THE INTERNATIONAL CRYOGENICS MONOGRAPH SERIES
General Editors: K. D. Timmerhaus, Alan F. Clark, and Carlo Rizzuto

Cryogenic Regenerative Heat Exchangers

Robert A. Ackermann

Cryogenic Regenerative Heat Exchangers

Robert A. Ackermann

General Electric Company
Schenectady, New York

PLENUM PRESS • NEW YORK AND LONDON

Library of Congress Cataloging-in-Publication Data

Ackermann, Robert A.
Cryogenic regenerative heat exchangers / Robert A. Ackermann.
p. cm. -- (The international cryogenics monograph series)
Includes bibliographical references and index.
ISBN 0-306-45449-1
1. Heat exchangers. 2. Cryogenic engineering. I. Title.
II. Series.
TJ263.A23 1997
621.402'2--dc21 97-13652
CIP

ISBN 0-306-45449-1

A Division of Plenum Publishing Corporation
233 Spring Street, New York, N. Y. 10013

http://www.plenum.com

10 9 8 7 6 5 4 3 2 1

Printed in the United States of America

Cryogenic Regenerative Heat Exchangers

THE INTERNATIONAL CRYOGENICS MONOGRAPH SERIES

Current volumes in this series

APPLIED SUPERCONDUCTIVITY, METALLURGY, AND PHYSICS OF TITANIUM ALLOYS • *E. W. Collings*
Volume 1: Fundamentals
Volume 2: Applications

CRYOCOOLERS • *G. Walker*
Part 1: Fundamentals
Part 2: Applications

CRYOGENIC PROCESS ENGINEERING
• *Klaus D. Timmerhaus and Thomas M. Flynn*

CRYOGENIC REGENERATIVE HEAT EXCHANGERS • *Robert A. Ackermann*

HELIUM CRYOGENICS
• *Steven W. Van Sciver*

MODERN GAS-BASED TEMPERATURE AND PRESSURE MEASUREMENTS
• *Franco Pavese and Gianfranco Molinar*

POLYMER PROPERTIES AT ROOM AND CRYOGENIC TEMPERATURES • *Günther Hartwig*

SAFETY IN THE HANDLING OF CRYOGENIC FLUIDS • *Frederick J. Edeskuty and Walter F. Stewart*

STABILIZATION OF SUPERCONDUCTING MAGNETIC SYSTEMS • *V. A. Al'tov, V. B. Zenkevich, M. G. Kremlev, and V. V. Sychev*

SUPERCONDUCTING ELECTRON-OPTIC DEVICES • *I. Dietrich*

THERMODYNAMIC PROPERTIES OF CRYOGENIC FLUIDS • *Richard T Jacobsen, Steven G. Penoncello, and Eric W. Lemmon*

Preface

This text provides a comprehensive discussion of the theory and design of regenerative heat exchangers for cryogenic applications. Previous texts have treated regenerators as a small subsection of the larger topic of continuous gas-to-gas heat exchangers. However, with the emergence of regenerative devices in cryogenic applications and the recent commercialization of many of these devices, a text devoted exclusively to regenerative heat exchangers is now appropriate.

Regenerators represent an important class of heat exchangers, in which heat exchange between two fluids occurs through an intermediate exchange with a matrix of high-heat-capacity material. This unique operation leads to a periodic heat exchange between two fluids, in place of the continuous heat exchange found in counterflow heat exchangers. This periodic nature of regenerators contributes to two important design characteristics: First, the use of a matrix material leads to a high-performance, compact geometry and, second, the periodicity of the flow leads to complex mathematical analyses based on time-dependent temperature and heat transfer parameters. The compact construction and periodic nature are important regenerator characteristics and are largely responsible for the many successful commercial applications they have enjoyed. The complexity of the analyses, however, has led to the need for costly development programs to design regenerators for new applications. This book explores these diverse characteristics with the intent of providing engineers with the theoretical background and data required to design a regenerator. In particular, we explore the requirements and use of regenerators in cryogenic applications, where the characteristics of high efficiency, small size, and low pressure drop have enabled small cryogenic refrigerators to achieve substantial commercial success over the last 25 years. This book is intended to provide engineers with both the theory and practical aspects of regenerator design so that they can effectively use them in the development of advanced cryogenic devices.

The text is organized into five chapters dealing with historical developments, theory, practical applications, design data, and numerical solutions.

