Lectures by Chair Professors of Tsinghua University
清 华 大 学 讲 席 教 授 课 程 讲 义

Elements of Vorticity Aerodynamics

涡量空气动力学原理

J. C. Wu (吴镇远)

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To my wife

张美英

Mei Ying Wu

whose love and support made this work possible

出版说明

“清华大学讲席教授课程讲义”系列出版物

近年来，清华大学聘请了在各个领域取得卓越成就的、国际知名的专家、学者作为讲席教授，就本学科前沿课题为研究生开设系列讲座或专门课程。这些系列讲座和课程为科研人员和研究生接触科学大师、了解学科研究前沿提供了很好的机会。

这些系列讲座和课程不仅具有新颖性、前沿性和国际水准，而且一般自成体系，具有很好的系统性。由于这些课程大多是临时性开设，只开设一次或若干年才能开设一次，很多研究生没有机会参加。因此，为了进一步扩大其影响，使清华大学和其他高等学校、科研院所中没有能够参加这些系列讲座和课程，但又对这些系列讲座和课程感兴趣的科研人员和研究生了解讲座和课程内容；同时也为了帮助学生提高听课效率，更好地掌握讲座和课程知识，清华大学出版社组织出版本“清华大学讲席教授课程讲义”(Lectures by Chair Professors of Tsinghua University)系列出版物。

本套出版物的编著主题希望能够体现以下原则：

1. 涉及学科前沿的重要发展趋势和研究成果；
2. 反映交叉学科、前沿学科的最新进展；
3. 具有理论或(和)应用方面的重要价值。

为了有利于体现这些原则，本套出版物在编著过程中，在具有一定的系统性和可读性的前提下，可以有思维和逻辑的适度跳跃；同时，本套出版物拟采取循序渐进的“递进”式出版程序以及灵活的出版方式：

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清华大学出版社

PUBLISHER'S PREFACE

Lectures by Chair Professors of Tsinghua University

In recent years, Tsinghua University appointed prominent experts and scholars with distinguished accomplishments in various disciplines as chair professors to offer lecture series or specialized courses. The lectures and courses provided excellent opportunities for researchers and graduate students to contact international masters in sciences and to gain understanding of the world's frontiers of scientific research.

The lectures and courses are not only innovative and at the forefront of international scientific advancements, they typically form self-contained entities and are systematically presented. Because most of these lectures are specially offered as one-time events, or once in several years, many graduate students do not have the opportunity to attend. For this reason, Tsinghua University Press established the present series of publications, *Lectures by Chair Professors of Tsinghua University*, to expand their influence, permitting interested researchers and graduate students at Tsinghua and at other universities and research institutes, who are unable to attend, to understand the contents of the lectures and, at the same time, enhancing the participating students' comprehension of the knowledge being conveyed.

The themes of this set of publication series are anticipated to conform to the following principles:

1. Delineations of important trends and research discoveries at the frontiers of scientific disciplines;

2. Deliberations of modern advancements in leading edge and multi-disciplinary fields;

3. Communications of developments of major importance in theories and/or in applications.

To facilitate the realization of these principles, the present series of publications, subjects to readability and organizational prerequisites, allows suitable leaps in conceptions and logic and uses a sequential procedure in publishing and an adaptive publication format.

1. Sequential publishing procedure:

- The first step: Based on detailed synopses (or lecture notes), publish in the form of lecture notes *Lectures by Chair Professors of Tsinghua University* (paperback).

- The second step: After lecturing 2 ~ 3 times, with incremental supplements, revisions, and enhancements of contents, publish in the form of textbooks *Texts by Chair Professors of Tsinghua University* (hard cover).

2. Adaptive publication format

- Publications in print version, paper media, or simplified text accompanied by multi-media video disk;
- Traditional written explanations of fundamental concepts, principles and methodology, or PowerPoint slides with corresponding explanatory notes;
- Texts in either English or Chinese.

