Rheological measurement

2nd ed.

A.A. Collyer, D.W. Clegg, editors.

Rheological Measurement

Second edition

Edited by

A. A. Collyer

Formerly senior lecturer in Division of Applied Physics, School of Science, Sheffield Hallam University, UK

and

D. W. Clegg

Formerly head of Division of Materials and Environmental Engineering, School of Engineering, Sheffield Hallam University, UK

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Contributors

M. Baumgärtel
BASF AG, ZU/AT-G615, 65056 Ludwigshafen, Germany.

J. H. Bland
Centre for Composite Materials Engineering, University of Newcastle upon Tyne, Newcastle upon Tyne NE1 7RU, UK.

D. V. Boger
Department of Chemical Engineering, University of Melbourne, Parkville, Victoria 3052, Australia.

T. O. Broadhead
53 Cambridge Crescent, Brockville, Ontario, Canada, K6V 6L7.

G. J. Brownsey
Institute of Food Research, Norwich Laboratory, Norwich Research Park, Colney, Norwich NR4 7UA, UK.

J. M. Dealy
Faculty of Engineering, McGill University, 817 Sherbrooke St. West, Montreal PQ, Canada, H3A 2K6.

A. J. Giacomin
Department of Mechanical Engineering, University of Wisconsin, 309 Mechanical Engineering Building, 1513 University Ave, Madison, WI 53706-1572, USA.

A. G. Gibson
Centre for Composite Materials Engineering, University of Newcastle upon Tyne, Newcastle upon Tyne NE1 7RU, UK.

W. Gleißle
Institut für Mechanische Verfahrenstechnik und Mechanik, Der Universität Karlesruhe (TH), Kaiserstrasse 12, D-76128 Karlesruhe, Germany.

R. K. Gupta
Department of Chemical Engineering, West Virginia University, 421 Engineering Science Building, PO Box 6102, Morgantown WV 26506-6102, USA.

Chang Dae Han
Department of Polymer Engineering, University of Akron, OH 44325-0301, USA.

G. Kotsikos
Centre for Composite Materials Engineering, University of Newcastle upon Tyne, Newcastle upon Tyne NE1 7RU, UK.

A. S. Lodge
210 Du Rose Terrace, Madison, WI 53705, USA.

M. E. Mackay
Department of Chemical Engineering, University of Queensland, St Lucia, Brisbane, Queensland 4072, Australia.

M. R. Mackley
Department of Chemical Engineering, University of Cambridge, Pembroke St., Cambridge CB2 3RA, UK.

G. Marin
Laboratoire de Physique des Matériaux Industriels, Université de Pau et des Pays de l'Adour, Avenue de l'Université, 64000 Pau, France.

M. Mours
BASF AG, Polymer Laboratory—Process Development, 67056 Ludwigshafen, Germany.

B. I. Nelson
Goettfert, 488 Lakeshore Parkway, Rock Hill, SC 29730, USA.

R. A. Pethrick
Department of Pure and Applied Chemistry, University of Strathclyde, Thomas Graham Building, 295 Cathedral St., Glasgow G1 1XL, UK.

R. L. Powell
Department of Chemical Engineering and Materials Science, University of California, Davis, CA 95616-5294, USA.

R. P. G. Rutgers
Department of Chemical Engineering, University of Cambridge, Pembroke St., Cambridge CB2 3RA, UK.

P. R. Soskey
322 Kelly Drive, Neshanic Station, NJ 08853-4044, USA.

T. Sridhar
Department of Chemical Engineering, Monash University, Clayton, Victoria 3168, Australia.

R. I. Tanner
University of Sydney, Sydney, NSW 2006, Australia.

S. Toll
Department of Polymeric Materials, Chalmers University, Gothenburg, Sweden.

B. Warburton
37a Oatlands Ave, Weybridge, Surrey, KT13 9SS, UK (formerly at the School of Pharmacy, University of London).

R. C. Warren
Weapons Systems Research Laboratory, Defence Research Centre Salisbury, GPO Box 2151, Adelaide, SA 5001, Australia.

H. H. Winter
Department of Chemical Engineering, University of Massachusetts, Amherst, MA 01003, USA.

K. F. Wissbrun
1 Euclid Ave, Apartment 4E, Summit, NJ 07901, USA.

Preface

In many cases rheological measurements are carried out in the simplest of geometries, but the interpretation involved in obtaining the rheological parameters of the test fluids from these measurements is surprisingly complex. When making measurements and obtaining from them parameters that describe the flow behaviour of the test fluids, the experimentalists need to understand the underlying theory and shortcomings of the measurement technique; they need to be aware of the likely microstructure of the fluid, and they need to appreciate how the fluid and the measuring system interact with one another. This interaction gives both the required rheological parameters and the artefacts which confuse the issue. The aim of this book is to concentrate on the techniques and the physical principles underlying them. This is achieved by emphasizing the points on which most workers in the field agree, and to let the authors deal with the contentious points according to their own beliefs and experience. This work represents a summary of the current thought on rheological measurement by experts in the various techniques.

This second edition of *Rheological Measurement* covers the main rheological measurement techniques, combining the main techniques included in the first edition and some additional, more sophisticated techniques originally covered in *Techniques in Rheological Measurement*. The extent of the work meant it was important to concentrate on measurement, so two important issues have been dropped in this edition: the rheology of two-phase flows and mathematical modelling of two-phase flows. Many books involving these subjects have been published since the appearance of the first edition. The other main change is the division into four parts: small strain measurements, large strain measurements, extensional and mixed flows and specialized rheometers. However, the main objective is unaltered; this edition covers the basic methods, how the measurements are taken, and what assumptions and interpretations are made to obtain valid data on the test fluids. Where possible, the authors have kept to the nomenclature recommended by the Society of Rheology (Dealy, J.M., 1994, Official nomenclature for material functions describing the response of a viscoelastic fluid to various shearing and extensional deformations, J. Rheol., **37**, 179–191). Additional nomenclature is given at the end of some chapters.

Rheology is a vast and exciting subject; it can scarcely be covered in a single book, albeit a large one, but the editors hope this work will be of considerable relevance and help to all establishments in which rheological measurements are carried out. For a good understanding of the coverage, readers should have a working knowledge of first-year degree-level mathematics. Materials scientists, engineers or technologists in industry, research laboratories or in academic institutions should find the book valuable in providing an up-to-date review of current rheological thought from experts around the world.

A.A. Collyer
Weston-Super-Mare
1997

D.W. Clegg
1997

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