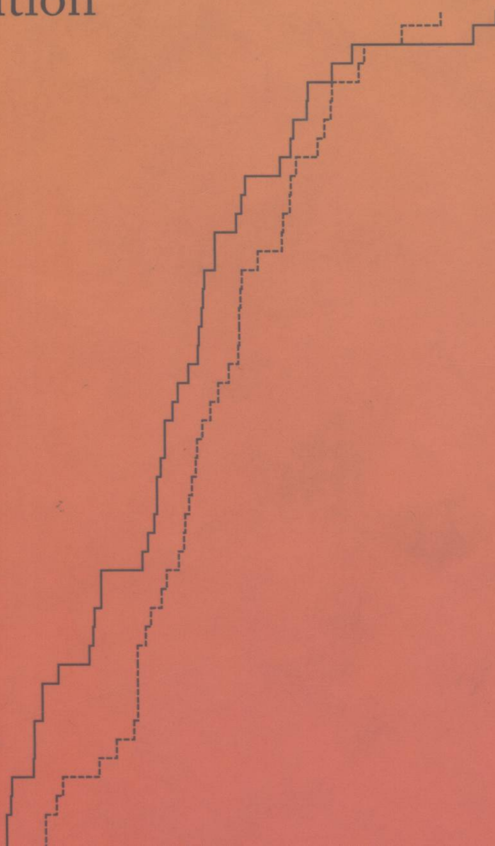
Frederick James

Statistical Methods in Experimental Physics

2nd Edition



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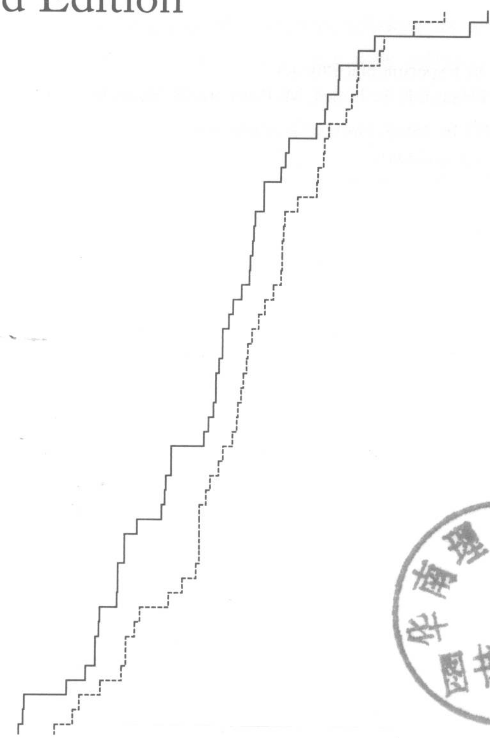

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PREFACE TO THE SECOND EDITION

Thirty years after the first publication of the first edition, it was decided that the continued demand justified the production of a new edition. The five original authors agreed that the new edition should reproduce as much as possible the complete text of the first edition, of course corrected for mistakes and modified to take into account recent developments. The original publisher ceded his rights to World Scientific Publishing Co. who kindly agreed to set the entire first edition in LaTeX to initiate the preparation of the new edition. Unfortunately, only one of the five original authors was ready to do this preparation, but the four others agreed to yield their rights in the interest of allowing the renewed availability of the book.

The new edition has required a considerable amount of work. For example, there are over 1000 formulae in the book, about half of which required modification, mostly for improved and consistent notation, but also to correct all the mistakes that have been reported over the years. In addition, the author of the second edition has had thirty years' additional experience in statistical data analysis, which necessarily translates into a better understanding of some problems and requires more than cosmetic changes in a few chapters. The overall result is that most of the text comes from the first edition, but the modifications are sufficiently important that the author of the second edition must take all the responsibility for the final text.

For the reader, the most striking difference between the editions will certainly be the improved typesetting. All the other benefits of computer preparation should make this edition much easier to read and more reliable than its predecessor.

F. E. James
July 2006, Geneva

PREFACE TO THE FIRST EDITION

This course in statistics, written by one statistician (W.T.E.) and four high-energy physicists, addresses itself to physicists (and experimenters in related sciences) in their task of extracting information from experimental data. Physicists often lack elementary knowledge of statistics, yet find themselves with problems requiring advanced methods, if adequate methods at all exist. To meet their needs, a sufficient course would have to be very long. Such courses do indeed exist [e.g. Kendall], only the physicists usually do not take the time to read them.

We attempt to give a course which is reasonably short, and yet sufficient for experimental physics. This obviously requires a compromise between theoretical rigour and amount of useful methods described.

Thus we are obliged to state many results without any rigorous proof (or with no proof at all); still we have the ambition to present more than just a cook-book of prescriptions and formulae. We omit the mention of many techniques which, in our judgement, seem to be of lesser importance to experimental physics.