Tsinghua University Press

PREFACE

Helmholtz' ground breaking vortex theorems in the mid-nineteenth century provided the tools for the momentous discoveries in theoretical aerodynamics by Prandtl and others nearly a century ago. Since then, studies of vorticity dynamics have received continual impetus from diverse applications in engineering, physics and mathematics. A number of books on vortex theory and vortex dynamics addressing a broad range of topics of theoretical and practical interests are presently available. In comparison with these books, the present monograph has a narrower focus. It is aimed at sharing my understanding of theoretical aerodynamics with the reader interested in the classical circulation theory and in the role of modern vorticity dynamics in theoretical aerodynamics involving unsteady and non-streamlined flows.

My lectures in a short course at Tsinghua University and at the Second Biennial Retreat on Vorticity Aerodynamics, both scheduled for September of 2004 in Beijing, presented an ideal occasion for preparing my notes: an audience and a firm target date. As it turned out, however, my initial estimate of required time was overly optimistic. In the end, to meet target dates, certain compromises had to be made.

One major compromise is the omission of a chapter discussing vorticity-based flow computations. While computational aerodynamics is one of my favorite subjects, time limitations prevented the inclusion of this subject as a component of this monograph. Topics discussed in the present work, however, form the core of vorticity-based computation methods. Detailed discussions of these methods are available in some of the references quoted in Chapters 1 and 3.

Other compromises involve editorial issues such as curtailing redundancies and adding figures, exercises, and more sample problems. Chapters of my notes were prepared more or less as independent articles, each with its own themes, references and introductory discussions, intermittently over an extended period of time. Efforts to tie the chapters together and to implement the obviously desirable improvements were ultimately limited by available time.

The present monograph is based essentially on my lecture notes completed in August of 2004. It is my intent to prepare a *Version* 2.0 of the monograph in the reasonable future. It is my hope that my colleagues would kindly provide commentaries and critiques about this initial version. One issue of special concern during my preparation of the present version is the proper discussion of certain viewpoints and strategies about vorticity aerodynamics, acquired and used over the years in my research and teaching. These viewpoints and strategies are obviously not the only ones that work; they are by no means a panacea for all applications of vorticity dynamics. I am, however, convinced that they are consistent, rational, and very effective within the perimeters defined in this monograph. Advocating these viewpoints and strategies is not meant to underplay the merits of alternative viewpoints and strategies, especially the classical ones. In this regard, I wish to acknowledge the special help of Professor J. Z. Wu, who kindly reviewed my draft manuscripts on very short notices.

During my teaching and research career, I had the good fortune of associations with many brilliant and marvelous individuals — teachers, colleagues, and former students — who provided indispensable inspiration for my work. I wish to take this opportunity to express my gratitude for their contribution to my understanding of vorticity aerodynamics.

I would be remiss not to mention again the love and support of my wife Mei-Ying Wu, especially during the past year, as the preparation of my notes took up more and more, eventually virtually all, of the time at my disposal.

吴镇远
J. C. WU
September 2004

CONTENTS

Chapter 1 INTRODUCTION

1.1 Preliminaries

The science of aerodynamics is often defined as *the branch of dynamics which treats of the motion of air and other gases and of the forces acting on bodies in motion through air or on fixed bodies in a current of air (or other gases)* [Webster 1953]. The aerodynamicist, however, views his discipline in a more focused context as *an applied science that deals with forces produced by air or other fluids on bodies moving through it.* He recognizes that, as a branch of fluid dynamics, aerodynamics deals with the motion of fluids. Yet, in aerodynamics, studies of fluid motions do not represent ends by themselves. Rather, such studies are a means for evaluating the aerodynamic force. Historically, successes in theoretical aerodynamics were linked to discoveries that permit the prediction of the aerodynamic force without requiring the knowledge of complete flow details. The focal subject — the force — distinguishes the discipline of aerodynamics from all other branches of fluid dynamics.

