Nonlinear Multivariate Analysis

Nonlinear Multivariate Analysis

Albert Gifi
University of Leiden
The Netherlands

JOHN WILEY & SONS
Chichester • New York • Brisbane • Toronto • Singapore

Copyright © 1990 by John Wiley & Sons Ltd.
Baffins Lane, Chichester
West Sussex PO19 1UD, England

All rights reserved.

No part of this book may be reproduced by any means,
or transmitted, or translated into a machine language
without the written permission of the publisher.

Other Wiley Editorial Offices

John Wiley & Sons, Inc., 605 Third Avenue,
New York, NY 10158-0012, USA

Jacaranda Wiley Ltd, G.P.O. Box 859, Brisbane,
Queensland 4001, Australia

John Wiley & Sons (Canada) Ltd, 22 Worcester Road,
Rexdale, Ontario M9W 1L1, Canada

John Wiley & Sons (SEA) Pte Ltd, 37 Jalan Pemimpin 05-04,
Block B, Union Industrial Building, Singapore 2057

Library of Congress Cataloging-in-Publication Data:

Gifi, Albert
Nonlinear multivariate analysis / Albert Gifi.
p. cm.
Includes bibliographical references.
ISBN 0 471 92620 5
1. Multivariate analysis. 2. Nonlinear theories. [illegible]
QA278.G535 1990
519.5'35—dc20

British Library Cataloguing in Publication Data:

Gifi, Albert
Nonlinear multivariate analysis.
1. Multivariate analysis. Applications of computer
systems. Programming
519.5'35'028851

ISBN 0 471 92620 5

Printed in Great Britain by Courier International, Tiptree Essex

FOREWORD

This book by Gifi is a nonstandard treatment of some of the more important parts of multivariate data analysis. A heavy emphasis has been placed on plotting observations and/or variables in low-dimensional Euclidean space. This clearly shows its psychometric origins, because of the obvious relationships with factor analysis and component analysis. Much less emphasis than usual is paid to regression and analysis of variance, and the various multivariate generalizations of the linear model.

There are other ways in which the book is nonstandard. It is organized, as many other multivariate data analysis books are, around various key techniques, such as principal component analysis, canonical analysis, and so on, but in this book these techniques are identified, much more than is usually done, with computer programs and the type of output they reproduce. A technique is defined, more or less, as a tool into which you feed data of a particular type and format, and which then reproduces output of a particular type and format. This very instrumental interpretation of data analysis is contrasted, in many places, with the model-oriented approach of classical statistics. Models do have a place in Gifi's philosophy: not as tentative approximations to the truth, but as devices to sharpen and standardize the data analytic techniques. At the same time the inferential methods of statistics are explicitly introduced as one of various ways in which the stability of data analysis tools can be investigated.

Of course we have to realize that there is, according to many statisticians, one appropriate way to analyse data: first formulate a model on the basis of prior knowledge, then compute the likelihood of the data given the model, and then estimate and test the appropriateness of the model by maximizing the likelihood function. There are some variations on this scheme, depending on what exactly this prior knowledge includes, but these variations are minor compared to the prescription seemingly advocated by Gifi. First choose a technique (implemented in a computer program) on the basis of the format of your data, then apply this technique, and study the output. The prior knowledge is not mentioned explicitly, but more importantly it is not isolated in any one particular time and place in the data analysis process. The classical recipe suggests that prior knowledge (theory, prejudice, experience, expecta-

ions) enters in the formulation of the model, but then it should not be used at all during the other stages of the process. These are supposed to be mechanical, almost as applying the gigantic computer program MAXLIK, which handles data of any type, or the even more gigantic program TBAYES, which takes data and prior information as input and transforms this to posterior information. In the book by Gifi many other places where prior information, and expertise, play a role in data analysis are indicated. Not all thought and creativity of the investigator, and the history of his science, is concentrated in the mysterious phase in which the model is formulated.

