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Freerk A. Lootsma

FUZZY LOGIC FOR PLANNING AND DECISION MAKING

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by

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PREFACE

This volume starts with the basic concepts of Fuzzy Logic: the membership function, the intersection and the union of fuzzy sets, fuzzy numbers, and the extension principle underlying the algorithmic operations. Several chapters are devoted to applications of Fuzzy Logic in various branches of Operations Research: PERT planning with uncertain activity durations, SMART and the AHP for Multi-Criteria Decision Analysis (MCDA) with vague preferential statements, ELECTRE using the ideas of the AHP and SMART, and Multi-Objective Optimization (MOO) with weighted degrees of satisfaction. Finally, earlier studies of colour perception illustrate the attempts to find a physiological basis for the set-theoretical and the algorithmic operations in Fuzzy Logic. The last chapter also discusses some key issues in linguistic categorization and the prospects of Fuzzy Logic as a multi-disciplinary research activity.

I am greatly indebted to the Department of Mechanical Engineering and Applied Mechanics, College of Engineering, University of Michigan, Ann Arbor, for the splendid opportunity to start the actual work on this book during my sabbatical leave from Delft (1993 - 1994); to LAMSADE, Université de Paris-Dauphine, where many ideas emerged during two winter visits (1989, 1990); to the International Institute for Applied Systems Analysis, Laxenburg, Austria, where I got further inspiration during a number of summer visits (1992, 1995, and 1996); and to the NISSAN Foundation in The Netherlands who enabled me to visit several Japanese universities (June 1996). Moreover, I gratefully acknowledge the stimulating support given by many colleagues in the International Society on Multi-Criteria Decision Making and in the European Working Group "Aide Multicritère à la Décision". To a large extent, the research has eventually been carried out as an integral part of the "Committee Beek" project TWI 90-06 "Multi-Criteria Decision Analysis and Multi-Objective Optimization" of the Delft University of Technology. Last, but not least, I happily remember my early experiences with network planning in the Operations Research Group of Philips Research Laboratories (1963 - 1970). Confronted with the inadequacy of probability theory in planning techniques, I much later felt a particular affinity with Fuzzy Logic as soon as it emerged in the literature. For a long period, however, I devoted my time and energy to crisp optimization.

On a more personal level I would like to thank many people in chronological order. In the early eighties, Manfred Grauer (IIASA, Laxenburg, Austria, now Universität-GH Siegen, Germany) cooperated with me to supervise the studies of Matthijs Kok (Delft University of Technology, now HKV Lijn in Water, Lelystad) who tenaciously worked on MOO with applications in the energy sector. Roger Cooke (Delft University of Technology), my nearest colleague in Operations Research, sharpened my views on Fuzzy Logic by his compelling devotion to probabilistic models. In the early phases of their career Peter van Laarhoven (Eindhoven University of Technology), Wytold Pedrycz (University of Manitoba, Winnipeg), and Guus Boender (Erasmus University, Rotterdam) explored with me the theory of Fuzzy Logic and its potential in MCDA. Leo Rog (Delft University of Technology) contributed significantly to the "Committee-Beek" project by the development of the REMBRANDT program for MCDA. Bernard Roy (LAMSADE, Université de Paris-Dauphine) gave me several opportunities to work in his laboratory and to develop ideas which unfortunately diverge from his views on MCDA. Joanne Linnerooth-Bayer (IIASA, Laxenburg, Austria) focussed my attention on the nature of environmental policies and brought up many aspects which are not yet fully treated in the present book. Josée Hulshof (Ministry of Health, Welfare, and Sport, The Hague) was the most successful project manager which I ever found when I participated in Operations Research projects. Pieter Bots (Delft University of Technology) appeared to be a very effective and supportive facilitator in that project, and Karien van Gennip (Delft University of Technology, now McKinsey & Company, Amsterdam) a student with an unusual feeling for administrative problems. With Jonathan Barzilai (School of Computer Science, Technical University of Nova Scotia, Halifax, Canada) I share the experience that it is hazardous to question the fundamentals of the established schools in MCDA. Panos Papalambros (MEAM, University of Michigan, Ann Arbor) was my host who generously gave me the opportunity to work on MCDA and MOO during my sabbatical leave in his Department. Rob van den Honert (University of Cape Town, South-Africa) and R. Ramanathan (Indira Gandhi Institute of Development Research, Bombay, India) spent long periods in Delft as Research Fellows in order to cooperate with me on group decision making and on issues of fairness and equity. Finally, I gratefully mention the discussions with Paul Hekkert (Delft University of Technology) about the judgement of art, as well as the advice given by Geert Booij (Free University, Amsterdam) about linguistic categorization.

