OLIVER M. O'REILLY

Intermediate Dynamics for Engineers

A Unified Treatment of Newton–Euler and Lagrangian Mechanics

CAMBRIDGE

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A UNIFIED TREATMENT OF NEWTON–EULER AND LAGRANGIAN MECHANICS

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INTERMEDIATE DYNAMICS FOR ENGINEERS

This book has sufficient material for two full-length semester courses in intermediate engineering dynamics. For the first course a Newton–Euler approach is used, followed by a Lagrangian approach in the second. Using some ideas from differential geometry, the equivalence of these two approaches is illuminated throughout the text. In addition, this book contains comprehensive treatments of the kinematics and dynamics of particles and rigid bodies. The subject matter is illuminated by numerous highly structured examples and exercises featuring a wide range of applications and numerical simulations.

Oliver M. O'Reilly is a professor of mechanical engineering at the University of California, Berkeley. His research interests lie in continuum mechanics and nonlinear dynamics, specifically in the dynamics of rigid bodies and particles, Cosserat and directed continuua, dynamics of rods, history of mechanics, and vehicle dynamics. O'Reilly is the author of more than 50 archival publications and *Engineering Dynamics: A Primer*. He is also the recipient of the University of California at Berkeley's Distinguished Teaching Award and three departmental teaching awards.

This book is dedicated to my adventurous daughter, Anna

Preface

The writing of this book started more than a decade ago when I was first given the assignment of teaching two courses on rigid body dynamics. One of these courses featured Lagrange's equations of motion, and the other featured the Newton–Euler equations. I had long struggled to resolve these two approaches to formulating the equations of motion of mechanical systems. Luckily, at this time, one of my colleagues, Jim Casey, was examining the elegant works [205, 207, 208] of Synge and his co-workers on this topic. There, he found a partial resolution to the equivalence of the Lagrangian and Newton–Euler approaches. He then went further and showed how the governing equations for a rigid body formulated by use of both approaches were equivalent [27, 28]. Shades of this result could be seen in an earlier work by Greenwood [79], but Casey's work established the equivalence in an unequivocal fashion. As is evident from this book, I subsequently adapted and expanded on Casey's treatment in my courses. My treatment of dynamics presented in this book is also heavily influenced by the texts of Papastavridis [169] and Rosenberg [182]. It has also benefited from my graduate studies in dynamical systems at Cornell in the late 1980s. There, under the guidance of Philip Holmes, Frank Moon, Richard Rand, and Andy Ruina, I was shown how the equations governing the motion of (often simple) mechanical systems featuring particles and rigid bodies could display surprisingly rich behavior.

There are several manners in which this book differs from a traditional text on engineering dynamics. First, I demonstrate explicitly how the equations of motion obtained by using Lagrange's equations and the Newton–Euler equations are equivalent. To achieve this, my discussion of geometry and curvilinear coordinates is far more detailed than is normally found in textbooks at this level. The second difference is that I use tensors extensively when discussing the rotation of a rigid body. Here, I am following related developments in continuum mechanics, and I believe that this enables a far clearer derivation of many of the fundamental results in the kinematics of rigid bodies.

I have distributed as many examples as possible throughout this book and have attempted to cite up-to-date references to them and related systems as far as feasible. However, I have not approached the exhaustive treatments by Papastavridis

[169] nor its classical counterpart by Routh [184, 185]. I hope that sufficient citations to these and several other wonderful texts on dynamics have been placed throughout the text so that the interested reader has ample opportunity to explore this rewarding subject.

Using This Text

This book has been written so that it provides sufficient material for two full-length semester courses in engineering dynamics. As such it contains two tracks (which overlap in places). For the first course, in which a Newton–Euler approach is used, the following chapters can be covered:

1. Kinematics of a Particle (Section 1.5 can be omitted)
2. Kinetics of a Particle
   Appendix on Tensors
6. Rotation Tensors
7. Kinematics of Rigid Bodies
8. Constraints on and Potentials for Rigid Bodies
9. Kinetics of a Rigid Body
11. Multibody Systems

The second course, in which a Lagrangian approach is used, could be based on the following chapters:

1. Kinematics of a Particle
2. Kinetics of a Particle
3. Lagrange's Equations of Motion for a Single Particle
4. Lagrange's Equations of Motion for a System of Particles
5. Dynamics of Systems of Particles
   Appendix on Tensors
6. Rotation Tensors (with particular emphasis on Section 6.8)
7. Kinematics of Rigid Bodies
8. Constraints on and Potentials for Rigid Bodies
9. Kinetics of a Rigid Body
10. Lagrange's Equations of Motion for a Single Rigid Body
11. Multibody Systems

In discussing rotations for the second course, time constraints permit a detailed discussion of only the Euler angle parameterization of a rotation tensor from Chapter 6 and a brief mention of the examples on rigid body dynamics discussed in Chapter 9.