The techniques actually discussed in the book have a very strong emphasis on geometry, in the sense that they make pictures of data. As we said, this places the techniques rather firmly in the tradition of factor analysis, or perhaps even more clearly in the tradition of multidimensional scaling. Another relationship with multidimensional scaling is that variables are transformed optimally, using monotone transformations or splines, in order to improve the fit of the low-dimensional representation. This is different from traditional factor analysis, in which the fit is improved by computing additional factors and then rotating them to simple structures. Gifi incorporates the additional factors in the nonlinear transformations, and keeps the pictures simple and preferably in two or at most three dimensions. The pictures are, of course, descriptive devices. In most cases they have to represent an enormous amount of information, and they can approximate that by showing the dominant relations in the data, its most coarse features. This may be useless, or even offensive, to people who want specific answers to specific questions. Again the psychometric (or social science) origins of Gifi's system become apparent here, because in the large scale survey type of investigations in which multivariate methods are likely to be used the questions are usually not very specific.

Gifi was one of the first investigators to use the bootstrap and jackknife to study the stability of multivariate representations. The reasons for this are clear: the amount of prior knowledge one needs to use these techniques is minimal, especially if one uses the perturbation interpretation discussed in Chapter 12 of the book. If we use the idea that a statistical technique is a tool, that can be applied in many different situations, although it is not necessarily optimal for any one specific situation, then this fits in rather nicely with the ideas behind bootstrapping, jackknifing, cross validating, and using randomization tests. The tools have their own quality control devices built in, as it were, and because of that they become more useful and more informative at relatively little extra cost.

The application of a computer program, or a set of related computer programs, as a statistical toolbox around which various applications can be

此为试读,需要完整PDF请访问: www.ertongbook.com

organized, is much more common now than it was during the Gifi project. We now have books built around GLIM, BMDP, the DataDesk, SPSS, and so on. The emphasis on computations to replace assumptions, and the use of resampling and Monte Carlo in that context, is very important in modern statistics. Optimal transformations occur with great regularity in the statistical journals these days, in combination with semi-parametric models, splines, smoothing, and Box–Cox families. Correspondence analysis (sometimes disguised as ACE) can now be discussed among consenting statisticians, without causing frothing at the mouth and the throwing of stones. It was not always like this! The Gifi project started as a one-person effort in 1968. It became a more-than-one-person effort in 1974 and a quite-a-lot-of-persons effort in 1978. It was generously supported by ZWO, the Netherlands Organization for the Advancement of Pure Research, but was frowned upon by official statisticians, and led to the types of semi-religious debates that typically occur in the foundations of statistics area. That this resistance was eventually largely overcome was not in the least due to internal developments in the discipline of statistics itself.

There are a few persons I would like to thank personally, on behalf of Albert Gifi, of course: in the first place John Van de Geer, who taught us the geometric approach to multivariate analysis that pervades the book, who provided us with a Department to work in, and with the freedom we needed; secondly, Doug Carroll and Joe Kruskal at Bell Laboratories in Murray Hill, who invited several of us to come over there and who provided access to many resources, tools and organizations; thirdly, Forrest Young and Yoshio Takane, who provided invaluable assistance at a very critical point in the development of these techniques; fourthly, Chris Haveman, of SPSS, who remembered from his days at the Computing Center of the University of Leiden that people there were doing useful things in multivariate data analysis; and finally, Richard Gill, formerly of the Center for Mathematics and Informatics in Amsterdam, who provided valuable support and connections to the world of academic statistics. There are innumerable other persons who provided various services at various points in time, and I would like to thank them collectively as well.

As explained elsewhere in the book, Gifi is not quite dead yet. Many of the topics mentioned in the book, but not worked out in detail there, have been taken up in dissertations and research monographs. The basic approach has been extended to path models and dynamic systems, and variations based on multinormal likelihood theory are also available. Links to multidimensional scaling have been strengthened, as have the links to classical multivariate statistical theory. However, the book, this book, is the only comprehensive,

programmatic, encompassing statement of the general Gifi philosophy, and the way in which it is implemented on the digital computer.