The task of finding theoretical solutions to practical aerodynamic flow problems was (and still is) formidably difficult. Bypassing flow details as much as possible was indeed the only strategy open to the pioneering aerodynamicist. Together with simplifications of the inviscid-fluid assumption, this strategy served as the springboard for the dazzling developments of classical aerodynamics a century ago by leading European scholars: Kutta, Joukouski, Lanchester, Prandtl, among others. The power of the resulting theory of lift and induced drag, called the circulation theory today, is well documented.

Also well recognized are the vortex theorems of Helmholtz, Kelvin, Stokes and others that paved the way for the development the circulation theory. Less well acknowledged, especially outside the community of aerodynamicists, is the fact that the efficacy of the circulation theory is limited for the most part to steady and quasi-steady aerodynamics involving attached flows over streamlined airfoils and wings. For convenience, such attached flows are referred to as streamlined flows in the present study. Efforts to extend the circulation theory to general non-streamlined and fully unsteady flows are by no means straightforward. Attempts to estimate unsteady forces using the circulation theory at times lead to well-known musings such as: *It is impossible for the bumblebee to fly*. The fact of course is the bumblebee does fly, but it does not rely on streamlined steady forces that the aerodynamicist understands so well. There exists a vast unexplored and potentially fertile territory — in non-streamlined and/or fully unsteady realms — where the aerodynamicist's understanding of the principles of flight is woefully lacking.

With the circulation theory, the steady aerodynamic force is predictable with amazingly little information about flow details. For the two-dimensional flow, there is the Kutta-Joukouski theorem stating that the lift on an airfoil is equal to the product of the fluid density, the flight speed, and the circulation. This theorem can be proved in several different ways. For the three-dimensional flow, the legendary Ludwig Prandtl [1921] used a simple model — the horseshoe vortex system — to represent the vorticity field in the flow and to create the lifting-line theory. The fact that this simple model produces the powerful lifting-line theory for predicting both the lift and the induced drag never ceased to marvel the aerodynamicist, students and leading scholars alike. In a recent article, for example, Kroo [2001] discussed a *force-free wake* interpretation of why *Prandtl's model works well with a very poor representation of the wake shape*. Rather unexpected is a remark by Max Munk [1981], who made important contributions to the lifting-line theory, first as Prandtl's student in Germany and subsequently as a distinguished scientist at the National Advisory Committee for Aeronautics (NACA) in the U. S.: *My principal paper on induced drag was still under the spell of Prandtl's vortex theory. Everything that Prandtl said was correct, but it was not the right approach.*

Munk was over 90 when he made the above quoted remark. Munk neither elaborated nor proffered an alternative, perhaps more correct, approach to the lifting-line theory. Advances during the past century, however, made it timely to revisit classical theories of aerodynamics from the modern viewpoint of the dynamics of the vorticity field in the viscous fluid, rather than from the viewpoint of the vortex in the inviscid fluid. In this Chapter, the general parameters of a revisit, undertaken by this writer intermittently over the past three decades, are described. In subsequent chapters, certain outcomes of the revisit are reviewed, attributes of vorticity dynamics as applied to aerodynamics are summarized, new interpretations of the classical circulation theory are provided, and a framework for additional studies of non-streamlined unsteady aerodynamics is advocated.

The foundation of the present study, as in all theoretical studies of fluid dynamics, is the first principles of fluid dynamics: the laws of mass and energy conservation and Newton's second law of motion. Many familiar assumptions are employed to simplify the description of the flow. Assumptions are indispensable since the first principles are intractable without them. An assumption, of course, should not be admitted merely because it simplifies, but should be justifiable as an approximation of the physical reality being described. On this basis, the inviscid-fluid assumption is not adopted for the present study from the outset. This powerful assumption is abandoned not because inviscid results are thought to be less powerful than commonly believed. It is simply that this assumption can only be viewed as an idealization, not as an approximation.

