a point of the reading that I purposely skip in the lecture. I also advertise the fact that I plan to use examples from the book as test problems. I have even made up fill-in-the-blank questions that consist of two or three sentences from the book with the conclusion of the sentences being the blank. (None of these are in the quizzes in this manual.) The essential thing is to get the students into the reading habit in Chapter 1 when the reading is easy, so that the transition to more mathematical thinking and more concise prose in Chapters 3 through 7 will not hit them squarely in the face.

NOTES ON USING THIS MANUAL

Chapter 2 of this manual contains a description of a lecture on each section of the book. These descriptions are largely taken from lecture notes I used when teaching from earlier versions of the book. The descriptions should give you a quick overview of the major content of each section and some ideas for the presentation of topics that are sometimes difficult to put across.

Chapter 3 of this manual contains all numerical answers to exercises in the book plus hints or solutions to all show, or prove, exercises.

Chapter 4 can be used in one of several ways: as a source of sample test questions, or as a source of actual tests. It can also be used as a basis for a self-paced precalculus course. It breaks the book into ten approximately equal units with a study guide and three sample quizzes for each unit. The units and quizzes closely follow those I used when I taught a self-paced course with the first version of the book. They are also similar to the units that the book was divided into for the modified self-paced course that I taught from the book most recently. This modified system is described in Chapter 4 of this manual. The sample final exam in Chapter 4 is based on the one I used most recently. Answers to all the quizzes and exam questions are included.

CHAPTER 2 NOTES ON INDIVIDUAL SECTIONS

SECTION 1.1 AN INTRODUCTION TO FUNCTIONS

Concepts: Function. Domain. Image. The coordinate plane. Graphs.

A good example of a function that coordinates with the book is the function given by $G(x) = \frac{1}{4}x$, which converts liquid measure in quarts to liquid measure in gallons. This is a good example for graphing. Since the natural domain of the function consists of the nonnegative reals, it allows you to start out by plotting a few integer points and then filling in more and more points until it is clear that the graph is a straight-line segment. I try to begin the lecture with an example of a function that has a nonnumerical domain but a numerical image--like the one that assigns weights to people--so that I can explain that one reason for being concerned with the concept of domain is so that we can explain why this function seems different from the liquid-measure conversion function. I usually devote about a third of the lecture to definitions and examples of functions and then two-thirds to graphs. I try to draw a graph of the square-root function and a function like the ones given in Exercise 7.

SECTION 12 LINEAR FUNCTIONS

Concepts: Linear functions. Slope. Straight lines. Zeros of functions. Equality of linear functions and intersections of straight lines. Increasing and decreasing functions.

One example that covers a lot of the concepts in this section is the function that gives distance from a certain point at a certain time. For example, I tell students to imagine that we are driving from Hanover, New Hampshire, to Boston, a distance of 135 miles at a constant rate of 50 miles per hour. I work out a rule for the function that gives our distance from Boston at time x, draw its graph, observe that it is a straight line with slope -50, and talk about linear functions for a while. Then I return to the main example and ask when we get to Boston, using this question to lead to a discussion of zeros of functions. We can continue the example by having a plane leave an hour later flying at 100 miles per hour and ask when the plane and car are the same distance from Boston. We can set two linear functions equal to each other to find this, and interpret the result to show that the plane catches up with the car 35 miles from Boston, after the car has been driving for two hours. Sometimes I also note that the distance to Boston is decreasing and discuss increasing and decreasing functions; sometimes I let the students read about this on their own.

SECTION 1.3 A POINT ON THE GRAPH AND THE SLOPE OR TWO POINTS ON THE GRAPH DETERMINE A LINEAR FUNCTION

Concepts: The rule for a linear function whose graph goes through (h,k) with slope m. The rule for a linear function whose graph goes through (a,c) and (h,k). Linear interpolation and extrapolation. Families of linear functions with a given slope or going through a given point.

A good example to begin this section is another one that has to do with driving. Assume that we've been driving on Interstate 75 toward Columbus, Ohio, for several hours at the constant rate of 50 miles per hour when we see a sign that says Columbus 100 miles. Our distance from Columbus is thus a linear function with slope -50 whose graph goes through the point (3,100). We find the equation of the function by writing $f(x) = -50x + b$ and substituting $x = 3$, $f(x) = 100$ to find b. Natural questions to ask are, How far from Columbus were we when we started? and When will we get to Columbus? This furnishes a good opportunity to distinguish between a zero of f and f(0).