I dedicate this book to my wife Erica and to my children Joanne, Auke, and Roger, who all tenderly tolerated the absent-mindedness of their husband and father.

Delft, April 1997.

CONTENTS

CHAPTER 1

INTRODUCTION

Decision making under uncertainty is as old as mankind. Even in the Antiquity some people suspected that uncertainty could be modelled via chance mechanisms. When the Roman consul Ceasar crossed the river Rubicon (49 BC) --- the first move in a hazardous attempt to defeat his rival Pompeius --- he spoke the famous words: "*Alea iacta est*". Indeed, the die had been cast, but what were the possible outcomes of his risky action: a rapid victory for Pompeius, a rapid victory for himself, a drawn-out civil war, or a peaceful settlement after some skirmishing? And did Ceasar subjectively assess the outcome probabilities before he decided to move?

1.1 TYPES OF UNCERTAINTY

Probability theory is a well-established mathematical theory now, designed to model precisely described, repetitive experiments with observable but uncertain outcomes. In the last few decades other types of uncertainty have been identified, however, and new mathematical tools are accordingly under study, in attempts to deal with situations which are not or cannot be covered by the classical tools of probability theory. The key notion is that uncertainty is a *matter of degree*. Thus, events occur with a particular degree of likelihood, elements have properties with a particular degree of truth, and actions can be carried out with a particular degree of ease. Roughly speaking, we distinguish the following types of uncertainty.

Randomness occurs when a precisely described experiment such as casting a die on a flat table has several possible outcomes, each with known probability (a perfect die with a homogenuous mass distribution) or with unknown probability (an inhomogenuous die). The outcomes of the experiment (the faces 1, 2, ..., 6) can unambiguously be observed. The experiment of casting the die can arbitrarily be repeated. Further experimentation will reduce the uncertainty: it will reveal the probability distribution of the outcomes of the die. **Probability theory** is concerned with the uncertainty of whether the respective outcomes will occur or not, that is, with their degree of likelihood.

Subjective probability theory models the degree of belief of a rational individual with respect to precisely defined, observable, but not necessarily repetitive phenomena. When a powerful technical component with a low failure probability must be tested, frequent experimentation and observation are, not conceptually but physically and/or economically, impossible. One may then ask experts to express their degree of belief in the component by an estimated failure probability or an estimated life-time distribution (Cooke, 1991).

Vagueness or imprecision arises as soon as the outcome of the experiment cannot properly be observed. A typical example is given by the situation arising after the experiment of casting a die with coloured faces, under twilight circumstances where colours cannot properly be distinguished. There are several possible outcomes, each with a particular degree of truth. Further experimentation will not reduce the uncertainty. Colour perception (blue-green in various gradations, orange as a simultaneous perception of red and yellow, etc.) illustrates that vagueness or imprecision may be due to the manner in which our neural system operates (Kay and McDaniel, 1978). In several languages there are special word endings to express vagueness or imprecision, as Table 1.1 illustrates. Note that the vagueness may be acceptable in our daily conversation. Professional work is more demanding, however. Thus, colours are carefully categorized and these categories are labelled with a so-called colour code (see the colour number of your car). For meteorological services a stormy wind has wind force 8 on the Beaufort scale (wind velocity in the category of 61 - 72 m/sec) between hard wind and storm with the respective wind forces 7 (50 - 60 m/sec) and 9 (73 - 85 m/sec).

