Lecture Notes in Physics

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Nonlinear Hydrodynamic Modeling: A Mathematical Introduction

Edited by Hampton N. Shirer

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To Daniel and Julia, two of the
greatest kids growing up in a nonlinear world.

PREFACE

These lecture notes provide an introduction to the nonlinear analysis of solutions to hydrodynamic models. Traditionally, at least in the atmospheric sciences, most theoretical study of the properties of the solutions is limited to a linear analysis. The reasoning is that finding the linear modes with the fastest growth rates should yield an adequate understanding of the observed flows. In recent years, however, it has become clear that significant fundamental progress is possible only if a nonlinear approach is taken. With this approach, the finitely many modes are sought that, in the long term at least, carry most of the energy. Although many introductory texts covering the requisite topics of bifurcation, stability, catastrophe theory, etc. exist (e.g., Iooss and Joseph, 1980, or Guckenheimer and Holmes, 1983), most are written from a mathematically formal perspective. An introductory survey of the subject using relatively simple language is needed to provide students of hydrodynamics with sufficient background to read these more mathematical treatments. It is possible to avoid much of the formal mathematical jargon because the necessary basic theorems depend on elementary concepts requiring only undergraduate-level knowledge of differential equations and linear algebra. It is this approach that we adopt here to motivate and describe, without proof, the results and applications of these theorems.

This monograph has a further purpose. As it becomes clear that nonlinear analysis of systems of equations is necessary, it also becomes apparent that the analysis would proceed more easily if the number of equations were small. The pioneer in this area, which is known as low-order modeling, is Professor Edward N. Lorenz of the Massachusetts Institute of Technology. In a series of studies covering a range of fluid responses, he has discovered much about the fundamental nonlinear properties of fluid flow by utilizing various low-order systems of ordinary differential equations. In this monograph, we use his 1963 model of Rayleigh-Bénard convection to illustrate many of the necessary nonlinear concepts and calculations. Although it is a simple matter to write a three- or five-coefficient system of equations, it is often a distinct challenge to write one that is physically relevant. It is natural, then, to ask whether any principles guiding model development could be created, principles whose application would be based on nonlinear mathematical theory. In this monograph, we discuss a preliminary list of seven such principles and we summarize many of the types of calculations that should be carried out to test whether the model results are likely to be physically relevant. The results of these calculations give us a wide variety of information, ranging from the preferred geometrical configuration of the flow to signals that the results are too restrictive and that certain degrees of freedom therefore must be added to the system. Thus we bring a new perspective to nonlinear modeling, a perspective embodied in the concept of metamodeling, or the study of the modeling process itself, and we argue that its application will allow us to create useful and relatively simple-to-analyze nonlinear models of fluid behavior.

The initial manuscript upon which this monograph is based was written in the fall of 1984 by the 11 participants in a graduate meteorology seminar on nonlinear hydrodynamics at The Pennsylvania State University. This seminar was organized by Professors John A. Dutton and Hampton N. Shirer; the students were Dr. Steven B. Feldstein, Mr. Ronald Gelaro, Mr. R. Wayne Higgins, Mr. Paul A. Hirschberg, Mr. Mark J. Laufersweiler, Mr. Jon M. Nese, Mr. Robert J. Pyle, Mr. Arthur N. Samel, and Mr. David J. Stensrud. Each participant was asked to give a lecture on one or more topics and then to organize their material into a chapter for a set of notes. These original chapters were used by the 14 participants in the fall of 1985 seminar, who gave lectures from and critically reviewed the initial manuscripts. On the basis of these reviews, the notes were completely reorganized and rewritten in the form presented here.

The chapters of the monograph are divided into four main groups--Chapters 1 to 3 introduce the basic concepts of modeling and review the creation, development, and analysis of nonlinear models; Chapters 4 to 10 discuss various aspects of time-independent, or stationary, solutions to the models; Chapters 11 to 14 review the behavior of temporally periodic solutions; and Chapters 15 to 18 survey more complicated temporal flows and the analysis of their properties.

