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GENERAL MECHANICS

TRANSLATED FROM THE SECOND FRENCH EDITION

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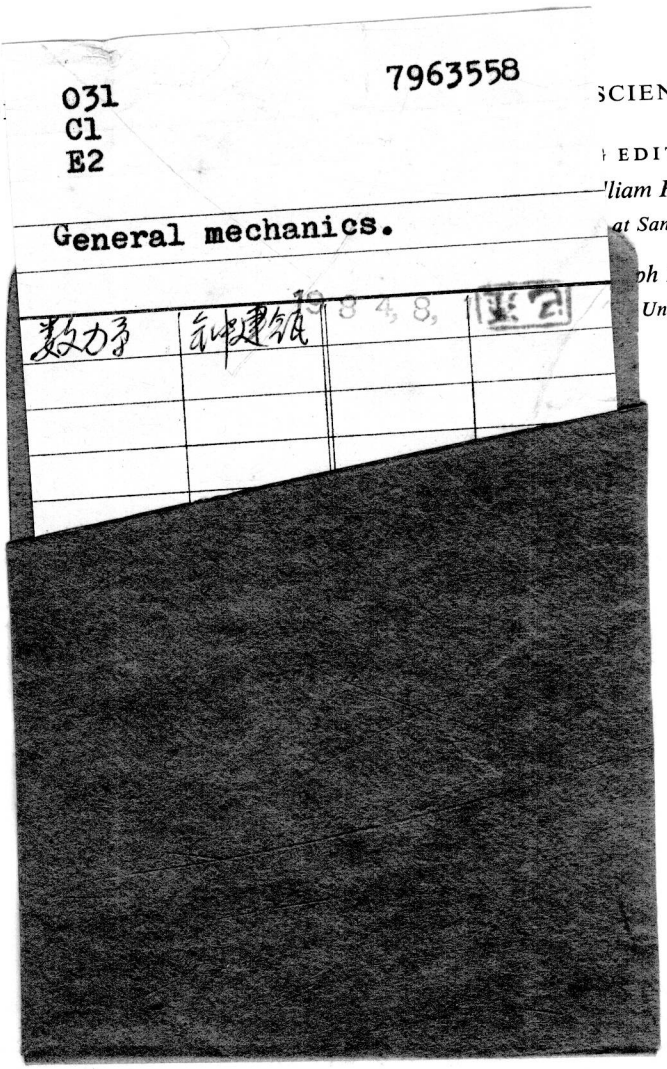

To the memory of my father

Translator's Preface

It is with great pleasure that I have been able to bring to fruition the translation of Professor Cabannes' authoritative text. Throughout the project I enjoyed the complete cooperation and understanding of Professor Cabannes, to whom I owe an immense debt of gratitude. His cheerful acceptance of my suggestions for minor changes of notation as well as his nearly instantaneous response to my questions were most effective in preventing delays. During the copyediting of the manuscript it was my good fortune to have the company of Professor Cabannes, who was then visiting Brown University.

I sincerely believe that readers of this English-language edition will find the book a welcome addition to the family of texts in mechanics. The point of view, especially in the presentation of the governing principles of dynamics, is thorough as well as refreshingly original. Professor Cabannes makes consistent use of vector and tensor notation with the result that he is able to discuss many interesting applications in a fairly compact volume. In particular, I appreciated the rigor with which he develops the theory of the gyroscope (Section 9.1.4) and his discussions of certain problems related to astronomy, such as an elementary theory of the tides (Section 3.3.3) and the precession of the Earth's equinoxes (Section 9.3.2). This book is further distinguished by the presence of two chapters which introduce the mechanics of fluids, a field where much current research is in progress. I am sure that many workers in this field will find it convenient to refer to these chapters, one expounding the microscopic aspect, the other the macroscopic aspect, for a quick review of the mathematical foundations of fluid mechanics. The author's skill in composing such a concise discourse on this subject reveals the fact that this is the domain where he has made many outstanding research contributions. I would also like to mention that this text contains about 200 problems, a majority of which are inspired by practical

engineering situations. The problems which comprise the third part of the book are presented with solutions.