On the other hand, we do introduce many theoretical concepts which may not seem immediately useful to the experimenter. This we think is necessary for two reasons. Firstly, the experimenter may need to know some theory or some "generalized methods" in order to design his own methods, experimental physics posing always novel questions. This is a justification for the stress on Information theory (Chapter 5), and for the attempt in Chapter 7 to define a "general" method of estimation. We hope that although the method the reader will arrive at may not be optimal, still it will be useful.

Secondly, the experimenter should be aware of the assumptions underlying a method, whether it be a standard method or his own. It is for this reason that we insist so much on the Central Limit Theorem, which is at the foundation of all "asymptotic" statistics (Chapters 3 and 7).

Quoting theorems, we also try to state their range of application, to avoid too careless use of some methods.

Among the underlying assumptions, especially important are the ones about the parent distributions of the data, since they will condition the results. In Chapter 4 we give a catalogue of useful ideal distributions; in real life they may have to be truncated (Sec. 4.3), experimental resolution may have to be folded in (Sec. 4.3), detection efficiency may have to be taken into account (Sec. 8.5). Moreover, the true distribution may not be known, in which case one is led to empirical distributions (Sec. 4.3), robust estimation (Sec. 8.7), and distribution-free tests (Chapter 11).

A very common tacit assumption in the everyday use of statistics is that the set of data is large enough for asymptotic conditions to apply. We try to distinguish clearly between asymptotic properties (usually simple whenever they are known) and finite sample properties (which are usually unknown). We also often give asymptotic expansions, in order to indicate how rapidly the asymptotic properties become true.

In general, we stress the various concepts of optimality. The justification for this is not only that this is the only way for a classical statistician to choose between different procedures, but also that experimental physicists handle ever increasing amounts of data, and therefore need increasingly optimal methods. However, there is an "optimal optimality", because the last bit of optimality can often be achieved only at great cost. This introduces the aspect of economy, which we try to stress on many occasions.

Facing the controversy between Bayesians and Anti-Bayesians ("classical" statisticians), we tend to favour the classical approach (because of professional bias), however keeping the reader partly informed about the Bayesian approach throughout. This attitude we justify as follows. In Chapter 6 we show how taking a decision from a limited amount of information leads to a fundamental indeterminacy: any decision depends on *a priori* assumptions. These assumptions being largely subjective by nature, we think that it is not the role of an experimenter to take decisions. His aim should be to summarize the results of his experiment for the rest of the physics community in such a way as to convey a maximum of information about the unknowns measured. In a certain sense this leaves to the general consensus the task to take decisions.

This is our motivation to the Information theory approach to Estimation. Logically, Test theory should then be Bayesian (since testing really is a decision). Our excuse for not being Bayesian in Test theory (Chapters 10, 11) is that physicists, as a matter of general practice, consider a confidence level as an objective measure of the "distance" of the experiment from the hypothesis tested.

A minor consequence of our professional bias is that in contrast to most (if not all) books on probability and statistics, we avoid using examples from gambling. Physicists often find it frustrating trying to convert such examples into physics; therefore, our examples are taken from physics (mainly high-energy physics). The theory is, of course, the same and gamblers should not be discouraged from converting our examples back into card games, dice, etc!

Let us finally point out that we do not discuss numerical optimization techniques, very important e.g. in the methods of maximum likelihood and least squares. The reasons are that there exist in our opinion excellent treatises of optimization, should the experimenter want to know the details of optimum-searching algorithms, and most physicists do have powerful optimization programs at their disposal (e.g. in the CERN Computer Program Library), which save them one more worry.

W. T. Eadie
D. Drijard
F. E. James
M. Roos
B. Sadoulet
December 1970, Geneva

CONTENTS

Chapter 1

INTRODUCTION

1.1. Outline

The subject of the following ten chapters can be divided into two main parts:

- Theory of Probability: Chapters 2–4
- Statistics: Chapters 5–11

The theory of probability is needed only to provide the necessary tools for statistics, which forms the main body of the course.

Chapters 5 and 6 define two general approaches to the choice of estimators: the *information* approach and the *decision theory* approach. The former consists essentially in maximizing the amount of information in the estimate, whereas the latter is based on minimizing the loss involved in making the wrong decision about the parameter value. In the limit of large data samples, the two approaches are equivalent, but where they differ we will try to point out the distinction.

Estimation of parameters is divided into three chapters, 7 and 8 dealing with point estimation, theory and practice, and 9 dealing with interval estimation. Tests of hypotheses are divided into general testing, Chapter 10, and goodness-of-fit tests, Chapter 11.

Our reference policy is as follows. We quote literature when we have omitted the proof of an important result, or when we want to give hints for further