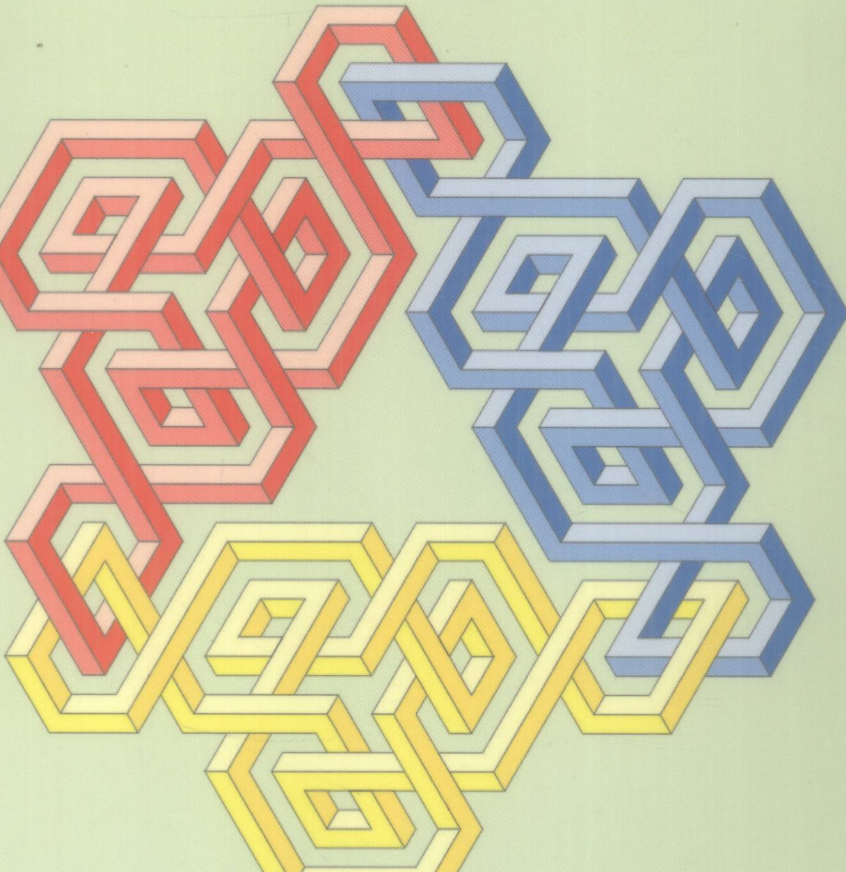
GYÖRGY DARVAS

SYMMETRY



BIRKHÄUSER

György Darvas

Symmetry

Cultural-historical and ontological aspects of science–arts relations

The natural and man-made world in an interdisciplinary approach

Translated from the Hungarian by David Robert Evans

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Preface

Hermann Weyl (1885–1955), who from the 1920s onwards turned the general phenomenon of symmetry into a subject of research in its own right, retired in 1951 to Europe from Princeton, where he spent his years in emigration in the company, among others, of John von Neumann (1903–1957) and Eugene P. Wigner (1902–1995). As a parting gesture, he held four lectures at the university in which he sought to summarize all there was to know about symmetry. The talks were written at the level of scientific discourse of the age, but Weyl did not address the representatives of individual disciplines — the material was accessible to those from all of the university's scholarly fields. The edited text of the lectures was published by the university in a separate volume. Weyl's *Symmetry* become a publishing sensation, being translated into some fifty languages, enjoying countless new editions, and is used in education throughout the world to this day.

Since Weyl's retirement there has been a huge upsurge in research into symmetry (and, I should add, into its absence or its violation). At almost the same time as Weyl's lectures, Buckminster Fuller patented his geodesic dome, containing hexagons, and ensuring a high level of symmetry (employing his principle of synergetics) and thereby great stability. The world first witnessed this structure in the form of the spatious dome constructed for the Montreal world exhibition, then later as the most stable sewing pattern for soccer balls, but this only really became a success following the discovery of the spherical carbon molecule fullerene, capable of stable bonding, in 1985. We know from the memoirs of J. Watson that — within a year of the publication of the Weyl volume — it was a symmetry consideration which led to the final discovery of the structure of the double helix. A year later Yang and Mills published their article about a new type of gauge invariance to describe the conservation of isotopic spin, which has become an inescapable foundation for physics ever since: no discovery in particle physics could have been made without it. During the 1950s, Eugene P. Wigner published a series of articles on the application of symmetries and conservation laws, turning these into a fundamental theory in physics, and, like some of those mentioned above, was rewarded with the Nobel prize. From this point forward, the history of physics became a series of discoveries of symmetries and symmetry breakings. Fivefold symmetry, which remained a mystery for centuries, became material reality in the form of the quasicrystals discovered in the 1980s. The concept of symmetry, as a method, became an element of heuristics, being transferred from one branch of science to the next as an idea to catalyse intellectual creativity, not to mention the interplay between arts and sciences, and the effects of the role it has played in various different cultures. We only have to think of the intellectual proximity of the eightfold way of Buddha to the classification of elementary particles and the use of the SU(3) group that describes their symmetry. Or consider the role of Japanese origami in designing structures used in spaceships, and the history of the discovery of the artificial retina, which combines branches of science formerly considered distant from one another (the theory of analogue and digital chips, ancient logic, and cerebral asymmetries). Neither have the results of symmetry research left untouched such areas of scientific research as the thermodynamics of chemical equilibria, psychology, brain research, education science, musicology and sociolinguistics.