Here we briefly summarize the major topics of each chapter. In Chapter 1, we discuss models and their purposes and we outline the seven modeling principles; four we regard as fundamental to all models, with the remaining three being intermediate their branching properties can be determined via a power series analysis of the differential system. Finally, in Chapter 14, we show how to extend the notion of asymptotic stability to one for temporally periodic solutions, and then we review the three principal types of temporal solutions that bifurcate from the periodic ones.

In the remaining four chapters of the monograph, we consider more complicated temporal, or chaotic, flows; these flows have been proposed to model one form of turbulence commonly observed in fluids. Several proposed routes along which a flow might evolve toward a turbulent state are discussed in Chapter 15. Some measures of the complicated structures of the chaotic solutions are introduced in Chapter 16. In Chapter 17, we present a quite general review of the properties of the solutions to some of the differential equations of atmospheric flows, and then we propose two ways that optimum models might be created. Finally, in Chapter 18, we return to a discussion of the elements of the modeling process that provide the necessary concepts underlying metamodeling.

An undertaking of this magnitude would not be possible without the tireless efforts of a large number of people. We are especially grateful to Professor Robert Wells whose advice led to major improvements in the discussion throughout the monograph. Professor Wells thoroughly read the entire manuscript and offered a large number of extremely helpful suggestions. In addition, his help was essential in implementing the Alexander-Yorke continuation method discussed in Chapter 10 as well as in calculating the Lyapunov dimensions in Chapter 16. We thank Professor John A. Dutton, who reviewed and offered many useful comments on each chapter, as well as Dr. Harry W. Henderson and Mrs. Tracy H. Hirschberg, who offered many critical comments on the material in some of the chapters. Also, Professor Peter Kloeden kindly gave us his notes from which Section 13.2 was written.

All of the authors helped to critique and proofread the manuscript; several, especially Mr. R. Wayne Higgins, Mr. Paul A. Hirschberg, Mr. Mark J. Laufersweiler, and Mr. Jon M. Nese, helped considerably with the host of other mundane tasks. Also, we thank the fall 1985 participants in the graduate seminar for their comments that led to greatly improved organization of the material: Ms. Shuyi Chen, Ms. Sharon Douglas, Dr. Steven B. Feldstein, Mr. R. Wayne Higgins, Mr. Paul A. Hirschberg, Mrs.

ones that must be incorporated eventually. We describe the modeling process in more detail in Chapter 2, where we show how solutions to a simple model of a physical system may possess the qualities outlined in the four fundamental principles. Then, in Chapter 3, we review the Galerkin method for creating low-order dynamical systems, or truncated spectral models, whose behavior we investigate in subsequent chapters.

Normally we begin analyzing nonlinear models by first studying their stationary solutions. We show in Chapters 4 and 5 how determination of the asymptotic stability properties of the stationary solutions may be used to identify the critical values of the forcing at which transitions or bifurcations between stationary solutions occur. Acceptable branching behavior is modeled by forms that satisfy certain physically imposed constraints, and we review these in Chapter 6. In order that the solutions to the models correspond to observed flows, some of the response parameters, such as cell aspect ratio, must take preferred values, and the methods used to find these values are covered in Chapter 7.

We begin in Chapters 8 and 9 to consider the three intermediate modeling principles, that is we investigate whether model behavior is sensitive to additional possible degrees of freedom. First, in Chapter 8, we show how to identify the forcing parameters necessary for describing adequately all types of transitions; in addition, we note how to identify the forcing effects that might serve as candidates for any of the identified parameters not already in the problem as it was originally posed. Hierarchies of transitions are observed in many laboratory flows, and we see in Chapter 9 how these can be modeled via secondary branching. We complete our consideration of stationary solutions in Chapter 10, where we present a numerical algorithm for determining all stationary solutions to a low-order model.