Chapter 1 reviews the historical development of cryogenic refrigeration and the use of regenerators in these devices. The history of regenerator designs is significant because it illustrates the broad spectrum of regenerator applications and their varied design features. The regenerative heat exchanger was first developed by Dr. Robert Stirling in 1816 for use in his hot air engine. From this important beginning, regenerators have achieved success in such diverse applications as high-temperature gas turbines and low-temperature cryogenic refrigerators. In cryogenic applications, the regenerator has been instrumental in the development of small refrigerators for military infrared systems and in commercial uses involving cryopumping and cooling of superconducting magnets in medical imaging systems. It is the emergence of these applications and the importance of cryogenic refrigerators in advancing this commercialization that provide the foundation for this book.

Chapter 2 presents the development of the thermal and fluid dynamic equations required to analyze regenerator performance. The discussion includes the development of the basic differential equations that govern the heat transfer and fluid flow characteristics of a regenerator. These equations describe the temperature distributions in the matrix material and fluid as functions of both space and time and lead to a complex set of differential equations for which no closed-form solutions exist. To provide insight into the operation of a regenerator, we present the concept of an ideal regenerator and make several assumptions that reduce the complexity of the differential equations. This simplification enables the definition of a set of design parameters that characterize basic regenerator performance. The chapter concludes with a review of several prominent matrix geometries and the development of the mathematical formulas that define the heat transfer and flow characteristics of these geometries. The geometries include woven screen, spheres, gaps, and wound foils.

Chapter 3 covers the classical solutions of the ideal regenerator equations and discusses the effects of longitudinal thermal conduction and variable matrix heat capacity on regenerator performance. The review of classical solutions presents Hausen's comparison of the temperature changes in a regenerator with those of a recuperative heat exchanger and discusses the important similarities and differences between these two types of heat exchangers. The review also covers Nusselt's derivation of an integral equation to describe the fluid temperature distributions based on a defined initial matrix temperature distribution and Iliffe's numerical solution of Nusselt's integral equation, which provides an important description of the cyclic matrix and fluid temperature distributions in a regenerator.

Because of the importance of variable specific heat on cryocooler performance at very low temperatures, below 10 K, we conclude the chapter by considering recent developments of the use of rare-earth intermetallic compounds in very-low-temperature applications. These materials exhibit a magnetic phase transition that manifests itself in a large increase in specific heat at the transition temperature. By using these materials, researchers for the first time have been able to extend the operational range of regenerative cryocoolers to below the normal boiling temperature of helium, a remarkable achievement that has great commercial potential in the field of superconductivity.

Chapter 4 presents regenerator design and performance data. The material is a compilation of data that have appeared in technical publications over the past 30 years, including a review of the test and data reduction methods most commonly used to obtain the heat transfer coefficient, the longitudinal thermal conductivity, and the heat transfer efficiency of porous materials. The data presented are for a variety of matrix geometries and materials, including coarse and fine bronze and stainless-steel woven wire mesh screens and lead spheres. The results include room-temperature data obtained by researchers at Stanford University during the 1950s and 1960s using a single-blow test rig and more recent cryogenic temperature data for temperatures down to 20 K obtained using periodic flow test apparatus. The objective of this chapter is to provide a comprehensive file of useful data for engineers to use in the design of regenerators for new cryogenic applications.

Chapter 5 deals with numerical techniques used to solve the complex nonideal performance equations and methods for optimizing regenerator performance. Numerical methods represent an open form of solution to the regenerator equations in which the differential equations are replaced with difference equations and solutions are obtained by a stepwise iterative process. The development of the difference equations, the solution of these equations, the necessary conditions for convergence, and the accuracy of the solutions are presented for a first-order model that assumes constant flow and pressure and zero fluid stored energy. It is shown that by describing the regenerator as a series of parallel flow elements with the fluid and matrix material represented as parallel flow streams, good accuracy, with reasonable amounts of computational time, can be achieved.

Regenerator optimization is illustrated with an example of the optimization of a regenerator for a Stirling cycle refrigerator. The criteria for optimizing the regenerator and the development of the equations relating the key parameters to regenerator performance are presented for this example. The equations are developed through a phasor representation of the pressure and volume variations in a Stirling cryocooler and the use of the Schmidt method of analysis to relate regenerator performance to such critical regenerator parameters as matrix geometry, length, diameter, and matrix porosity.

The final chapter, Chapter 6, presents a discussion of regenerators used in cryogenic refrigeration equipment. The application and performance of a regenerative heat exchanger in low-temperature equipment is presented by considering Stirling, Gifford–McMahon, pulse tube, and magnetic cryocoolers. The operation of the regenerator in each of these devices is discussed, and the importance of the regenerator to the overall efficient operation of the cryocooler is examined.