Los Angeles, 22 April, 1989 Jan de Leeuw

PREFACE

'It's difficult to understand why statisticians commonly limit their inquiries to Averages, and do not revel in more comprehensive views. Their souls seem as dull to the charm of variety as that of the native of one of our own flat English counties, whose retrospect of Switzerland was that, if its mountains could be thrown into its lakes, two nuisances would be got rid of at once. An average is but a solitary fact, whereas if a single other fact be added to it, an entire Normal Scheme, which nearly corresponds to the observed one, starts potentially into existence. 'Some people hate the very name of statistics, but I find them full of beauty and interest. Whenever they are not brutalised, but delicately handled by the higher methods, and are warily interpreted, their power of dealing with complicated phenomena is extraordinary. They are the only tools by which an opening can be cut through the formidable thicket of difficulties that bare the path of those who pursue the Science of man.'

Francis Galton (1889), *Natural Inheritance*, London: Macmillan (from D.W. Forrest (1974), *Francis Galton: The Life and Work of a Victorian Genius*, New York: Taplinger Publishing Co.)

This volume is a revised version of a mimeographed text that was brought into limited circulation in June 1981. At that time it served as reading material for the post-doctoral course *Nonlinear Multivariate Analysis*, prepared by the Department of Data Theory and attended by approximately fifty participants from Belgium and The Netherlands. A previous Dutch version (Albert Gifi, Niet-lineaire Multivariate Analyse, Leiden, 1980) served a similar purpose for a course held in March 1980. Since then a large number of other courses and

seminars, both at home and abroad, have been built around the 'Gifi system' (a name coined by Jan de Leeuw in 1984 when the endeavour showed signs of losing momentum), and major parts of the software have been developed into reliable and portable procedures. In the course of time the 1981 version of the text saw five completely unrevised impressions.

Who is Albert Gifi? From the preface of the 1981 text we quote: 'The text is the joint product of the members of the Department of Data Theory of the Faculty of Social Sciences, University of Leiden. Albert Gifi is their nom de plume. The portrait, however, of Albert Gifi shown here is that of the real Albert Gifi to whose memory this book is dedicated, as a far too late recompense for his loyalty and devotion, during so many years, to the Cause he served.'

Since the composition of a department changes over time it is proper to lift the veil of anonymity a little bit. The members of the Gifi team who contributed to the original texts and programs are Bert Bettonvil, Eeke van der Burg, John van de Geer, Willem Heiser, Jan de Leeuw, Jacqueline Meulman, Jan van Rijckevorsel, and Ineke Stoop. Assistance in organizing the Gifi courses was provided by Steef de Bie, Judy Knip, and Dré Nierop. Editorial assistance in an earlier enterprise to revise the 1981 text was provided by Peter van der Heijden and Adriaan Meester. Throughout the years Peter Neufeglise has supported our group by lending his patient technical assistance to many problems on numerous computer systems; he has also been responsible for software maintenance. Renée Verdegaal contributed to the development of OVERALS, only a rough outline of which was documented in the 1981 book. Peter Verboon contributed to the present volume by redoing numerous graphics, and Ivo van der Lans assisted in proof-reading. Gerda van den Berg and Patrick Groenen took care of the final stage of converting the software into SPSS procedures.