Most of the exercises at the end of each chapter are highly structured and are intended as a self-study aid. As I don't intend to publish or distribute a solutions manual, I have tailored the problems to provide answers that can be validated. Some of the exercises feature numerical simulations that can be performed with MATLAB or MATHEMATICA. Completing these exercises is invaluable both in terms of comprehending why obtaining a set of differential equations for a system is important and for visualizing the behavior of the system predicted by the model. I also strongly recommend semester projects for the students during which they can delve into a specific problem, such as the dynamics of a wobblestone, the flight of a Frisbee, or the reorientation of a dual-spin satellite, in considerable detail. In my courses, these projects feature simulations and animations and are usually performed by students working in pairs who start working together after 7 weeks of a 15-week semester.

Image Credit

The portrait of William R. Hamilton in Figure 4.6 in Subsection 4.11.3 is from the Royal Irish Academy in Dublin, Ireland. I am grateful to Pauric Dempsey, the Head of Communications and Public Affairs of this institution, for providing the image.

Acknowledgments

This book is based on my class notes and exercises for two courses on dynamics, ME170, ENGINEERING MECHANICS III, and ME175, INTERMEDIATE DYNAMICS, which I have taught at the Department of Mechanical Engineering at the University of California at Berkeley over the past decade. Some of the aims of these courses are to give senior undergraduate and first-year graduate students in mechanical engineering requisite skills in the area of dynamics of rigid bodies. The book is also intended to be a sequel to my book *Engineering Dynamics: A Primer*, which was published by Springer-Verlag in 2001.

I have been blessed with the insights and questions of many remarkable students and the help of several dedicated teaching assistants. Space precludes mention of all of these students and assistants, but it is nice to have the opportunity to acknowledge some of them here: Joshua P. Coaplen, Nur Adila Faruk Senan, David Gulick, Moneer Helu, Eva Kanso, Patch Kessler, Nathan Kinkaid, Todd Lauderdale, Henry Lopez, David Moody, Tom Nordenholz, Jeun Jye Ong, Sebastién Payen, Brian Spears, Philip J. Stephanou, Meng How Tan, Peter C. Varadi, and Stéphane Verguet. I am also grateful to Chet Vignes for his careful reading of an earlier draft of the book.

Many other scholars helped me with specific aspects of and topics in this book. Figure 9.1 was composed by Patch Kessler. Henry Lopez (B.E. 2006) helped me with the roller-coaster model and simulations of its equations of motion. Professor Chris Hall of Virginia Tech pointed out reference [118] on Lagrange's solution of a satellite dynamics problem. Professor Richard Montgomery of the University of California at Santa Cruz discussed the remarkable figure-eight solutions to the three-body problem with me, Professor Glen Niebur of the University of Notre Dame provided valuable references on Codman's paradox, Professor Harold Soodak of the City College of New York provided valuable comments on the tippe top, and Professors Donald Greenwood and John Papastavridis carefully read a penultimate draft of this book and generously provided many constructive comments and corrections for which I am most grateful.

Most of this book was written during the past 10 years at the University of California at Berkeley. The remarkable library of this institution has been an invaluable resource in my quest to distill more than 300 years of work on the subject matter in this book. I am most grateful to the library staff for their assistance and the taxpayers for their support of the University of California.

Throughout this book, several references to my own research on rigid body dynamics can be found. In addition to the students mentioned earlier, I have had the good fortune to work with Jim Casey and Arun Srinivasa on several aspects of the equations of motion for rigid bodies. The numerous citations to their works are a reflection of my gratitude to them.

This book would not have been published without the help and encouragement of Peter Gordon at Cambridge University Press and would contain far more errors were it not for the editorial help of Victoria Danahy. Despite the assistance of several other proofreaders, it is unavoidable that some typographical and technical errors have crept into this book, and they are my unpleasant responsibility alone. If you find some on your journey through these pages, I would be pleased if you could bring them to my attention.