Table 1.1 *Terms which suggest degrees of truth in several languages.*

English	French	German	Dutch
reddish	rougeâtre	rötlich	roodachtig
bleuish	bleuâtre	bläulich	blauwachtig
rainy	pluvieux	regnerisch	regenachtig
stormy	orageux	stürmisch	stormachtig
misty	brumeux	neblich	mistig

The mathematical theory of **fuzzy sets** (Zadeh, 1965), alternatively referred to as **fuzzy logic**, is concerned with the degree of truth that the outcome belongs to a particular category, not with the degree of likelihood that the outcome will be observed.

Ambiguity arises when a verbal statement has a number of distinct meanings so that only the context may clarify what the speaker really wants to say. Hot food may be warm or spicy. A statement like "*the bridge is open*" does not specify whether the draw-bridge is open for road traffic or for shipping.

Possibility theory models the degree of ease for a particular event to occur. The literature offers several examples to show the potential applications of possibility theory in actual decision making (Zimmermann, 1996). The physical ability of a given person to eat x eggs, for instance, where x stands for a positive integer, can be modelled via a possibility distribution over the integer values x = 1, 2, ... The distribution function equals 1 at x = 0, and decreases monotonically to zero when x increases. A high possibility does not imply a high likelihood, but if an event is practically impossible it is also unlikely to occur (Dubois and Prade, 1988). Thus, the physical ability of a given person to eat x eggs is an upper bound for the probability that he will eat x eggs at a given moment. The passability of a road in a swampy area can be expressed via a possibility distribution over the weights of cars and lorries. With a small and light car the driver can easily muddle through, but with a heavy lorry he/she may easily get stuck. Similarly, the vulnerability of a river dike can be modelled via a possibility distribution over the water levels in the river. The dike can more or less easily be damaged by high waves (that depends, for each water level, on a combination of factors such as the direction and the force of the wind and the duration of the flood so far) whereafter it may collapse. Note that human beings seem to have an open eye for possibilities. The Romance languages have an astonishing amount of words ending on -able or -ible to suggest that particular events can more or less easily happen: human beings are more or less vulnerable, objects are more or less breakable, roads more or less passable, methods more or less workable or feasible, etc. The Germanic languages have similar words ending on -bar or -baar to suggest that there are various degrees of easiness. Some examples are given in Table 1.2.

Table 1.2 *Terms which suggest degrees of easiness in various languages.*

English	French	German	Dutch
breakable	cassable	brechbar	breekbaar
feasible	faisable	ausführbar	uitvoerbaar
passable	praticable	begehbar	begaanbaar
vulnerable	vulnérable	verletzbar	kwetsbaar

Risk is not a particular type of uncertainty but rather a mixture. Even under good weather and road conditions, an experiment like driving a car has several possible outcomes, some of them very serious but very unlikely. The outcomes (safe and timely arrival, death immediately or shortly after an accident, invalidity in various gradations) cannot precisely be classified into a small number of categories so that it is also difficult to specify their probabilities.

In the above overview we started from a type of uncertainty that can empirically be analyzed (simulation) and whereby predictions can be made about future outcomes (such as the breakdown of components). Uncertainty is not necessarily related to the future, however. In medical diagnosis, for instance, we can use the conditional probabilities of the symptoms to decide what the most likely disease of the patient now is or what the most likely cause of his/her disease was. Note that the symptoms may be vaguely described.

1.2 VAGUENESS AND FUZZINESS

Fuzzy-set theory is a mathematical theory designed to model the vagueness or imprecision of human cognitive processes. Ever since the pioneering work of Zadeh (1965) it has been heavily criticized because it has no well-established mathematical or empirical methods to model graded human judgement. Thus, it could be a mathematical edifice constructed in order to manipulate numbers that have no basis in the psychological reality (Smithson, 1987). On the other hand, fuzzy logic provides appropriate models for the ability of human beings to categorize things, not by verifying whether they satisfy some unambiguous definitions, but by comparing them with prototypical (characteristic) examples of the categories in question. The class of birds is not crisply defined by abstract rules, but vaguely described by typical examples. Human beings can easily decide to which degree certain animals (robins, eagles, crows, oistriches, bats) are typical for the class of birds or to which degree certain tools (guns, knives, clubs, whips, scissors) are typical for the class of weapons. Such a mode of operation seems to be workable in human communication. Similarly, most chairs have four legs, but in the course of time the designers came up with many variations. This is rarely embarrassing for human observers. They can easily agree that some pieces of furniture are more typical for the class of chairs than others.