In March 1987 the present editorial team started with the final revision. First we merely aimed at two things: (a) correcting typing errors and (b) getting the distribution of sections and paragraphs more even (while reading we could perhaps just as well make some notes for an index). However, soon more desiderata popped up, like having a more uniform level of presentation and getting the notation consistent across chapters. We also thought of making fresh drawings of the figures, checking all analyses with updated programs, and so on. These ambitions almost led to a dead end, until we seized the opportunity – starting in December 1987 – to stay a couple of times in the cultural centre 'De Pauwhof' in Wassenaar, where we found the right atmosphere to fully concentrate on the job. We decided to put Chapter 4 (old) after Chapter 8 (old) for reasons of balance, and we adjusted all notation and cross-referencing as well as we could. We wrote a number of short intro-

ductions and added extra explanations where the original text really was obscure. An important decision was made to refrain from inserting new insights and/or references to new work in the body of the text. In our opinion this would have resulted in an unbalanced mixture of the original version with a possible start of a completely new book. We chose to add a section called 'Epilogue' to each chapter; the epilogues give some guidance to new developments, especially when it is inspired by – or relevant to – the Gifi system, without trying to be exhaustive. When the sign '➳' appears in the text comments on the current topic may be found in the last section of the chapter.

After eight years of informal circulation Gifi's work has now reached the present form. We are in debt to students, colleagues, and friends for many helpful corrections and suggestions during this time. Especially the detailed remarks by Shizuhiko Nishisato, Jos ten Berge, Charley Lewis, and Ivo Molenaar were very welcome. A final word of thanks is due to Susañña Verdel, who was involved in rounds and rounds of word processing, and who finished the job with great accuracy and care.

The Editors

De Pauwhof,
Wassenaar, The Netherlands
9 May, 1989

Willem Heiser
Jacqueline Meulman
Gerda van den Berg

CONTENTS

Foreword v

Preface ix

CHAPTER 1 CONVENTIONS AND CONTROVERSIES IN MULTIVARIATE ANALYSIS 1

1.1 CONTENT ANALYSIS OF MVA BOOKS 2
1.1.1 Roy (1957) 2
1.1.2 Kendall (1957, 1975) 2
1.1.3 Anderson (1958) 4
1.1.4 Cooley and Lohnes (1962, 1971) 5
1.1.5 Morrison (1967, 1976) 5
1.1.6 Van de Geer (1967, 1971) 6
1.1.7 Dempster (1969) 6
1.1.8 Tatsuoka (1971) 8
1.1.9 Harris (1975) 8
1.1.10 Dagnelie (1975) 10
1.1.11 Green and Carroll (1976) 10
1.1.12 Cailliez and Pagès (1976) 11
1.1.13 Giri (1977) 12
1.1.14 Gnanadesikan (1977) 12
1.1.15 Kshirsagar (1978) 13
1.1.16 Thorndike (1978) 14

1.2 CORRESPONDENCE ANALYSIS OF TABLES OF CONTENT 15

1.3 A SHORT SUMMARY AND SOME PROBLEMS 19

1.4 DATA ANALYSIS AND STATISTICS 21
1.4.1 Tukey's definition of data analysis 21
1.4.2 Benzécri's definition of data analysis 25
1.4.3 Robust statistics 27
1.4.4 Exploration and confirmation 29
1.4.5 Inference 32