The example of Fahrenheit and centigrade temperature scales given in the book is the best example I know of a linear function determined by two points on its graph. I usually work this example out in class. Then I point out to the students that they can solve similar problems by computing the slope using the two points on the graph and then finding the formula with the point-slope method. I try to deemphasize formulas in this section of the book; the only formula I work out is $f(x) = m(x - h) + k$.

I try to do an interpolation example with something like square roots: I note that between $x = 4$ and $x = 9$, the graph of the square-root function looks a lot like a straight line. Then I work out the equation of the straight line through (4,2) and (9,3) and use it to get approximations to $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, and $\sqrt{8}$. I check the value for $\sqrt{5}$ (2.2) by squaring it and show that its square is closer to 5 than the next closest possible one-decimal place approximation to $\sqrt{5}$ (2.3). Getting one-decimal-place accuracy from such a gross approximation seems to impress the students.

SECTION 1.4 RELATIONSHIPS THAT AREN'T LINEAR OR AREN'T FUNCTIONS

Concepts: Hyperbolas (optional). Circles. The distance formula. Nonlinear functions. Power and root functions. Direct variation.

The practical example in the book is complicated enough that I usually spend about a third of the class working it out. In the process, I develop the formula for the distance between two points. I then apply the distance formula to get the equation of a circle and discuss relationships that aren't functions. As final examples of nonlinear functions, I sketch graphs of either the squaring and square-root functions or the cubing and cube-root functions. In preparation for Chapter 5, I comment on the similar shape and relative orientation of the two graphs.

SECTION 2.1 LINEAR FUNCTIONS OF TWO VARIABLES AND LINEAR INEQUALITIES

Concepts: Linear functions of two variables. Linear inequalities. Manipulation of linear inequalities. Graphs of linear inequalities. Intersections of straight lines. Convexity.

All of Chapter 2 is intended to be optional. However, I make every effort to include it in my course, for two reasons. First, it gives the student valuable experience with manipulating inequalities and solving equations. Second, it gives the student an insight into the process of restating a work problem mathematically. For Section 2.1 I concentrate on the process of translating English statements into inequalities and inequalities into graphs. I state a linear programming problem: my favorite problem for class has to do with designing a new cereal which satisfies certain nutritional requirements but which won't be priced out of the market.

The cereal company is developing a new high-protein cereal that is to have at least 8% protein. They plan to use soy flour, which has 20% protein, oat flour, which has 8% protein, and wheat flour, which has 4% protein, as the major ingredients. For proper rising and consistency, at least half the flour must be wheat flour. Because soy flour has a strong flavor, the amount of soy flour should be no more than one-third the amount of wheat flour. Also, for the sake of flavor, the ratio of soy flour to oat flour should be no more than 2. Using x to denote the number of ounces of oat flour in a pound and y to denote the number of ounces of soy flour, these conditions give the following inequalities:

$$0.08x + 0.2y + 0.04(16 - x - y) \geq 16(0.08)$$

or

$$y \geq 4 - \frac{1}{4}x$$

$$x + y \leq 8$$

$$y \leq \frac{1}{3}(16 - x)$$

$$y \leq 2x$$

These inequalities bound a region that you can sketch fairly nicely: the corners are

$$(4,4),\ (\frac{16}{3}, \frac{8}{3}),\ (\frac{16}{9}, \frac{32}{9}) \text{ and } (\frac{8}{3}, \frac{16}{3})$$

There is a fixed cost of 20¢ per pound to prepare the cereal; in addition, wheat flour costs 1.5¢ per ounce, oat flour 2¢ and soy flour 2.5¢.

My approach is to write down the cost function first and then to point out that until we know something about the domain, we will not be able to figure out what the minimum value of the cost function will be. I spend the rest of the period translating the English to inequalities and graphing the inequalities, emphasizing the five basic rules promulgated after Example 2.1. I also remind the students that we are trying to understand what the domain of the profit function is. Generally I have time when I get done to quote the main theorem of Section 3 and find the minimum cost, to observe that the graph is convex, and to work out one of the regions in Exercise 1.

SECTION 2.2 SYSTEMS OF LINEAR EQUATIONS: GAUSSIAN ELIMINATION

Concepts: Gaussian elimination. Systems of equations with more than two variables. Equivalent systems of equations.

