THE MANY-BODY PROBLEM

An Encyclopedia of Exactly Solved Models in One Dimension

edited by

Daniel C Mattis

Department of Physics
University of Utah
USA

World Scientific
Singapore • New Jersey • London • Hong Kong

Published by

World Scientific Publishing Co. Pte. Ltd.

P O Box 128, Farrer Road, Singapore 9128

USA office: Suite 1B, 1060 Main Street, River Edge, NJ 07661

UK office: 73 Lynton Mead, Totteridge, London N20 8DH

The author and publisher would like to thank the following publishers of the various journals and books for their assistance and permission to include the selected reprints found in this volume:

Academic Press (*Ann. Phys.*)
American Association of Physics Teachers (*Amer. J. Phys.*)
American Inst. of Physics (*J. Math. Phys., J. Appl. Phys., J. Chem. Phys., Phys. Fluids*)
The American Physical Society (*Phys. Rev., Rev. Modern Phys., Phys. Rev. Lett.*)
Cambridge Univ. Press (*Proc. Camb. Phil. Soc.*)
Elsevier Science Publishers (*Physica, Phys. Lett., Nuclear Phys., Phys. Rep.*)
John Wiley & Sons, Inc. (*Comm. Pure Appl. Math.*)
Inst. of Physics (*J. Phys.: Condensed Matter, Proc. Phys. Soc. (London), J. Phys. C: Solid State Phys.*)
Physical Soc. of Japan (*J. Phys. Soc. Japan*)
Prog. Theor. Phys. (*Kyoto*)
Rocky Mountain Math. Consortium (*Rocky Mountain J. Math.*)
The Royal Soc. (*Proc. Roy. Soc. (London)*)

THE MANY-BODY PROBLEM
An Encyclopedia of Exactly Solved Models in One Dimension

ISBN 981-02-0975-4

Printed in Singapore.

The Many-Body Problem

Preface

This book deals with exactly solved models in the many-body physics of one spatial dimension (1D). One of the first, if not *the* first many-body problem in all of modern physics to have found an exact solution, is the magnetic chain solved by Bethe's *Ansatz* in 1931. This showed the way to solve dozens of *other* physical models.

Bethe's highly detailed paper (in a new English translation) is one of over 80 original and review papers which are reprinted in their entirety in the present book, as are the 1955 Fermi, Pasta, Ulam report on paradoxical dynamics of anharmonic chains, Lieb and Wu's solution of the Hubbard model in 1968, the 1979 discovery of fractional charge in organic polymers by Su, Schrieffer and Heeger, and a number of other seminal publications in one-dimensional physics which make up an encyclopedic tour of this special world.

It is, of course, almost impossible to separate out a second dimension. If we analyze the time-dependence of some model, *time* provides a second dimension, or if we solve for the ground state of the anisotropic Heisenberg Hamiltonian in 1D, we are in effect calculating the ground state of the transfer matrix of some corresponding system in two *spatial* dimensions, and thereby the free energy of a two-dimensional model. But there is more, beneath the surface.

A number of many-body problems in *three* dimensions can be accurately modeled in 1D. For example, in chapter 4 we mention the problem of the anomalous shape of the X-Ray absorption threshhold in metals, and the Kondo problem, as important topics in three-dimensional physics which have been solved by specialized one-dimensional methods. It is also significant that Lanczos' method for the reduction of arbitrary Hermitean matrices to tridiagonal form is functionally equivalent to the reduction of arbitrary Hamiltonians to one-dimensional chains with on-site and nearest-neighbor interactions;[1] this reduction has been useful in the study of surface normal modes, random lattices, etc. Thus, while most of the techniques espoused in the various chapters of this book are peculiarly one-dimensional and apply to polymers or other one-dimensional systems, yet a significant number of applications are to the wide world of physics.

[1] See D. C. Mattis, "How to reduce practically any problem to one dimension," in *Physics in One Dimension*, eds. J. Bernasconi and T. Schneider, Springer-Verlag, Berlin, New York, 1981.

The organization into chapters, and some of the textual material and reprints are taken from an earlier book, *Mathematical Physics in One Dimension*, by E. H. Lieb and D. C. Mattis, Academic Press, New York, 1965, which has been out-of-print and unavailable for some time. The new book differs from its predecessor in a number of important ways. Classic discoveries which once had to be omitted owing to lack of space—such as the unpublished "report" by Fermi, Pasta and Ulam on lack of ergodicity of the linear chain, or Bethe's original ansatz paper—can now be incorporated thanks to improved book-binding technology. Many applications which did not even exist in 1965 (some of which were in fact spawned by the publication of Lieb & Mattis) can now be included. Among these, we have surveyed a number of important developments such as:

—classical *and* quantum theory of the electron plasma,
—the exact solution of the Hubbard model,
—the concept of spinons,
—the Haldane gap in magnetic spin-one chains,
—bosonization and fermionization, and intermediate statistics,
—solitons and the approach to thermodynamic equilibrium,
—the attempts at exact *quantum* statistical mechanics of interacting particles,
—localization of normal modes and eigenstates in disordered chains, etc.