The exclusion of the inviscid-fluid assumption, as it turns out, does not invalidate many useful inviscid theories. Instead, by excluding this assumption from the outset, advantages of treating the kinematic aspect of vorticity dynamics separately from the kinetic aspect are brought into focus. It then becomes evident that vorticity kinematics, unlike vorticity kinetics, is not linked to the viscosity of the fluid. A defining strategy of the present study is to identify, differentiate, and analyze these two aspects individually as much as possible before merging the results. This strategy offers remarkable advantages in aerodynamic analysis because it helps to clarify certain conceptual difficulties and paradoxes associated with several familiar assumptions of classical aerodynamics, including the inviscid-fluid assumption.

1.2 Differential Equations, Initial and Boundary Conditions

The present report is concerned with external aerodynamics. The time-dependent incompressible flow of an infinite viscous fluid, primarily air and water, in three-dimensional space relative to a finite and rigid solid body immersed and moving in the fluid is selected as the *reference flow problem*. To underscore external aerodynamic applications, the solid body is at times referred to as a wing. The flow can be steady, quasi-steady, or fully unsteady. The wing is not constrained to operate at a small angle of attack in a steady streamlined environment. It may flap either while flying forward or while hovering. It experiences a lift, a drag, and possibly also a side force.

To formulate the reference flow problem mathematically, the region occupied by the fluid is denoted R_f and that occupied by the wing R_s. The solid-fluid interface is denoted S. The combined region of the fluid and the wing, denoted R_∞, is infinite and unlimited. Both the wing and the fluid are at rest at the initial time level $t = 0$.

Subsequent prescribed motion of the wing during the time period $0 < t < t_1$ induces a corresponding motion of the fluid. The viscosity of the fluid is assumed uniform for simplicity.

The fluid region R_f is assumed to be simply connected, bounded internally by the closed surface S and externally by S_∞, a closed surface on which every point is infinitely far from S. The flow in R_f is then described by the following set of two equations:

$$\nabla \cdot \mathbf{v} = 0 \tag{1.1}$$

$$\frac{\partial \mathbf{v}}{\partial t} = -(\mathbf{v} \cdot \nabla)\mathbf{v} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{v} + \nabla q \tag{1.2}$$

where **v** is the flow velocity, p, ρ, and ν are respectively the pressure, the density, and the kinematic viscosity of the fluid, and ∇q represents a conservative external body force.

Equation (1.1) is a statement of the law of conservation of mass for an incompressible flow. This equation is called the continuity equation because it is based on the assumption that the fluid is a continuous medium: *a continuum* [Shapiro 1953]. The continuum assumption is also a part of (1.2), the Navier-Stokes momentum equation, which is a mathematical statement of Newton's second law of motion.

Equations (1.1) and (1.2) comprise a set of four scalar equations containing four scalar unknown field variables: the pressure and the three components of the velocity vector. In fluid dynamics, the fact that the number of equations and the number of unknown field variables are equal is often considered important. For example, with the addition of the energy equation, the energy density of the fluid is an additional field variable. For the compressible flow, the mass density also becomes an unknown field variable. Some authors therefore conclude that an additional equation, the equation of state, is required. This conclusion is correct. The number of scalar differential equations describing a physical problem, however, needs not be equal to the number of scalar field variables. In fact, in the vorticity-dynamic formulation of the flow problem, these numbers are unequal.

To complete the mathematical formulation of the flow problem, values of **v** in R_f at the initial time level, $t = 0$, and on the boundaries S and S_∞ for the time period $0 < t < t_1$ need to be specified. With the flow initially at rest, the initial condition is,

$$\mathbf{v}(\mathbf{r}, t = 0) = 0 \text{ in } R_f \tag{1.3}$$

where **r** is the position vector.

With the wing motion prescribed for the time period $0 < t < t_1$, one writes