Unique operating features of cryocoolers that produce deviations from ideal regenerator theory are discussed.

The author is indebted to Ms. Karin Ostrom for her outstanding job of editing and preparing the manuscript. The author also wishes to acknowledge the support of the General Electric Company and the staff of GE Corporate Research and Development for their help in locating and obtaining the research material required to write this book.

Robert A. Ackermann

List of Symbols

PRIMARY SYMBOLS

A_{ff}	Fluid axial free flow area
A_{m}	Matrix thermal conduction heat transfer area
A_{n}	Thermal conduction heat transfer area normal to the flow; for a regenerator $A_{\mathrm{n}} = A_{\mathrm{m}}$; also, fluid cross-sectional area normal to the flow
A_{r}	Total regenerator frontal area
A_{s}	Matrix total heat transfer area
A_{wl}	Wall thermal conduction heat transfer area
C_{c}	Cold fluid heat capacity rate
C_{h}	Warm fluid heat capacity rate
C_{m}	Matrix heat capacity rate
C_{max}	The larger of C_{c} and C_{h}
C_{min}	The smaller of C_{c} and C_{h}
c_p	Specific heat at constant pressure (isobaric)
c_v	Specific heat at constant volume (isochoric)
D_{d}	Displacer diameter
D_{h}	Hydraulic diameter ($D_{\mathrm{h}} = 4r_{\mathrm{h}}$)
D_{r}	Regenerator diameter
d	Screen wire diameter or sphere diameter
E	Modulus of elasticity
e	Specific enthalpy
f	Fluid coefficient of friction
fr	Frequency
G	Mass flow rate per unit free flow area
h	Heat transfer coefficient
h_{f}	Energy loss related to friction
Ie	Regenerator thermal inefficiency
K_{f}	Fluid thermal conductivity
K_{m}	Matrix thermal conductivity
K_{wl}	Regenerator wall thermal conductivity

k	Adiabatic process constant
L	Regenerator length
M	Molecular weight
M_f	Mass of the fluid
M_m	Mass of the matrix material
$\dot{m}$	Mass flow rate
$\bar{\dot{m}}_\mathrm{c}$	Mean flow rate into and out of the expansion space
N_s	Number of screens in a regenerator packing
n	Screen mesh size
P	Thermodynamic state pressure
$\bar{P}$	Average cycle pressure
P_c	Compression space pressure
P_e	Expansion space pressure
P_max	Maximum cycle pressure
P_mean	Mean cycle pressure
P_min	Minimum cycle pressure
P_s	System pressure
Pa	Pressure ratio
p	Fluid pressure; a function of time and position
p_so	System pressure amplitude
$\dot{Q}$	Heat flow rate
Q_c	Heat conduction
Q_f	Heat transported by the fluid
Q_h	Heat transfer between the fluid and matrix material
Q_l	Thermal losses
Q_net	Net cryocooler refrigeration, also denoted as Q_r and Q_ref
Q_r	Cryocooler available refrigeration, equivalent to Q_net and Q_ref
Q_ref	Available refrigeration
Q_reg	Regenerator thermal loss
Q_rej	Rejected heat
Q_η	Frictional heating
R_m	Matrix thermal resistance
R_t	Total regenerator thermal resistance
R_wl	Regenerator wall thermal resistance
r	Radial coordinate
r_h	Hydraulic radius
S	Expander stroke
T_a	Ambient room temperature
T_c	Regenerator cold-end temperature
T_c	Counterflow heat exchanger cold stream temperature
T_cr	Critical temperature
T_cs	Compression space temperature

T_{e}	Expansion space temperature
T_{f}	Regenerator fluid temperature
$(T_{\mathrm{f}})_{\mathrm{c}}$	Regenerator fluid temperature during cooling period
$(T_{\mathrm{f}})_{\mathrm{h}}$	Regenerator fluid temperature during heating period
$(T_{\mathrm{f}})_{\mathrm{in}}$	Fluid inlet temperature
$(T_{\mathrm{f}})_{\mathrm{out}}$	Fluid outlet (exhaust) temperature
T_{h}	Counterflow heat exchanger hot stream temperature
T_{m}	Regenerator matrix temperature
$(T_{\mathrm{m}})_{\mathrm{c}}$	Regenerator matrix temperature during cooling period
$(T_{\mathrm{m}})_{\mathrm{h}}$	Regenerator matrix temperature during heating period
$(T_{\mathrm{m}})_{0}$	Initial matrix temperature at the start of the heating and cooling periods
T_{w}	Regenerator warm-end temperature
T_{wl}	Regenerator wall temperature
t	Time
t_{s}	Individual screen thickness in a packed screen matrix
U	Overall heat transfer coefficient
V_{cs}	Compression space volume
V_{e}	Expansion space volume
V_{hx}	Heat exchanger volume
V_{m}	Volume occupied by the matrix material
V_{r}	Regenerator total volume
$V_{\mathrm{r}v}$	Regenerator void volume
v	Specific volume
v_{eo}	Expansion space volume amplitude
W	Heat transfer area per unit length for a counterflow heat exchanger $= (A_{\mathrm{s}}/L)$ for a regenerator
$\dot{W}$	Mechanical power
$\bar{W}$	Averaged axial flow velocity
W_{p}	Weight of the packed matrix material
W_{pv}	Gross mechanical refrigeration produced by a cryocooler
$W_{\Delta v}$	Refrigeration loss related to pressure drop
$\mathbf{w}$	Velocity vector
X_{d}	Displacer position
X_{p}	Compressor piston position
z	Axial coordinate