This section provides an alternative approach to solving systems of linear equations that is more systematic than the method introduced in Chapter 1. If you expect your students to go on to a course in linear algebra or to a finite mathematics course that covers row reduction of matrices, then this section will be very helpful to them. I generally cover this section, but not Section 2.4, because matrices are introduced in our other courses. I generally approach this section in a somewhat theoretical manner, proving that a system of equations in n variables has a unique solution if and only if it can be reduced to a system of n equations in which one equation has one variable, one has two variables, one has three, and so on. I then apply Gaussian elimination to solve a system of equations. I make a point of assigning Exercise 11.

SECTION 2.3 LINEAR PROGRAMMING

Concepts: Maximum and minimum values of a linear function of two variables whose domain is a convex polygon.

I include this section in my course whenever I have time because I like its problems. Generally I spend between a third and a half of a period going over the proof that a linear function of two variables with a convex polygon for a domain has a maximum and minimum value and these values occur at corners of the polygon. Then I review whatever motivating example I used when I covered Section 1.4, and work out one of the word problems in the exercises that I don't plan to assign.

SECTION 2.4 MATRICES AND SYSTEMS OF LINEAR EQUATIONS

Concepts: Matrix representations of systems of linear equations. Elementary row operations on matrices. Inconsistent and dependent systems of equations. Row-reduced echelon form.

I do not usually cover this section because it is part of our finite math and calculus courses. I would suggest concentrating on examples, beginning by solving a system of equations in the usual notation and in matrix notation side by side. It might be a good idea to use two days, spending one day solving systems of equations that do have solutions and introducing the concept of row-reduced echelon form. On the second day, you might then deal with inconsistent systems and on systems with a different number of equations and unknowns. It would be a good idea to emphasize that the matrix approach not only helps us not write variables but also helps us organize our thinking along the lines of row reduction. I would conclude from appropriate examples that row reduction is not only a method for finding solutions but also a tool for discovering whether there are any solutions at all. The homework exercises have been set up so that systems which have solutions have relatively nice ones, so you can avoid complicated arithmetic by choosing your examples from among the unassigned exercises!

SECTION 3.1 WHY QUADRATIC FUNCTIONS ARE INTERESTING

Concepts: Quadratic functions. Parabolas. Symmetry vertex of a parabola. Maximum and minimum values and local maximum and minimum values of simple quadratic functions.

When I lecture on this section, I like to discuss the garden example and sketch the graph of the appropriate quadratic function. This allows me to show why we are interested in finding maximum and minimum values and leads naturally into a discussion of symmetry, vertices, and the general shape of parabolas. I then remind the students about the graph of the squaring function and related functions briefly and save enough time to do a problem like Example 3.4. Since virtually all this material is review to almost all the students, I usually concentrate on graphs and motivation of minmax and zeros problems.

SECTION 3.2 FACTORING QUADRATIC FUNCTIONS

Concepts: Factoring. Finding zeros by factoring.

This section is a review section for most of my students and therefore I tell the class to read it and I assign the exercises with daggers. I discuss it briefly when we cover section 3.4. I spend extra time with those students who cannot do the exercises on a one to one basis. If most of your class needs to cover factoring in detail, I would suggest that you do one example of each general rule before you state the rule, and then do another example of the same type using the rule. Emphasize that factoring is not an end in and of itself, but a tool to deal with the important problem of finding zeros. The general rules I have in mind are those for factoring $ax^2 + bx$, $x^2 + (a + b)x + ab$, $x^2 + 2bx + b^2$, $x^2 - a^2$, and finally the mode of attack on something like $ax^2 + bx + c$.

SECTION 3.3 DRAWING GRAPHS AND FINDING EXTREME VALUES BY COMPLETING THE SQUARE

Concepts: Vertices and axes of symmetry for graphs of general quadratic functions. Completing the square. Graphing quadratic functions. Maximum and minimum values of quadratic functions.

I usually begin with a discussion of graphing a quadratic function whose rule is in completed square form, including vertices and axes of symmetry. I then attempt to put the rule for the area function for the garden of the previous section in completed square form and show how you can read the maximum value of a function and the domain value that yields this maximum from the completed square form of the rule. Finally I do several examples of completing the square, doing at least one with the "quick" method outlined at the end of the section, and sketch several graphs.

SECTION 3.4 ZEROS OF QUADRATIC FUNCTIONS AND SYSTEMS OF NONLINEAR EQUATIONS

Concepts: Factoring. Finding zeros by completing the square. Systems of equations involving quadratic expressions.