Contents

PART ONE

DYNAMICS OF A SINGLE PARTICLE

1 Kinematics of a Particle

1.1 Introduction

One of the main goals of this book is to enable the reader to take a physical system, model it by using particles or rigid bodies, and then interpret the results of the model. For this to happen, the reader needs to be equipped with an array of tools and techniques, the cornerstone of which is to be able to precisely formulate the kinematics of a particle. Without this foundation in place, the future conclusions on which they are based either do not hold up or lack conviction.

Much of the material presented in this chapter will be repeatedly used throughout the book. We start the chapter with a discussion of coordinate systems for a particle moving in a three-dimensional space. This naturally leads us to a discussion of curvilinear coordinate systems. These systems encompass all of the familiar coordinate systems, and the material presented is useful in many other contexts. At the conclusion of our discussion of coordinate systems and its application to particle mechanics, you should be able to establish expressions for gradient and acceleration vectors in any coordinate system.

The other major topics of this chapter pertain to constraints on the motion of particles. In earlier dynamics courses, these topics are intimately related to judicious choices of coordinate systems to solve particle problems. For such problems, a constraint was usually imposed on the position vector of a particle. Here, we also discuss time-varying constraints on the velocity vector of the particle. Along with curvilinear coordinates, the topic of constraints is one most readers will not have seen before and for many they will hopefully constitute an interesting thread that winds its way through this book.

1.2 Reference Frames

To describe the kinematics of particles and rigid bodies, we presume on the existence of a space with a set of three mutually perpendicular axes that meet at a common point P. The set of axes and the point P constitute a reference frame. In Newtonian mechanics, we also assume the existence of an inertial reference frame. In this frame, the point P moves at a constant speed.

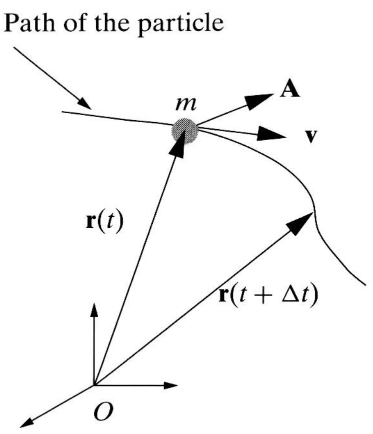

Figure 1.1. The path of a particle moving in $\mathbb{E}^3$. The position vector, velocity vector, and areal velocity vector of this particle at time t and the position vector of the particle at time $t + \Delta t$ are shown.

Depending on the application, it is often convenient to idealize the inertial reference frame. For example, for ballistics problems, the Earth's rotation and the translation of its center are ignored and one assumes that a point, say E, on the Earth's surface can be considered as fixed. The point E, along with three orthonormal vectors that are fixed to it (and the Earth), is then taken to approximate an inertial reference frame. This approximate inertial reference frame, however, is insufficient if we wish to explain the behavior of Foucault's famous pendulum experiment. In this experiment from 1851, Léon Foucault (1819–1868) ingeniously demonstrated the rotation of the Earth by using the motion of a pendulum.* To explain this experiment, it is sufficient to assume the existence of an inertial frame whose point P is at the fixed center of the rotating Earth and whose axes do not rotate with the Earth. As another example, when the motion of the Earth about the Sun is explained, it is standard to assume that the center S of the Sun is fixed and to choose P to be this point. The point S is then used to construct an inertial reference frame. Other applications in celestial mechanics might need to consider the location of the point P for the inertial reference frame as the center of mass of the solar system with the three fixed mutually perpendicular axes defined by use of certain fixed stars [80].

For the purposes of this text, we assume the existence of a fixed point O and a set of three mutually perpendicular axes that meet at this point (see Figure 1.1). The set of axes is chosen to be the basis vectors for a Cartesian coordinate system. Clearly, the axes and the point O are an inertial reference frame. The space that this reference frame occupies is a three-dimensional space. Vectors can be defined in this space, and an inner product for these vectors is easy to construct with the dot product. As such, we refer to this space as a three-dimensional Euclidean space and we denote it by $\mathbb{E}^3$.

* Discussions of his experiment and their interpretation can be found in [62, 138, 207]. Among his other contributions [215], Foucault is also credited with introducing the term "gyroscope."