The book has been centered about the limited number of *exactly soluble mathematical models of the many-body problem.* An annotated Bibliography is arranged with a sense of historical continuity, and separated into the various topics. It will serve to organize a vast literature which is not usually repertoried. As such it is intended not just as a guide and reference to the classic early literature, but also as an indicator of contemporary trends. Explanatory material in each chapter and a number of didactic review articles on selected topics ($s = 1$ chains, solitons, etc.) should help make the reprints accessible to graduate students and budding professionals in applied mathematics, in theoretical physics, and in allied fields. The idea is to highlight the main thrusts of original research, situate them in an historical context, and explain obscure techniques in cases where textbooks are presently unavailable.

The chapters are organized according to subject matter: Statistical Mechanics, Chains of Random and/or Anharmonic Oscillators, Electron Energy Bands, The Many-Fermion and The Many-Boson Problems, Magnetism, and finally, Time-Dependent Phenomena and Approach to Thermodynamic Equilibrium. But several common topics run from one chapter to the next: solitons are introduced in chapter 2, but their principal applications are in chapter 7. The random chain is studied in chapter 2 (lattice vibrations) and chapter 3 (electron energy bands in disordered alloys). Bethe's ansatz is germane to chapters 4, 5, and 6, although Bethe's paper best fits chapter 6 where it appears. "Bosonization" or "fermionization" are two of several topics overlapping chapters 4 and 6.

The choices of all the topics and papers new since 1965 have been dictated by the concerns and interests of the present author, who assumes sole responsibility for any and all errors in interpretation or selection. The original, successful, Lieb and Mattis book was a tough act to follow. Here, the principal new emphasis is on the many-body problem as defined by its condensed-matter applications, and despite some overlap, field theory and pure mathematics are therefore given short shrift. While I have tried to reprint a number of *important* discoveries in condensed matter theory, lack of space frequently allowed only a few of the subsequent publications based on these seminal works to be reproduced. (The Bibliography should be useful in tracing the others.)

For example, the reader will find among the reprinted works a new translation of Bethe's original paper (the original is in difficult German, oft-quoted but little read!), its application to the bose gas by Lieb and Liniger, the extension by Yang and Yang to the bose gas at finite temperature, and also the exact solution of the one-dimensional Hubbard model by Lieb and Wu, using Bethe's method. But we omit the large number of "exact" large U expansions of the solutions to the Hubbard model which have been published. The latest of these are listed in the Bibliography, and the reader who consults them will (hopefully!) find in them references to earlier efforts.

I have incorporated brand new developments, such as "Haldane's gap" in the energy spectrum of integer spin antiferromagnets. In this instance because of the fog of controversy still surrounding it, several additional articles which touch on this topic are reprinted including a masterful review of this field by Affleck.

The successful solutions of the Kondo problem originating with Andrei and independently, with Wiegmann (each based on slightly different interpretations of Bethe's ansatz, as applied to a one-dimensional reduction of the original three-dimensional problem) presented a special case. It is a fascinating topic which is nevertheless only briefly touched upon in chapter 4 because two easily obtainable, recent, lengthy and comprehensive review articles by Andrei, Wiegmann, and their collaborators, have made inclusion of this topic in the present volume redundant.

In nonlinear dynamics, I have included the original, unpublished, report by Fermi, Pasta and Ulam (with invaluable help by their computer programmer, Ms. M. Tsingou) on the unexpectedly non-ergodic behavior of a chain of anharmonic oscillators. Using the theory of solitons, it has become possible to resolve their paradox. A review article by Joseph Ford ties it all together. Toda's invention of the lattice which bears his name and its eventual solution by Hénon and Flaschka using the methods of Gardner, Greene, Kruskal and Miura are all included. Less well known are the other nonlinear dynamical systems introduced by Calogero and by Sutherland, and their extensions to quantum mechanics, which we have also reproduced.

New additions to the collection of reprints include studies of short-range order and electrical conductivity in disordered 1D metals. The Bibliography will also provide newcomers an entry into some recent trends in this field.

I sincerely hope that a new generation of readers will approve the updated contents and format and will find this book useful and fun to read.

Daniel C. Mattis
Salt Lake City, Utah
July, 1992

Acknowledgements

I wish to express profound gratitude to a friend and colleague, Prof. Elliott H. Lieb of Princeton University, for his gracious permission to reprint his papers and to recycle some of the material which we developed jointly many years ago. I regret that his numerous other commitments prevented him from participation in this new venture. Thanks are due to the editors and staff of World Scientific Publishing Co., especially to Dr. K. K. Phua and Ms H. M. Ho, for their helpfulness and extraordinary support in this complex project. Many colleagues have helped me personally or by correspondence, including Drs. W. J. L. Buyers, J. Carmelo, J. Ford, J. D. Johnson, A. Lenard, B. S. Shastry, B. Sutherland and F. Wilczek. I thank them each and all. To the authors and publishers of the reprinted articles, I express gratitude for their permission to reproduce their papers, thus helping make the one-dimensional world a better workplace.

Note to the Readers

Each chapter is followed by a Bibliography. Works preceded by an asterisk (*) are also listed in the main Table of Contents, and are reprinted in full following the introductory material to each chapter.

The detailed Table of Contents, which lists the topics in each chapter and the titles and authors of the reprinted articles, is designed to take the place of a separate author or subject index.

Contents