Potentially, fuzzy-set theory is an important branch of Operations Research, providing tools to quantify imprecise verbal statements and to classify outcomes of decision-analytical experiments. Usually, when decisions are prepared a considerable amount of imprecise information with a quantitative connotation is transmitted via natural language. Well-known examples are the frequency indicators like: almost never, rarely, sometimes, often, mostly, and almost always. They are meaningful albeit in a particular context only. Since decisions are invariably made within a given context, graded judgement should also be considered within a particular framework. The question of whether a given man is short, somewhat taller, rather tall, tall, or very tall depends on the situation: do we compare him with healthy West-European adults grown up in a fourty years period of peace and affluence, or with adults in an African country disrupted by war and famine? Mutual understanding of what the context is seems to be possible by common experience and education of human beings. Mutual understanding of the gradations of human judgement could also be due to the physiological information-processing system of human beings.

We note in passing that the examples of the previous paragraph reveal a linguistic mechanism to code the gradations of human judgement. The qualifying terms like almost, rather, somewhat,...., the so-called hedges, enable us to express degrees of truth in situations where a black-or-white statement would be inadequate.

Classification or categorization in general and quantification of verbal judgement are opposite activities. The calculated probability of an accident, for instance, does not immediately tell us whether the proposed action is perfectly safe, reasonably safe, somewhat risky, hazardous, or reckless, although these words, not the numbers themselves, convey the message which is required for actual decision making. The opposite of such a classification is that we assign (within a given context) probabilities to concepts such as perfectly safe, reasonably safe, etc.

Although fuzzy-set theory has been criticized for being probability theory in disguise, it is easy to understand now that the two theories are concerned with two distinct phenomena: with observations that can be classified in vaguely described (imprecise) categories only, and with experiments such that the outcomes can be classified into well-defined (crisp) categories. Let us illustrate this in a somewhat different manner: in the evening twilight a die with coloured faces is fuzzy, even if it lies on the table. Probability emerges as soon as we carry out the experiment of casting the die. In essence, fuzzy-set theory is concerned with our ability to categorize things and to label the categories via natural language which, despite its vagueness, lubricates human cooperation. Note how trade unions can frustrate cooperation by working-to-rule actions.

1.3 PHILOSOPHICAL ISSUES

The almost ideological debate between the adepts of probability theory and fuzzy-set theory reveals that the conflict has deep roots. Indeed, the fact that fuzzy-set theory models degrees of truth leads to a confrontation with our scientific tradition. Fuzzy logic agrees that an element may with a positive degree of truth belong to a set and with another positive degree of truth to the complement of the set, whereby it violates the law of non-contradiction (a statement cannot be true and not-true at the same time). Fuzzy logic also violates the law of the excluded middle (a statement is either true or false, *"tertium non datur"*). And indeed, the world around us is not a world of black-and-white. It is full of gray shades, most things have a degree of blackness and a degree of whiteness. The reason why we have so long accepted the mismatch between the black-and-white world of science and the shaded world of our common experiences is that we usually round off (Kosko, 1994). The mismatch can also be found in our laws. Is an 18 years old person a child or an adult? The law gives an unambiguous answer, which is nevertheless applicable in a limited number of cases only. Even older people may be somewhat childish, however, and younger people may be highly mature. Note that probability theory never challenged the traditional bi-valent logic. It has its roots in gambling, where the rules and the outcomes are unambiguous, and it is still valid under these casino conditions only.

Fuzzy logic has no difficulties with century-old paradoxes such as the problem of the heap of sand. If we remove one grain of sand, it is still a heap. If we remove a second grain, it is still a heap, etc. At which stage, however, does the heap lose the property of being a heap? There is no crisp threshold between a heap and a non-heap. Our classical, bi-valent logic which is based on the notion that statements are either true or false has unsurmountable difficulties here. Fuzzy logic, however, assigns a degree of truth or a truth value $t(n)$ to the statement that n grains of sand constitute a heap. The sequence of truth values converges to 0 when n goes to 0, and that solves the paradox.