Certainly other types of solutions are possible, and in Chapters 11 to 14 we consider temporally periodic ones. In Chapter 11, we discover how the creation of these solutions is signaled by a type of stability exchange known as Hopf bifurcation; in addition, we review the acceptable forms of these branching solutions. As with stationary solutions, the observed characteristics of the periodic solutions can be related to the preferred values of certain response parameters, and we discuss how to find their values in Chapter 12. Although periodic solutions cannot always be obtained analytically, we show in Chapter 13 that some of

Tracy H. Hirschberg, Mr. Mark J. Laufersweiler, Mr. Jon M. Nese, Mr. Robert J. Pyle, Mr. Shou-Ping Wang, Mr. Morris L. Weisman, and Mr. Chidong Zhang.

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Mrs. Delores Corman and Mrs. Nancy Warner typed and retyped the apparently endless versions of each chapter. We deeply appreciate their skillful work and good humor throughout the lengthy project. Also, we are indebted to Mr. Vic King who excellently drafted the large number of figures.

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HAMPTON N. SHIRER
December 1986

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CHAPTER 1

MODELING: A STRATEGY FOR UNDERSTANDING

JOHN A. DUTTON

Science is the process by which humans attempt to comprehend, predict, and perhaps control the environment in which they find themselves. It proceeds on a number of levels, sometimes sequentially and sometimes simultaneously. Modeling is the heart of the scientific enterprise, for it links together the phases of the scientific endeavor.

The basic strategy of science is to convert facts or intuition gained by observing objects or natural processes in our surroundings into conceptual and quantitative structures that encourage and permit extrapolation of knowledge to new circumstances. We effect this strategy by constructing models of the phenomena that capture our interest.

Models, then, in a preliminary description, are devices that mirror nature in some sense by embodying empirical knowledge in forms that permit inferences, preferably quantitative, to be derived from them. Many branches of science have evolved to a level in which the models employed are explicitly mathematical, perhaps systems of differential equations, that can be treated in an axiom and theorem formulation. In others, true theoretical study is not yet possible because the essential state variables have not yet been perceived or invented.

This chapter is an introduction to the main concepts of modeling, which is a way of codifying knowledge about our world; in a companion chapter, Chapter 18, we examine metamodeling, which is the study of the process of modeling itself.

We shall first examine the role of modeling in scientific progress, and then consider modeling in the context of the Earth Sciences, emphasizing the dynamics of fluid systems -- the atmosphere and ocean -- in part because modeling is more advanced in atmospheric and oceanic sciences than it is in the other Earth Sciences. We shall find that although enough has been done that we can take a formal view of our accomplishments, many challenges remain. The hope and motivation of the present approach is that the formal study of models and modeling may stimulate the development of higher-level and more effective strategies for future work.

1.1 Modeling Motivations and Issues

Scientific study of a natural object or phenomenon proceeds through a number of usually distinct but interactive phases. They can be described as (see box below for accompanying definitions):

- Exploration and discovery
- Observation
- Formulation of descriptive or empirical models
- Development and verification of theories or inferential models concerning the state variables and their evolution
- Simulation and prediction of phenomenological behavior
- Modification of phenomenological behavior in the physical world

The first two phases may be described, perhaps loosely, as pre-modeling stages; the next two explicitly involve the development, refinement, and verification of models. The last two stages are made possible by the existence of reliable and efficacious models.

The interaction of the phases of science is illustrated in Fig. 1.1. Empirical knowledge is developed by exploration, discovery, and observation, and is then codified in descriptive models or by theoretical assimilation and summary. Quantitative understanding embodied in theories can be converted into models that foster the quantitative inferences of simulation and prediction. The attempt to

DEFINITIONS

Theory --

A speculative or established explanation accounting for known facts or phenomena, often expressed in the physical sciences in a symbolic or mathematical form that emphasizes the evolution of state variables describing the system of interest.

Descriptive Model --

A representation of structure or process in a descriptive, graphical, or statistical form.

Inferential Model --

An implementation of a theory in a form that fosters inference, especially quantitative, about a specific collection of phenomena or processes.