MICROELECTRONICS AND SIGNAL PROCESSING

APPLICATIONS OF WALSH AND RELATED FUNCTIONS

With an Introduction to Sequency Theory

K.G. BEAUCHAMP

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MICROELECTRONICS AND SIGNAL PROCESSING

Series editors: **P. G. Farrell,** University of Manchester, U.K.
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The topic of microelectronics can no longer be treated in isolation from its prime application in the processing of all types of information-bearing signals. The relative importance of various processing functions will determine the future course of developments in microelectronics. Many signal processing concepts, from data manipulation to more mathematical operations such as correlation, convolution and Fourier transformation, are now readily realizable in microelectronic form. This new series aims to satisfy a demand for comprehensive and immediately useful volumes linking the microelectronic technology and its applications.

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In preparation

3. *Secure Speech Communications:* H. J. BEKER and F. C. PIPER

Preface

Some years ago the writer had occasion to survey the range and feasibility for computer applications of Walsh functions. This later formed the material of a book published in 1975 in Academic Press's Techniques of Physics series under the title "Walsh Functions and Their Applications". Now, almost a decade later, it is pertinent to consider the subject area again and to record the progress and development that have taken place in the intervening period.

A number of these developments have been quite extensive, and in two areas, Boolean logic analysis and non-sinusoidal communication, substantial research fields have opened up which enable new techniques to be applied to the solution of problems previously inviolate to other methods of attack.

An overall impression of the work of the preceding decade is that of a broadening in the range of applicability together with a wider availability of different orthogonal transformations relevant to signal processing and communications. Not only are the several alternative orderings of the Walsh function seen to provide their own particular solution to a varied range of problems, but the specific features of Haar, slant, hybrid, cosine, sine and block series are being recognised as part of a set of processing tools now available to match against the characteristics of the problem. The availability of fast transformation algorithms for all of these functions and the use of the microprocessor as a system-processing component are two of the reasons for these developments.

It is not possible in a book of modest size to consider in any depth all the very many applications of Walsh and related functions that have been demonstrated throughout the physical sciences in recent years. Some quite important applications are described at a fairly elementary level simply because to do otherwise would result in the exclusion of several other equally important subjects. Others are omitted, not because of their lack of relevance but because their description would entail a mathematical treatment too extensive to be pursued in a book concerned with an overall view. One example lies in the use of sequency functions for systems and control, and another in the valuable contributions that have been made to dyadic theory and logic analysis in various countries during the past decade.

Instead a broad treatment of the main lines of development has been attempted, with the emphasis on understanding the principles involved. It is hoped that sufficient detail is included together with comparison of alternative methods so that the reader can assess the relevance to his own problems. To this end a considerable list of references is given at the end of each chapter and a list of further references and bibliographies, arranged by subject matter, is included at the end of this book.

The book consists of two parts. The first takes the form of a tutorial in sequency theory (Chapters 1–3) and gives the background essential for understanding the applications part which follows. Chapter 4 forms a bridging 'hardware' chapter between the earlier theoretical chapters and the application chapters (5–8).

Chapters 5 and 6 are concerned with signal processing in one and two dimensions. In this latter area the pace of development has increased in recent years, with sequency methods playing a significant role. It is likely that in the newer areas of robotic vision and satellite surveillance significant further progress will be made.

Chapter 7 describes applications in communications, and it is interesting to note the considerable progress that has been made recently in non-sinusoidal communications and radar, which has led to several commercial devices now becoming available.

Finally, Chapter 8 attempts to summarise another quite new field for analysis which is of considerable importance in the design and testing of integrated logic systems and is already producing significant practical results.

Selection and assembly of material for this book have relied considerably on the help given by very many people and organisations.

The writer would like to express his particular thanks for the assistance given by Professor H. Harmuth of the Catholic University of America and by Dr. S. Hurst of the University of Bath. Appreciation is also expressed to the following who have contributed in various ways to this book: Professor P. Besslich of Bremen University, Dr. B. Durgen of the University of Vermont, Dr. W. Chen of the U.S. Army Topographical Laboratory, Dr. M. Hussain of Kuwait, Professor M. Karpovsky of Boston University, Professor R. Kitai of McMaster University, Mr. C. Nicol of British Telecomm, Professor R. Redinbo of Rensselaer Polytechnic Institute and Dr. C. Yuen of the University of Hong Kong. Finally, acknowledgement is extended to the Royal Society for travel support and to the Institution of Electrical Engineers for library assistance.