I begin with a review of practical problems that have led us to ask for zeros of quadratic functions, and do a problem that asks when a ball hits the ground or what the dimensions of a garden with a fixed perimeter must be in order to have a certain area. I set the example up so that I can solve it by factoring--or use the example in the book. Then I change the problem slightly so that we discover we cannot solve the new problem by factoring. Then I ask if our inability to factor means that there are no zeros. I suggest that we sketch a graph to see if it crosses the x-axis and complete the square in order to sketch the graph. (Obviously I pick the numbers for the new version of the problem in advance by starting with the completed square form I want.) Once we have seen from the graph that the function has a zero, I point out that the zero can be virtually read off from the completed square form of the rule. Then I do several examples to convince the students that completing the square is a quick and effective way to find zeros.

Finally I show one example of how to find out when two quadratic functions are equal or when a quadratic and a linear function are equal. (Two bus companies, one with a linear rule and one with a quadratic rule, do quite nicely.) I point out that this shows that systems of equations that are not linear can be important. Exercise 11 follows up on this idea.

SECTION 3.5 THE QUADRATIC FORMULA AND COMPLEX NUMBERS

Concepts: The quadratic formula. Complex numbers. Complex conjugates. Real and imaginary parts of complex numbers. Absolute values of complex numbers.

I begin by asking students to think about their homework exercises to see if they felt that there was a way to short-circuit the process of completing the square. I then apply the process of completing the square to derive the quadratic formula and apply the formula to several of their homework exercises. Next I do a problem along the following lines: "A child throws a stone from ground level at a squirrel 20 feet high in a tree. The stone travels at 32 feet per second when the child lets go of it. When does the stone hit the squirrel?" Fortunately for the squirrel, the time when the stone hits it is a complex number! After pointing out how much the squirrel likes complex numbers, I treat complex numbers in a matter-of-fact way without much fanfare. I observe that any quadratic function has complex zeros, so that it should be interesting to study complex numbers. Since you add, subtract, and multiply complex numbers in a perfectly obvious way, I ask how to divide one complex number into another. (Some examples of student response are: "Divide the real parts and imaginary parts into each other." "Write x instead of i and divide them like polynomials.") This leads into a discussion of complex conjugates. Finally I cover graphing complex numbers long enough to show that a number and its conjugate are symmetric with the real axis and the length of the arrow representing a complex number is the square root of the number multiplied by its conjugate. I have found Exercise 12 to be particularly helpful for developing the students' appreciation of complex numbers.

SECTION 3.6 CONIC SECTIONS

Concepts: Quadratic relations. Parabolas. Hyperbolas. Ellipses. Symmetry. Asymptotes.

I usually include problems in the homework for Section 4 that involve drawing graphs of hyperbolas and ellipses, because I rarely have the time to cover this section. My lecture plan for this section would be to point out the large number of quadratic relations we've already studied and then suggest a systematic study of all quadratic relations. After working out the special case of parabolas that have the axis as their axis of symmetry, I'd show how hyperbolas are related to the main example of Section 4. I explain why some hyperbolas have the x-axis as their axis of symmetry and finally work out the graph of an ellipse. I would probably let the students read through the material on translation of conic sections on their own.

SECTION 4.1 THE ARITHMETIC OF FUNCTIONS

Concepts: Sums, products, differences, and quotients of functions. Polynomial functions. Summation notation. Rational functions. Inverse and joint variation.

The way I deal with this section depends on whether or not I plan to cover Sections 4.5 and 4.6 on rational functions. If I cannot cover 4.5 and 4.6, I point out that the quadratic functions we've studied are special kinds of polynomial functions and that polynomial functions are built from power functions by addition and multiplication by constant functions. I point out that the product of two polynomial functions is a polynomial function, and then ask about the quotient. I try to get to this point as fast as possible to have a bit of time to spend showing that the sum and difference of rational functions are rational functions, so as to be able to ask the students to practice adding and subtracting fractions. I let the students learn about the graph of a sum of two functions on their own, but I try to work in a few examples of summation notation along the way.

If I know I will be able to cover Sections 4.5 and 4.6, then I play up the theory of the arithmetic of functions a bit. I talk about graphing the sum of two functions, and work out an example like the graph of $\frac{1}{x + 1} - \frac{1}{x - 1}$. Then I show that the function has the rule $-2/(x^2 - 1)$ and interpret this in the graph. I discuss how variation and surface area problems lead us to interesting functions.