GREEK SYMBOLS

α	Matrix porosity
β	Area density
Γ	Regenerator matrix geometric factor

γ	Fluid specific weight
Δp_{f}	Friction pressure drop
$\Delta p_{\mathrm{r}v}$	Regenerator void volume pressure drop
ΔT	Temperature difference
$\Delta \bar{T}$	Average period temperature difference
ε	Efficiency (effectiveness) for a recuperative heat exchanger
ε_{r}	Regenerator thermal efficiency
η	Reduced time
Θ_{f}	Reduced fluid temperature
$(\Theta_{\mathrm{f}})_0$	Constant fluid inlet reduced temperature
Θ_{m}	Reduced matrix temperature
$(\Theta_{\mathrm{m}})_0$	Constant initial matrix reduced temperature
θ	Angular cylindrical coordinate
κ	Thermal conduction parameter; also compressibility coefficient
Λ'	Heating number of heat transfer units, $\mathrm{NTU}_{\mathrm{h}}$
Λ''	Cooling number of heat transfer units, $\mathrm{NTU}_{\mathrm{c}}$
λ_{c}	Cooling flow period
λ_{h}	Heating flow period
μ	Fluid dynamic viscosity
ξ	Reduced length
Π	Reduced period
ρ	Density
ρ_{f}	Fluid density
$\bar{\rho}_{\mathrm{f}}$	Mean fluid density through the regenerator
$(\rho_{\mathrm{f}})_{\mathrm{e}}$	Fluid density in the expansion space
ρ_{m}	Density of the matrix material
σ	Normal stress
σ_{r}	Ratio of free flow area to regenerator frontal area
τ	Fluid shear stress
ω	Matrix rotational speed for rotary regenerator, or flow reversal rate for stationary regenerator

DIMENSIONLESS PARAMETERS

$\dfrac{C_{\min}}{C_{\max}}$	Fluid heat capacity ratio $\left[\dfrac{(\dot{m}c_p)_{\min}}{(\dot{m}c_p)_{\max}}\right]$
C_{r}	Matrix capacity ratio $\left[\dfrac{C_{\mathrm{m}}}{C_{\min}} = \dfrac{1}{\lambda}\dfrac{(Mc_p)_{\mathrm{m}}}{(\dot{m}c_p)_{\min}}\right]$

NTU	Total regenerator number of heat transfer units $\left(\frac{A_s U}{C_{\min}}\right)$ where $\frac{1}{A_s U} = \left[\frac{1}{(hA_s)_h} + \frac{1}{(hA_s)_c}\right]$
$\mathrm{NTU_c}$	Number of heat transfer units for the cooling period $\left(\frac{hA_s}{\dot{m}c_p}\right)_c$
$\mathrm{NTU_h}$	Number of heat transfer units for the heating period $\left(\frac{hA_s}{\dot{m}c_p}\right)_h$
Nu	Nusselt number $\left(\frac{hD_h}{K_f}\right)$
Pr	Prandtl number $\left(\frac{\mu c_p}{K}\right)_f$
Re	Reynolds number $\left(\frac{GD_h}{\mu}\right)$
St	Stanton number $\left(\frac{h}{Gc_p}\right)_f$
Θ_f	Dimensionless fluid temperature ratio $\left[\frac{T_f - T_c}{T_w - T_c}\right]$
Θ_m	Dimensionless matrix temperature ratio $\left[\frac{T_m - T_c}{T_m - T_c}\right]$
Π	Hausen's reduced period $\left[\frac{hA_s}{(Mc_p)_m}\lambda\right]$

Contents