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J R Leigh

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JR Leigh

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Control Theory

Second Edition



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To the colleagues who have enriched my professional life

Introduction to the second edition

In the ten years that have passed since the first edition of this book was published, the main developments in the subject of control have been:

- Within control theory proper, the rise of H_∞ and similar approaches, allowing a combination of practicality, rigour and user interaction to be brought to bear on complex control problems and helping to close the often discussed gap between control theory and practice.
- The rise of artificial intelligence (AI) techniques such as a neural networks that, within a computer intensive context, have become inextricably linked into the control subject.
- The rise in the availability of comprehensive software packages particularly designed for solving control related problems.

In this new edition, I have added two additional chapters devoted to H_∞ approaches and to AI approaches, respectively. I have also added a chapter that, placed at the end of the book, briefly reviews the development of control, so forming something of a context for what has gone before.

In addition to these major changes, I have reviewed and, where necessary, revised all the earlier material. In the spirit of the first edition, I have added ten additional diversionary ‘Interludes’ and of course taken the opportunity to update and enhance the references and suggestions for further reading. I very much hope that the resulting new edition is well placed to satisfy the aims of the first edition, which were as stated in the following section.

The structure, content and purpose of the book

This book is drastically different from other control books. It follows no well-trying formula but, thinking as it goes, imitates in a sense the author's discussions with students, supervisees and colleagues. Most of these discussions were interesting because they were concerned with concepts too general or too simple to be included in standard textbook material or alternatively they were too detailed, esoteric or unfinished to be there.

The book is structured around a few limited concepts that are central to control theory. The concepts are presented with a minimum of detail and, once sufficient work has been done to establish ideas, the reader is pointed off to specific references.

The treatment is augmented by more detailed interludes. These interludes appear in a different typescript and although they are always relevant to their context, they are not necessarily so easy to follow as the mainstream text. However, if they are skipped over, this will not be detrimental to understanding the main thread of the book.

The first three chapters, quite deliberately, contain no mathematics at all. It is intended that these chapters can form a useful introduction to control theory for a wide class of readers. These chapters largely answer the questions:

- What is control theory?
- What are the main ideas?
- What are the features that make the subject so fascinating and absorbing?

The features of the book may be summarised:

- Emphasis on concepts.
- Follow up for the reader by reference links from the text to easily available standard books.
- The first three chapters are entirely non-mathematical.
- The large number of interludes stimulates interest. Appearing in a distinctive typescript, they may be omitted without detriment in a first reading of the mainstream text.
- Very extensive annotated bibliography.

The policy for citation of references within this book is worthy of explanation:

Control is, in general, an integrated rather than disparate subject. Many of the references cited in this text are relevant to a number of different sections and chapters of the book and, on this basis, it is appropriate that references are cited as part of the whole work rather than by individual chapter. A complete list of references is therefore given in Chapter 19. However, the reader will also note that Chapters 8, 16 and 17 contain references that are not only of general use, but are of primary importance to the chapter in which they appear.

The intended readership for the book is:

- Students working at any level on control engineering. Despite the multiplicity of available control books at all levels, students still struggle to understand basic concepts. This book is intended as their companion and friend.
- Students of science, computing, mathematics and management. The book will supply these students with the main concepts of control, thus supporting the auxiliary control courses that are attended by these students.
- Industrialists, managers and professionals in a wide variety of fields. A large number of professionals from a wide variety of fields wish to understand the fundamentals and the potential of control, to an extent that will demystify the subject and that will allow them more effectively to assess the benefits of control to their particular areas.
- Engineers already familiar with control. They could actually find the book enjoyable, paralleling the enjoyment that I have obtained from writing it.

Every worthwhile discipline has a strong structure and underlying principles and is possessed of a continuous striving towards improved coherence so that what, at first sight, appeared to be isolated phenomena take their place in the structure in a consistent way. Thus, the science of physics has been brought, by generations of dedicated development, to its present well-unified state.

Here, we are concerned with the structure, principles and context of control theory.

Control theory is a very powerful body of knowledge indeed. It allows the synthesis of systems having specified characteristics. It can model and include within its control loops any complex object (for instance, an aircraft) that needs to be so included. It can produce adaptive solutions that change automatically as circumstances change. It can combine with pattern recognition, with expert systems and with artificial intelligence (AI) in general. It makes use of computer power to identify problems, to solve problems, to validate solutions and to implement the final solutions. Control has an impressive track record of successful applications across aircraft, ships, satellite and missile guidance, process industries (chemicals, oil, steel, cement, etc.), pharmaceuticals, domestic and computer goods (automatic cameras, etc.), public utilities (e.g. all aspects of electrical generation and supply), automatic assembly, robotics, prosthetics and increasingly it lends its basic ideas to other disciplines.

Control theory is built up around a few very simple ideas: such ideas as feedback loop and stability. The writing of this book has been motivated by a belief that it is

absolutely vital to obtain a robust understanding of these few simple ideas and not allow them to be submerged below a cloud of techniques or numerical detail.

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