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E. N. Economou

Green's Functions in Quantum Physics

Third Edition

Springer

Eleftherios N. Economou

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With 60 Figures

Springer

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Preface to the Third Edition

In this third edition the book has been expanded in three directions:

1. Problems have been added at the end of each chapter (40% of which are solved in the last section of the book) together with suggestions for further reading. Furthermore, the number of appendices (marked with a grey stripe) has been substantially enlarged in order to make the book more self-sufficient. These additions, together with many clarifications in the text, render the book more suitable as a companion in a course on Green's functions and their applications.
2. The impressive developments of the 1980s and 1990s in mesoscopic physics, and in particular in transport properties, found their way – to a certain extent – in the new Chaps. 8 and 9 (which also contain some of the material of the old Chap. 7). This is a natural expansion, since Green's functions have played an important role as a theoretical tool in this new field of physics, a role that continues in nanoregime research (see, e.g., recent publications dealing with carbon nanotubes). Thus, the powerful and unifying formalism of Green's functions finds applications not only in standard physics subjects such as perturbation and scattering theory, bound-state formation, etc., but also at the forefront of current and, most likely, future developments.
3. Over the last 15 years or so Green's functions have found applications not only in condensed matter electronic motion but in classical wave propagation in both periodic and random media; photonic and phononic crystals are the outcomes of this line of research whose underlying basic theoretical principles are summarized in Sect. 7.2.4.

I would like to thank Ms. Mina Papadakis and Dr. Stamatis Stamatiadis whose help was invaluable during the writing and typesetting of this drastically revised third edition of my book.

Heraklion, Crete, March 2005 *E. N. Economou*

Preface to the Second Edition

In this edition, the second and main part of the book has been considerably expanded so as to cover important applications of the formalism of Green's functions.

In Chap. 5 a section was added outlining the extensive role of the tight-binding (or, equivalently, the linear combination of atomiclike orbitals) approach to many branches of solid-state physics. Some additional information (including a table of numerical values) regarding square and cubic lattice Green's functions were incorporated.

In Chap. 6 the difficult subjects of superconductivity and the Kondo effect are examined employing an appealingly simple connection to the question of the existence of a bound state in a very shallow potential well. The existence of such a bound state depends entirely on the form of the unperturbed density of states near the end of the spectrum: if the density of states blows up, there is always at least one bound state. If the density of states approaches zero continuously, a critical depth (and/or width) of the well must be reached in order to have a bound state. The borderline case of a finite discontinuity (which is very important to superconductivity and the Kondo effect) always produces a bound state with an exponentially small binding energy.

Chapter 7 has been expanded to cover details of the new and fast-developing field of wave propagation in disordered media. The coherent potential approximation (a simple but powerful method) is presented with an extensive list of references to the current literature. Then the electrical conductivity is examined both because it is an interesting quantity in its own right and because it plays a central role in demonstrating how disorder can create a qualitatively different behavior. Since the publication of the first edition of this book, significant advances in the field of random media have taken place. An effort has been made to present in a simple way the essential points of these advances (for the reader with a casual interest in this subject) and to review the current literature (for the benefit of the reader whose research activities are or will be related to the field of disordered systems).

In this edition, each chapter is preceded by a short outline of the material to be covered and concluded by a summary containing the most important equations numbered as in the main text.

I would like to thank A. Andriotis and A. Fertis for pointing out to me several misprints in the first edition. I would also like to express my gratitude to Exxon Research and Engineering Company for its hospitality during the final stages of this work.

Heraklion, Crete, January 1983 *E. N. Economou*

Preface to the First Edition

This text grew out of a series of lectures addressed to solid-state experimentalists and students beginning their research career in solid-state physics.

The first part, consisting of Chaps. 1 and 2, is a rather extensive mathematical introduction that covers material related to Green's functions usually included in a graduate course on mathematical physics. Emphasis is given to those topics that are important in quantum physics. On the other hand, little attention is given to the important question of determining the Green's functions associated with boundary conditions on surfaces at finite distances from the source. The second and main part of the book is, in my opinion, the first attempt at integrating, in a systematic but concise way, various topics of quantum physics, where Green's functions (as defined in Part I) can be successfully applied. Chapter 3 is a direct application of the formalism developed in Part I. In Chap. 4 the perturbation theory for Green's functions is presented and applied to scattering and to the question of bound-state formation. Next, the Green's functions for the so-called tight-binding Hamiltonian (TBH) are calculated. The TBH is of central importance for solid-state physics because it is the simplest example of wave propagation in periodic structures. It is also important for quantum physics in general because it is rich in physical phenomena (e.g., negative effective mass, creation of a bound state by a repulsive perturbation) and, at the same time, simple in its mathematical treatment. Thus one can derive simple, exact expressions for scattering cross sections and for bound and resonance levels. The multiple scattering formalism is presented within the framework of the TBH and applied to questions related to the behavior of disordered systems (such as amorphous semiconductors). The material of Part II is of interest not only to solid-state physicists but to students in a graduate-level course in quantum mechanics (or scattering theory) as well.

In Part III, with the help of the second quantization formalism, many-body Green's functions are introduced and utilized in extracting physical information about interacting many-particle systems. Many excellent books have been devoted to the material of Part III (e.g., Fetter and Walecka: Quantum Theory of Many-Particle Systems [20]). Thus the present treatment must be viewed as a brief introduction to the subject; this introduction may help the solid-state theorist approach the existing thorough treatments of the subject and the solid-state experimentalist become acquainted with the formalism.

I would like to thank the "Demokritos" Nuclear Research Center and the Greek Atomic Energy Commission for their hospitality during the writing of the second half of this book.

Athens, Greece, November 1978 *E. N. Economou*

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Contents

Part I Green's Functions in Mathematical Physics

Part II Green's Functions in One-Body Quantum Problems