In closing, I would like to express my appreciation to my colleague Professor J. Kestin for many helpful suggestions, to Mr. W. Z. Sadeh for assistance with the figures, and to Mrs. M. Craven who typed the manuscript. Finally, I must acknowledge the forbearance of my wife and children, who lacked husband and father during the innumerable hours I devoted to this task.

S. P. S.

Preface

Mechanics is concerned with the study of motion. A course in general mechanics is an exposition of those ideas and principles which serve as the point of departure for this book. The principles are formulated from experience. By means of these principles, real phenomena may be explained and predicted. Mechanics is thus an experimental science.

To go from hypotheses to conclusions it is necessary to use mathematical concepts founded on abstract axioms. Certain of these axioms are verified with great precision by natural occurrences; these we admit as rigorously true and then utilize for the solution of concrete problems. In this sense, mechanics constitutes an example of the application of mathematics.

The analysis of a problem in mechanics can be divided into five stages:

1. A physical system becomes the object of our curiosity.
2. We conceive a mathematical model of the system.
3. We analyze the mathematical model and reach certain conclusions.
4. The conclusions are interpreted in physical terms.
5. The results are compared with experiment.

The second stage consists in constructing a simple mathematical model and is often a delicate matter. The real system is indeed very complex and we have to distinguish in this system between those properties which are fundamental and those which are secondary. Common sense resulting from centuries of experience guides us in making this distinction. The concepts of point mass, rigid body, force vector, all of which are mathematical abstractions, were developed for the purposes of this stage.

The third stage involves solving the differential equations associated with the mathematical model. Indeed, this stage will be considered completed when, for

the model being considered, the coordinates of each element have been expressed as functions of time and the parameters which define the initial positions and velocities. At present, electronic computing machines make it possible to solve these equations as precisely as one could wish. However, a numerical solution does not reveal the mathematical structure of the problem, nor does a collection of numerical solutions. This structure is very important to know and can be learned only by an examination of the differential equations themselves.

The fourth stage is a return to the concrete problem, and the fifth stage belongs to the domain of experimental physics. It is always important to know if the conclusions of a mathematical theory do or do not agree with experiment.

The first two chapters of this book, Kinematics and Kinetics, develop the essential preliminary ideas. The three-dimensional Euclidean space having been taken as the model for physical space, kinematics is the study of motion, independently of the causes producing it; it is based on the concepts of length, defined as the Euclidean distance, and time. Kinetics introduces subsequently the concept of mass. The fundamental law of classical mechanics and its immediate corollaries are presented in the third chapter. With a view toward synthesis, I have made a single statement which is valid both for motions with bounded accelerations and motions with velocity discontinuities. The definition of *sidereal time* has been based on the Earth's rotation and that of *ephemeris time* has been based on its translation. Universal time, according to which our daily life is regulated, is based on the rotation and translation of the Earth. The chronology and frames of reference which may be considered as absolute are defined; a concise examination of the principles of special relativity reveals limitations in the concepts of absolute space and time.

Some classical problems are solved in the fourth chapter. The concept of a constraint is specified exactly at that point. This concept, which is not absolutely essential, is presented with a view toward concrete applications.

Chapter 5 is devoted to the notions of work and power and is organized in a manner which needs some explanation. To neglect internal stresses in the mechanics of the rigid body is certainly a simplification, but a source of difficulty as well. As a result, certain demonstrations, such as that of the theorem of virtual power, lose clarity. To avoid this trouble, I felt obliged to introduce certain notions from the mechanics of continuous media, such as the representation of internal mechanical actions by a tensor field and the definition of the work done by these actions. It is precisely the vanishing of this work for every rigid-body displacement which allows us to disregard the internal stresses in rigid-body mechanics. Beghin's convention, which amounts to considering only those virtual velocity fields which rigidify an imaginary, but arbitrary fragmentation of a material system, then allows us to deduce from the virtual-power theorem the maximum amount of information possible within the framework

of rigid-body mechanics. The theorem of virtual power is of fundamental importance in analytical mechanics.

