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M. M. Postnikov

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Riemannian Geometry

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《国外数学名著系列》(影印版)序

要使我国的数学事业更好地发展起来，需要数学家淡泊名利并付出更艰苦地努力。另一方面，我们也要从客观上为数学家创造更有利的发展数学事业的外部环境，这主要是加强对数学事业的支持与投资力度，使数学家有较好的工作与生活条件，其中也包括改善与加强数学的出版工作。

从出版方面来讲，除了较好较快地出版我们自己的成果外，引进国外的先进出版物无疑也是十分重要与必不可少的。从数学来说，施普林格(Springer)出版社至今仍然是世界上最具权威的出版社。科学出版社影印一批他们出版的好的新书，使我国广大数学家能以较低的价格购买，特别是在边远地区工作的数学家能普遍见到这些书，无疑是对推动我国数学的科研与教学十分有益的事。

这次科学出版社购买了版权，一次影印了 23 本施普林格出版社出版的数学书，就是一件好事，也是值得继续做下去的事情。大体上分一下，这 23 本书中，包括基础数学书 5 本，应用数学书 6 本与计算数学书 12 本，其中有些书也具有交叉性质。 这些书都是很新的，2000 年以后出版的占绝大部分，共计 16 本，其余的也是 1990 年以后出版的。这些书可以使读者较快地了解数学某方面的前沿，例如基础数学中的数论、代数与拓扑三本，都是由该领域大数学家编著的“数学百科全书”的分册。对从事这方面研究的数学家了解该领域的前沿与全貌很有帮助。按照学科的特点，基础数学类的书以“经典”为主，应用和计算数学类的书以“前沿”为主。这些书的作者多数是国际知名的大数学家，例如《拓扑学》一书的作者诺维科夫是俄罗斯科学院的院士，曾获“菲尔兹奖”和“沃尔夫数学奖”。这些大数学家的著作无疑将会对我国的科研人员起到非常好的指导作用。

当然，23 本书只能涵盖数学的一部分，所以，这项工作还应该继续做下去。更进一步，有些读者面较广的好书还应该翻译成中文出版，使之有更大的读者群。

总之，我对科学出版社影印施普林格出版社的部分数学著作这一举措表示热烈的支持，并盼望这一工作取得更大的成绩。

王　元

2005 年 12 月 3 日

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Preface

The original Russian edition of this book is the fifth in my series "Lectures on Geometry." Therefore, to make the presentation relatively independent and self-contained in the English translation, I have added supplementary chapters in a special addendum (Chaps. 30–36), in which the necessary facts from manifold theory and vector bundle theory are briefly summarized without proofs as a rule.

In the original edition, the book is divided not into chapters but into lectures. This is explained by its origin as classroom lectures that I gave. The principal distinction between chapters and lectures is that the material of each chapter should be complete to a certain extent and the length of chapters can differ, while, in contrast, all lectures should be approximately the same in length and the topic of any lecture can change suddenly in the middle. For the series "Encyclopedia of Mathematical Sciences," the origin of a book has no significance, and the name "chapter" is more usual. Therefore, the name of subdivisions was changed in the translation, although no structural surgery was performed. I have also added a brief bibliography, which was absent in the original edition.

The first ten chapters are devoted to the geometry of affine connection spaces. In the first chapter, I present the main properties of geodesics in these spaces. Chapter 2 is devoted to the formalism of covariant derivatives, torsion tensor, and curvature tensor. The major part of Chap. 3 is devoted to the geometry of submanifolds of affine connection spaces (Gauss–Weingarten formulas, etc.).

In Chap. 4, Cartan structural equations in polar coordinates are deduced. The second half of this chapter is devoted to locally symmetric affine connection spaces. Globally symmetric spaces are considered in Chap. 5 (and the beginning of Chap. 6). In particular, their coincidence with symmetric space in the Loos sense is proved. In the major part of Chap. 6, the general theory is illustrated by examining Lie groups. In Chap. 7, the language of categories and functors is explained (this material is set in a smaller font), and also the main theorems on the relation between Lie groups and Lie algebras are presented in essence without proofs. In Chaps. 8 and 9, these theorems are generalized to the case of symmetric spaces; in Chap. 10, they are generalized to the case of finite-dimensional Lie algebras of vector fields.

Chapters 13 and 14 are mainly devoted to the theory of elementary surfaces. The main focus is on their isothermal coordinates and minimal surfaces. In Chap. 15, the main properties of the curvature tensor are established. The main topic of Chap. 16 is the Gauss–Bonnet theorem. In Chap. 17, its generalizations to Riemannian spaces of large dimension are presented without proof. In the same chapter, the Ricci tensor of a Riemannian space is considered, and Einstein spaces are introduced.

Chapter 18 is devoted to conformal transformations of a metric. The main focus is on the case where $n = 2$. In the first half of Chap. 19, isometries and Killing fields are considered; the rest of this chapter is devoted to the specialization of constructions in Chap. 3 to the case of submanifolds of a Riemannian space. In Chap. 20, certain specific classes of submanifolds (locally symmetric and compact ones) are considered, and consideration of the theory of hypersurfaces is started; all of Chap. 21 is devoted to this topic.

Chapters 22 and 23 are devoted to spaces of constant curvature, Chap. 24 is devoted to four-dimensional Riemannian spaces, and Chaps. 25 and 26 are devoted to invariant metrics on Lie groups. Chapter 27 is devoted to the Jacobi theory of the second variation, and the last two chapters are devoted to its applications (in particular, the Mayers theorem and the Cartan–Hadamard theorem are proved). Chapter 29 concludes with the proof of the Bochner theorem on the finiteness of the isometry group of a compact Riemannian space with a negative-definite Ricci tensor (the general topological theorem on the compactness of the isometry group of an arbitrary compact metric space, which is needed for this proof, is also proved).

M. M. Postnikov Moscow, January 2000

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