There is another interesting paradox, Russell's barber, which shows what the truth value $^1/_2$ can actually mean. The barber shop sign says that the barber shaves a man in the town if, and only if, he does not shave himself. So, who shaves the barber? If he shaves himself, then by definition he does not, but if he does not shave himself, then by definition he does. So, he does and he does not. Gaines (1983) proposed to interpret this paradox as follows. Let S be the statement that the barber shaves himself, and not-S that he does not. Then, since S implies not-S, and not-S implies S, the two statements are logically equivalent, and they should accordingly have the same truth values. Hence,

$$t(S) = t(\text{not-}S) = 1 - t(S),$$

which yields $t(S) = {}^1/_2$. This is the midpoint of the so-called truth interval [0, 1]. Rounding off is impossible and paradox occurs (Kosko, 1994).

1.4 FUZZY CONTROL

Fuzzy logic, the name of which now appears on Japanese cameras, washing machines, refrigerators, and other domestic appliances, seems to have a promising future in the design of control mechanisms. The first really exciting application of fuzzy logic was realized in 1987, when the Sendai railway started its operations. Sendai, a Japanese city of 800,000 inhabitants in Northern Honshu, has an advanced subway system (McNeill and Freiberger, 1993). On a single North-South route of 13.6 km and 16 stations, the train glides smoother than any other train because of its sophisticated control system. So, fuzzy logic did not come of age at universities (Kosko, 1994) but in industry and in the commercial market. The debate between fuzzy logic and probability theory will not be solvėd by theoretical arguments but by the successes in industrial design, development, production, and sales.

Why would control systems benefit so much from fuzzy logic? The simple reason is that fuzzy controllers follow the example of the human controller who categorizes his/her observations (the speed is rather high, rather low, etc.) whereafter he/she issues vague commands to the system under control (slow down, or accelerate slightly, etc.). A fuzzy air conditioner, for instance, employs a number of rules of the form

if temperature is cold **then** motor speed must be fast;

if temperature is just right **then** motor speed must be medium;

etc. The system obviously checks to which of the categories "cold", "cool", "just right", "warm", or "hot" the temperature belongs, whereafter the motor speed is properly adjusted if it does not sufficiently belong to the required category "stop", "slow", "medium", "fast", or "blast" (Kosko, 1994). The temperature in this example is alternatively referred to as a linguistic variable which can only assume a verbally defined "value". In general, a control system is characterized by how it transforms input quantities into output quantities. An intelligent control system emits appropriate problem-solving responses when it is faced with problem stimuli which are usually imprecise. Moreover, an intelligent system learns from past experience, it generalizes from a limited number of experiences which are mostly imprecise, and it creates new input-output relationships. The processing of imprecise information is typically the domain of fuzzy logic.

1.5 SCOPE OF THE PRESENT VOLUME

Since we want to concentrate on model formulation with fuzzy logic in the domain of planning and decision analysis we only devote a limited attention to the basic concepts. Chapter 2 is concerned with the membership function of a fuzzy set, the union and the intersection of fuzzy sets, the definition of fuzzy numbers, the arithmetic operations on fuzzy numbers, and the extension principle. For a more extensive treatment of the basic concepts we may refer the reader to Fodor and Roubens (1994), for instance.

Chapter 3 illustrates how effective the transition from a probabilistic to a fuzzy model may be. We shall extensively be dealing with network planning, one of the oldest and most successful tools of Operations Research. Originally designed to plan the development of the Polaris missile of the US Navy (1958), the Project Evaluation and Review Technique (PERT) enabled the users to process uncertain activity durations via a probabilistic model: the beta distribution. In many projects, however, the activity durations are uncertain, not only because of random events such as rainfall during the construction of buildings and motorways, but also because the concept of completion is not precisely defined. Thus, new activities may start even before the preceding ones are completely finished. The fuzzy network-planning model, with fuzzy numbers to model the uncertain durations, may therefore be more acceptable in practice than the original, probabilistic PERT.