1.5 DATA ANALYTIC PRINCIPLES OF THIS BOOK 33
1.5.1 Model and technique 33
1.5.2 Gauging 34
1.5.3 Stability 36

此为试读,需要完整PDF请访问: www.ertongbook.com

	1.6	SPECIFIC PROBLEMS OF MVA	39
		1.6.1 The multinormal distribution	39
		1.6.2 Tabular analysis	42
		1.6.3 Discrete MVA	45
		1.6.4 Causal analysis	47
	1.7	DEFINITION OF MVA	49
		1.7.1 Asymmetric role of rows and columns	49
		1.7.2 Linear, monotone, and nonlinear MVA	51
		1.7.3 Bivariate and multivariate MVA	52
	1.8	SOME IMPORTANT INGREDIENTS	53
		1.8.1 Join and meet problems	53
		1.8.2 Optimal scaling and alternating least squares	58
		1.8.3 Dimensionality and data massage	60
	1.9	EPILOGUE	61
CHAPTER	**2**	**CODING OF CATEGORICAL DATA**	**65**
	2.1	THE COMPLETE INDICATOR MATRIX AND ITS PROPERTIES	67
	2.2	QUANTIFICATION	69
	2.3	THE INCOMPLETE INDICATOR MATRIX	70
	2.4	MISSING DATA	73
	2.5	THE REVERSED INDICATOR MATRIX	76
	2.6	THE INDICATOR MATRIX FOR A CONTINGENCY TABLE	77
	2.7	GROUPING OF CATEGORIES	78
	2.8	GROUPING OF VARIABLES	78
	2.9	EPILOGUE	80
CHAPTER	**3**	**HOMOGENEITY ANALYSIS**	**81**
	3.1	HOMOGENEITY OF VARIABLES	81
	3.2	HISTORICAL PRELIMINARIES OF DIFFERENTIAL WEIGHTING	82
	3.3	MAXIMIZING HOMOGENEITY BY LINEAR WEIGHTING: PRINCIPAL COMPONENTS ANALYSIS	87
	3.4	ALTERNATING LEAST SQUARES ALGORITHMS FOR LINEAR WEIGHTING	87
		3.4.1 Normalized scores algorithm	88
		3.4.2 Normalized weights algorithm	92
		3.4.3 Adaptation for heterogeneous variances	95
		3.4.4 Multiple solutions	98

	3.5	LINEAR WEIGHTING FOR K SETS OF VARIABLES	100
	3.6	MORE HISTORICAL COMMENTS ON PCA	102
	3.7	MAXIMIZING HOMOGENEITY BY NONLINEAR TRANSFORMATION: NONLINEAR PCA	104
	3.8	CATEGORICAL PCA: HOMALS	106
		3.8.1 Properties in terms of the SVD	107
		3.8.2 Centering	109
		3.8.3 Normalization	112
		3.8.4 Contribution of variables:discrimination measures	113
		3.8.5 Geometrical properties	118
	3.9	RELATIONS BETWEEN HOMALS AND LINEAR PCA	120
	3.10	THE RELATIONSHIP BETWEEN HOMALS AND TOTAL CHI-SQUARE	127
	3.11	AN ILLUSTRATION: HARTIGAN'S HARDWARE	128
	3.12	HOMALS WITH INCOMPLETE INDICATOR MATRIX	135
	3.13	REVERSED INDICATOR MATRIX	142
	3.14	EPILOGUE	149
CHAPTER	**4**	**NONLINEAR PRINCIPAL COMPONENTS ANALYSIS**	**151**
	4.1	METRIC PRINCIPAL COMPONENTS ANALYSIS	151
	4.2	NONMETRIC PRINCIPAL COMPONENTS ANALYSIS	153
	4.3	THEORY OF JOIN LOSS	158
	4.4	THEORY OF MEET LOSS	167
	4.5	GEOMETRY OF MEET LOSS	172
	4.6	THE PRINCALS PROGRAM	177
	4.7	AN EXAMPLE COMPARING MULTIPLE AND SINGLE NOMINAL TREATMENT	179
	4.8	AN EXAMPLE WITH PREFERENCE RANK ORDERS	183
	4.9	AN EXAMPLE WITH DISCRETIZATION OF CONTINUOUS VARIABLES	187
	4.10	EPILOGUE	191
CHAPTER	**5**	**NONLINEAR GENERALIZED CANONICAL ANALYSIS**	**193**
	5.1	PREVIOUS WORK	194
	5.2	LOSS FUNCTION AND NORMALIZATION OF OVERALS	202