Preceding the study of analytical mechanics, Chapter 6 presents a review of the properties of differential equations. In particular, linear systems, elliptic functions, and oscillations are studied. The role of the equation $\ddot{x} + f(x) = 0$, which governs free, undamped oscillations, is a central one. Chapter 7, Analytical Mechanics, is confined to a study of Lagrange's equations and of two applications: the stability of equilibria and small displacements. Liapunov's theorem, presented without proof, defines exactly the limitations of the linearization procedures applied to the equations.

Chapter 8 treats the mechanics of the point mass or particle and contains some developments which are useful in theoretical physics. Motions under the action of a central force are discussed in detail for the most important force fields and an entire section is devoted to the problem of two interacting particles. The motion of a charged particle in a magnetic field is studied and then applied to the theories of the electron microscope and the polar auroras. The last chapter of Part I deals with the mechanics of the rigid body. There I present a rigorous theory of the gyroscope and indicate some industrial applications of this device. Certain subjects related to astronomy, such as the gyroscopic compass, the precession of the equinoxes, and the time equation, are also discussed.

The subjects treated in the first nine chapters, which have been collected under the title of General Mechanics, constitute only one segment of what is usually called Classical Mechanics. I do not pretend to have given here an encyclopedic account, but I do believe that I have treated the essentials. I have deliberately bypassed particular questions which would have their rightful place in more specialized treatises, such as the canonical equations and the mechanics of filaments. At the level for which this text is intended, the teaching of mechanics, I believe, will remain largely classical in nature. Efforts toward modernization bear mainly on the presentation and the applications given. Consequently, students may wonder what room is left for fundamental research in mechanics. The second part of this book aims at answering this question.

Part II consists of two introductory chapters on the mechanics of fluids. In each chapter, I endeavor to attack problems in their modern form and to expose the reader to some current topics of research. In the first chapter, the distribution of matter is assumed discrete. The number of particles being very large, the methods of statistical mechanics are applied. In the second chapter, the distribution of matter is assumed continuous. The equations of motion are then set down for the most general case of a medium which is viscous and thermally and electrically conducting.

In the elementary presentation adopted in this book, I make no claim to originality. My purpose is to give assistance to students and practitioners of engineering and physics by presenting well-known problems clearly and precisely.

The most outstanding of my predecessors are cited in the bibliographies at the end of each of the two main parts. Each chapter closes with a series of exercises; in many cases I give either the answer to the question posed or some hints on the solution. A few problems taken from examinations for the French "license" are given, with solutions, in Part III of the text.

The editing of a book is always a thankless task. Unlike other writers, a professor has the opportunity to put the manuscripts which evolve from his course notes to the test of oral presentation and thus to improve them with the help of his students. The first edition of this book was the result of six years of teaching. When the oral lectures take printed form and are distributed more widely, the timid reactions of the students are augmented by the remarks and highly pertinent suggestions of the professor's colleagues. This second edition owes much to the professors who teach general mechanics in the Faculty of Sciences and the Schools for Engineers. To all those who have participated thus in the second edition, I wish to express my sincere gratitude. I especially want to thank my colleague and friend, Paul Germain, who helped me to sharpen the presentation of the theorem of virtual power. I wish also to acknowledge the help of Messrs. Cartan, Casal, Gatignol, Roseau, and Thibault.

Finally, and above all, I want to thank my colleague and friend, Professor S. P. Sutera, Professor of Engineering at Brown University, for the devotion and competence with which he has assumed the heavy burden of translating the French text into English and thereby making possible the American edition. In the course of his work he detected and corrected several printing errors and a few inaccuracies, with the result that this translation unquestionably surpasses the original.

HENRI CABANNES

PART ONE

General Mechanics

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