If I know I won't be able to cover Sections 4.5 and 4.6, then I make sure that I assign as many rational functions problems as seem reasonable--i.e., one out of 9 and 10, both 12 and 13, and perhaps both 6 and 7 as well. If I know I'll cover Sections 4.5 and 4.6, then I pick a more balanced set of problems. Problems 14 and 15 work out very nicely, by the way.

SECTION 4.2 POLYNOMIAL FUNCTIONS AND THEIR GRAPHS

Concepts: Polynomial functions. Graphs of power functions. The general shape of the graph of a polynomial function.

I like to spend the first third of this lecture working out the volume problem that begins the section. I draw a careful graph of this function on a domain that includes some points to the left of $x = 0$ and some to the right of $x = 10$. I use the graph to motivate the idea that the graph of a polynomial of degree n cannot "change direction" more than n times. I point out to the students that maxima and minima are not located symmetrically between zeros as they were for quadratic functions. In fact, the volume function is a good example for all the general rules for graphing given later on in the section. If I have the time after I get done, I work out the graph of $(x^2 - 1)(x^2 - 4) = x^4 - 5x^2 + 4$ to contrast to the graph drawn in the text.

SECTION 4.3 ZEROS OF POLYNOMIAL FUNCTIONS AND POLYNOMIAL EQUATIONS

Concepts: Factoring polynomials, if c is a zero then $x - c$ is a factor. Long division. Rational zeros theorem.

I usually leave the introduction to this section for the students to read, and I work out a word problem like the one in the book that asks us to cut corners out of a 4-inch square and fold up the sides to make a box with a volume of 4 cubic inches. I work out this problem the way the book does, and note that the rational zeros theorem says that if c is a zero, then $x - c$ is a factor. Then I work out one of the exercises I haven't assigned from Exercise 4. I like to assign Problems 7, 8, and 9; each has its own way of making the student think a bit. Exercise 8 seems to generate the most questions. Students seem relieved when they realize that if $f(10)$ is negative and $f(20)$ is positive, then somewhere they should expect to find a zero!

SECTION 4.4 AN INTRODUCTION TO THE THEORY OF ZEROS OF POLYNOMIAL FUNCTIONS

Concepts: Rational zeros theorem. Remainder theorem (Euclidean algorithm). Factor theorem. The number of zeros of a polynomial of degree n. Factorization of arbitrary polynomials into linear and quadratic factors.

I've never had the luxury of covering this section in class. If I did, I would work out the proofs of what I call the remainder theorem (usually called the Euclidean algorithm) and what I call the factor theorem. I'd also like to discuss either Exercise 6 or 7 in class and assign the other one.

SECTION 4.5 RATIONAL FUNCTIONS AND THEIR GRAPHS

Concepts: Rational functions, asymptotes, limits. Graphing a constant divided by a linear function. Graphing a quotient of a polynomial function by a linear function (using long division). Quotients of linear functions. Graphing a constant divided by a quadratic function.

I generally begin this section with one of the graphing problems from Exercise 5 or 8. This leads me to long division, graphing a quotient of a constant by a linear function, and adding two graphs. I then graph $\frac{1}{x^2}$, $\frac{1}{x^2+1}$ and $\frac{1}{x^2-1}$ quickly, and if there is time I work out a problem like number 11.

SECTION 4.6 THE ARITHMETIC OF RATIONAL FUNCTIONS

Concepts: Rules for multiplying, dividing, adding and subtracting fractions. Lowest terms for a rational function. Partial fractions.

I spend about the first third of the lecture subtracting fractions like $1/(x+1) - 1/x$, or $1/(x-1)^2 - 1/x^2$, that cause the most trouble in calculus. I then pass on to partial fractions, approaching the subject as a method for writing a rational function as a sum of functions that are simpler to graph. I usually restrict myself to denominators that factor into nonrepeated linear functions.

SECTION 5.1 COMPOSITION OF FUNCTIONS

Concepts: Composition of functions; visualizing function composition. The composition of linear functions is linear. Composition is associative but not commutative.

I usually begin this example by pointing out that there are 3 teaspoons in a tablespoon and 16 tablespoons in a cup. Then I ask for the teaspoon equivalent of 1/4 of a cup and 1/3 of a cup. I work out the