	5.3	OVERALS AS A SPECIAL CASE OF HOMALS: THE USE OF INTERACTIVE VARIABLES	204
	5.4	MISSING DATA	207
	5.5	ALGORITHM CONSTRUCTION	208
	5.6	AN EXAMPLE: EFFECTS OF RADIOACTIVITY ON FISH	209
	5.7	EPILOGUE	214
CHAPTER	**6**	**NONLINEAR CANONICAL CORRELATION ANALYSIS**	**217**
	6.1	PREVIOUS WORK	217
	6.2	THEORY	220
	6.3	THE CANALS PROGRAM	221
	6.4	EXAMPLE 1: ECONOMIC INEQUALITY AND POLITICAL STABILITY	223
	6.5	EXAMPLE 2: PREDICTION OF A SCHOOL ACHIEVEMENT TEST	232
	6.6	EPILOGUE	238
CHAPTER	**7**	**ASYMMETRIC TREATMENT OF SETS: SOME SPECIAL CASES, SOME FUTURE PROGRAMS**	**241**
	7.1	MULTIPLE REGRESSION AND MORALS	241
	7.2	DISCRIMINANT ANALYSIS AND CRIMINALS	243
	7.3	MULTIVARIATE ANALYSIS OF VARIANCE AND MANOVALS	244
	7.4	PATH ANALYSIS AND PATHALS	245
	7.5	PARTIAL CANONICAL CORRELATION ANALYSIS AND PARTALS	247
	7.6	SOME EXAMPLES	248
	7.7	EPILOGUE	253
CHAPTER	**8**	**MULTIDIMENSIONAL SCALING AND CORRESPONDENCE ANALYSIS**	**255**
	8.1	HOMOGENEITY AND SEPARATION	255
	8.2	MINIMUM DISTANCE ANALYSIS OF HOMOGENEOUS GROUPS OF OBJECTS	258
		8.2.1 Scaling the row objects associated with F	259
		8.2.2 Scaling the column objects associated with F	264
		8.2.3 Joint homogeneity analysis of F	264

8.3 CORRESPONDENCE ANALYSIS 265
8.3.1 Joint analysis of rows and columns 267
8.3.2 The chi-square metric 268
8.3.3 The centroid principle and reciprocal averaging 272
8.3.4 Relation with homogeneity analysis of two indicator matrices 272

8.4 CONTINGENCY AND CORRELATION 273

8.5 THE PROGRAM ANACOR 276
8.5.1 An example with a simple contingency table 277
8.5.2 An example with a similarity table 280
8.5.3 An example with a multidimensional contingency table 283

8.6 THE PROGRAM ANAPROF: ANALYSIS OF PROFILE FREQUENCIES 289
8.6.1 An example with binary survey data 290
8.6.2 Asymmetric treatment of response categories 294

8.7 SOME GAUGING RESULTS FOR BINARY DATA 295

8.8 EPILOGUE 301

CHAPTER 9 MODELS AS GAUGES FOR THE ANALYSIS OF BINARY DATA 303

9.1 SOME GENERAL FORMULAS 303

9.2 MONOTONE LATENT TRAIT MODELS 304
9.2.1 Holomorph items 305
9.2.2 The Guttman scale 307
9.2.3 The Spearman hierarchy 314
9.2.4 The latent distance model 319
9.2.5 The Rasch model 321

9.3 NONMONOTONIC LATENT TRAIT MODELS 322

9.4 ORDER ANALYSIS OF BINARY MATRICES 324

9.5 DICHOTOMIZED MULTINORMAL DISTRIBUTIONS 326

9.6 EPILOGUE 328

CHAPTER 10 REFLECTIONS ON RESTRICTIONS 329

10.1 THE CLASS OF RESTRICTIONS CLOSED UNDER LINEAR TRANSFORMATIONS 330

10.2 EQUALITY CONSTRAINTS 331

10.3 OTHER LINEAR CONSTRAINTS 343

10.4 ZEROS AT SPECIFIC PLACES 346

10.5 NONLINEAR RESTRICTIONS 348

10.6 EPILOGUE 349

CHAPTER 11 NONLINEAR MULTIVARIATE ANALYSIS: PRINCIPLES AND POSSIBILITIES 351

11.1 JOIN AND MEET 352
11.1.1 Matrix formulation 354
11.1.2 Further analysis of meet loss 356
11.1.3 Further analysis of join loss 359
11.1.4 Solving meet and join problems 360
11.1.5 Some extensions of the join and meet framework 361
11.1.6 Extensions to infinite-dimensional space 363