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Abbreviations and Symbols

A–D	Analog-to-digital
CAD	Computer-aided design
CAL	Directly symmetrical Walsh function
CCD	Charge-coupled device
CMOS	Complementary metal–oxide–silicon
cos	Cosine
CT	Cosine transform
C–T	Cooley–Tukey
D–A	Digital-to-analog
DCT	Discrete cosine transform
DFT	Discrete Fourier transform
DHT	Discrete Haar transform
DMA	Direct memory access
DPCM	Differential pulse-coded modulation
DSM	Digital sequency multiplex
DST	Discrete sine transform
DWT	Discrete Walsh transform
ECD	Electrocardiograph
ECL	Emitter-coupled logic
EEG	Electroencephalograph
EPROM	Erasable programmable read-only memory
exp	Exponential
FCT	Fast cosine transform
FDM	Frequency division multiplex

FFT	Fast Fourier transform
FHT	Fast Haar transform
FST	Fast slant transform
FT	Fourier transform
FWT	Fast Walsh transform
HAD	Hadamard-ordered Walsh function
HAR	Haar function
HAW	Hadamard–Walsh function
HT	Haar transform
Hz	Hertz (cycles per second)
KLT	Karhunen–Loève transform
LSI	Large-scale integration
MSE	Mean-square error
MUX	Multiplexer
PAL	Paley-ordered Walsh function
PCM	Pulse-coded modulation
pel	Picture element
PLA	Programmable logic array
PROM	Programmable read-only memory
PSF	Point-spread function
RAD	Rademacher function
RAM	Random access memory
R–M	Reed–Muller
ROM	Read-only memory
SAL	Inversely symmetrical Walsh function
SAW	Surface acoustic wave
SDM	Sequency division multiplex
sin	Sine
SLA	Slant function
SLT	Slant transform
ST	Sine transform
TDM	Time division multiplex
VLSI	Very large-scale integration
WAL	Sequency-ordered Walsh function
WHT	Walsh–Hadamard transform
WT	Walsh transform
Zps	Sequency (zero crossings per second)
a_n	Fourier spectral coefficient
$\mathbf{A}$	Unitary matrix
b	Binary digit
b_n	Fourier spectral coefficient
B	Bandwidth
$\mathbf{BW}$	Besslich Rademacher–Walsh matrix

c	Phase velocity, velocity of light (3×10^8 m/s)
C	Channel capacity (bits per second)
$\mathbf{C}$	Covariance matrix
$C(i, t)$	A set of orthogonal signal carriers
$CT(f)$	Discrete cosine transform
$\mathbf{D}$	Diagonal matrix
e	2.71828
f	Frequency
f_c	Clock frequency
$f(t)$	Function of t
$f(x)$	Boolean logic function
$\underline{F}$	As $F(x)$ with logic values 0 and 1 replaced by $+1$ and -1
$\bar{F}$	As $F(x)$ with logic values 0 and 1 replaced by -1 and $+1$
$F(\omega)$	Filter response
$F(x)$	Binary vertices of a truth table
$\mathbf{F}$	Fourier transform matrix
$\mathbf{G}$	Filter weights matrix
h_k	Weighting coefficient
$\mathbf{H}$	Hadamard matrix
$\mathbf{Ha}$	Haar matrix
i	Current; series coefficient
$i(x, y)$	Input image spatial domain
$i(\omega)$	Input image frequency domain
$\mathbf{I}_m$	Identity matrix
$\mathrm{Im}(k)$	Imaginary value of k
j	$\sqrt{-1}$; series coefficient
k	Channel, constant
K	Kernel, filter weight, constant
m	Minterms of a logic function
n	Ordering number
N	Number of terms equalling 2^p
$o(x, y)$	Output image in spatial domain
$o(\omega)$	Output image in frequency domain
p	$\mathrm{Log}_2\, N$
$P_{aw}(k)$	Averaged Walsh power spectral coefficient
$P_F(k)$	Fourier power spectral coefficient
$P_H(k)$	Haar power spectral coefficient
P_N	Noise power
P_S	Signal power
$P_W(k)$	Walsh power spectral coefficient
$\mathbf{P}$	Permutation matrix
r	Rademacher–Walsh series
R_i	Rademacher–Walsh spectral coefficients

$\mathrm{Re}(k)$	Real value of k
$R_F(\tau)$	Correlation coefficient in real time
$R_W(\tau)$	Correlation coefficient in dyadic time
$\mathbf{RW}$	Rademacher–Walsh matrix
s_i	Discrete signal
$\hat{s}_i$	Estimated discrete signal
s_{ij}	Two-dimensional discrete signal
$S(t)$	An orthogonal series
$ST(f)$	Discrete sine transform
t, T	Time
T	Threshold
v	Velocity (meters per second)
$\mathbf{W}$	Walsh matrix
x_i	Sampled function of time; Boolean logic value
$\bar{x}_i$	Complemented Boolean logic value
$x_i(b)$	Binary function of time
$\mathbf{x}_{ij}$	Two-dimensional image matrix
$x(t)$	Continuous function of time
$X_c(k)$	CAL transform coefficient for x_i
$X_{ct}(n)$	Cosine transform coefficient for x_i
X_n	Transformed value of x_i
$X_{m,n}$	Transformed value of x_{ij}
$X_s(k)$	SAL transform coefficient for x_i
$X_{st}(k)$	Sine transform coefficient for x_i
$Z(\tau)$	Logical convolution
d, Δ	Increment
β	Angle
ϵ	Dielectric constant
η	Relative bandwidth
θ	Angle; normalised time (t/T)
λ	Wavelength
μ	Magnetic permeability
π	3.14159
σ	Conductivity
τ	Time delay
$\tau(\omega)$	Frequency transfer function
ϕ	Angle
ω	$2\pi f$, angular frequency
$\circledast$	Dyadic convolution operator
$\oplus$	Modulo-2 addition
$\otimes$	Kronecker product
$\leftrightarrow$	Transform operator

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Part One

Theory and Practice