11.2 CHOICE OF BASIS 363
11.2.1 Splines 365
11.2.2 An empirical gauge for recovery of rational functions 370
11.2.3 Interactive variables 376
11.2.4 Infinite dimensionality 377

11.3 SOME INFINITE-DIMENSIONAL GAUGES 381

11.4 ANALYSIS OF STOCHASTIC PROCESSES 386

11.5 ALTERNATIVE CRITERIA 387
11.5.1 Unitarily invariant norms 388
11.5.2 Least squares or other 389

11.6 EPILOGUE 390

CHAPTER 12 THE STUDY OF STABILITY 391

12.1 ANALYTICAL STABILITY 392
12.1.1 Eigenvalue problems 392
12.1.2 Eliminating a variable in HOMALS 395
12.1.3 Merging categories in HOMALS 397
12.1.4 Eliminating a subject in HOMALS 397
12.1.5 Problems that are not eigenvalue problems 399

12.2 ALGEBRAIC STABILITY 400
12.2.1 Eigenvalue problems with restrictions 400
12.2.2 Applications of the previous results 403
12.2.3 Discretization 404
12.2.4 Interactive variables 406
12.2.5 Other techniques for algebraic stability 406

12.3 REPLICATION STABILITY 408
12.3.1 The delta method 408
12.3.2 General theory of randomization methods 415
12.3.3 Approximation properties of the jackknife 418
12.3.4 Approximation properties of the bootstrap 420
12.3.5 Comparison of jackknife and bootstrap 423

12.4 EPILOGUE 424

CHAPTER 13 THE PROOF OF THE PUDDING 425

13.1 MULTIPLE CHOICE EXAMINATION 425
13.1.1 One-dimensional HOMALS with four categories per variable 428
13.1.2 One-dimensional HOMALS with merged wrong categories 430
13.1.3 HOMALS on the transposed data matrix 430

13.2 CONTROVERSIAL ISSUES 431
13.2.1 One-dimensional HOMALS with all 37 variables 434
13.2.2 Likert scales 434
13.2.3 Guttman scales 440
13.2.4 Two-dimensional HOMALS with 28 variables 443
13.2.5 PRINCALS 445

13.3 AS YEARS GO BY 447
13.3.1 One-dimensional HOMALS with all variables 449
13.3.2 One-dimensional HOMALS for subgroups of individuals 452
13.3.3 Various applications of MORALS 454
13.3.4 Comparing HOMALS and PRINCALS analyses 459
13.3.5 Bootstrapping one-dimensional HOMALS 461

13.4 PARLIAMENT SURVEY 465
13.4.1 Preference rank orders, 1968 466
13.4.2 Political preference and issue positions, 1972 470
13.4.3 Relation between position on issues and party allegiance, 1972 477

13.5 CRIME AND FEAR 485
13.5.1 Overview of analyses 485
13.5.2 Multiple join solutions with all variables 487
13.5.3 Single join solutions with all variables 492
13.5.4 Single meet solutions over all variables 496
13.5.5 Single meet solutions with the CRIME and FEAR variables 497
13.5.6 Rank orders for the categories of the background variables 499

13.6 EPILOGUE 502

APPENDIX A NOTATION 503

APPENDIX B MATRIX ALGEBRA 507

B.1 IMAGES 507

B.2 HYPERELLIPSOIDS 510

B.3 INVARIANT DIRECTIONS 513

B.4 SINGULAR VECTORS AND SINGULAR VALUES 514

此为试读,需要完整PDF请访问: www.ertongbook.com