The last half century of research into symmetry has extended our knowledge manifold. Not only has the content of this knowledge become enriched — so has the concept of symmetry itself. When in the 1990s I began holding special interdiscipliniary symmetry lectures for students at the Faculty of Sciences at L. Eötvös University in Budapest, I had to confront the question of how to summarize all that we can and should know about symmetry today in a single semester — in about the same depth as Weyl described the knowledge of his day in his four lectures. Half a century ago, interpretations of symmetry were dominated by crystallography and crystallographic analogies. To be true to the proportions we see in science today, I can devote no more than two out of fifteen chapters to this approach. Over the years the lectures have developed and become more polished, and the proportions have also changed. I have been greatly helped by consultations with colleagues, with members of what was the International Symmetry Society and what is now the International Symmetry Association, and by correspondence with the authors of articles as editor of the journal *Symmetry: Culture and Science*. Also of great importance were questions and responses from students, and what I learned from examination discussions. This is what provided the material for this volume. For their very useful suggestions and proof-reading of the manuscript, I would like to express particular gratitude to Szaniszló Bérczi, as well as to László Beke, József Cseh, Gábor Gévay, István Hargittai and Ervin Hartmann. It is my pleasure to thank David Robert Evans for his great contributions to the English text.

In the course of the half century mentioned above, both the specialist literature on symmetry and the array of artistic interpretation embodied by works of art have expanded in unprecedented measure. In almost all disciplines, works have appeared discussing the symmetries and violations of symmetry in that field. As specialization has increased, so too has the number of writings discussing the points of interdependence. It was inevitable that results in one particular area based on symmetry considerations would inspire other fields of research. Interest in such work has also increased. It is no accident that D. Hofstadter's monograph *GEB* has become one of the best-read works of the last two decades. Hofstadter addresses the question of what is common in the intellectual legacy of "*G*ödel, *E*scher, *B*ach". Of course, his all-embracing work shows that all three personalities representing the main thrust of its line of thought embody the intellectual legacy of the unity of humanity, a legacy we can only truly appreciate when we make the boundaries between disciplines and art-forms, which have criss-crossed human culture in a largely artificial fashion, both transparent and traversable. It transpired that one of the key means for this could be a phenomenon, a concept, a method that is present in all of them. One such means is symmetry.

Without either denying or accidentally repeating the spirit and content of valuable earlier works dedicated to the presentation of the holistic way of thinking, I set myself the objective of writing a book which switches the perspective, putting symmetry at the focus of discussion, in the light of the scholarly knowledge we have at our disposal today. In the course of this it relies on the factual material gathered by its predecessors, keeping to its own set of proportions to present those facts in a different light and group them in an alternative way. In some chapters I have allowed my own personal observations to be expressed.

One of the book's goals is to present the unity and interdisciplinary nature of human culture. It attempts — in contrast to the division of reality into different disciplines by school education — to introduce the reader to the alternative view of the world provided by the holistic approach. In the interests of this, it emphasizes three types of possibility for bridging the split elements of that culture. First, between the various scholarly disciplines. Second, by presenting the interplay between arts and sciences. And third, with examples of ways in which the different cultures of various ages and geographical regions have influenced each other to produce new intellectual achievements.

In the course of this, the book discusses three approaches to symmetry: first, as a phenomenon; second, as a concept with varying content characterizing a group of phenomena; third, as an operation (or rather a well-defined group of operations) which is at once a method. How can symmetry operations serve as a method? They can represent a method for implementing analogies, for example. For the observation of common elements in various ages, cultures and branches of knowledge, which are invariant in the face of their differences, and for their implementation elsewhere. I would like to draw particular attention to the strengthening role of heuristics in this regard.

The discussion of symmetry as a subject in its own right gives us a particular slice of scientific endeavour, one which cannot be fitted within the framework of any traditional discipline. The mode of discussion is partly historical. In addition to the history of art, science and ideas, its historical nature also presents itself in its method. This can be seen in the way it repeatedly makes use of philosophical analyses in the course of the discussion.

In choosing my subject, I could not avoid taking certain constraints into consideration. On the one hand, adherence to the aforementioned proportions; on the other, overall length. I could not attempt a repetition of the whole of the rich literature concerning symmetry, or indeed the discussion or even mention of every single phenomenon associated with symmetry or symmetry violation. I had to select, and this selection reflects my own choice. This is what I thought it important, here and now, to say about symmetry. In considerable measure, my choice rested on my experiences noted above.

Through the chosen examples, I try to present a cross-section of the most significant areas of the interpretation and implementation of symmetry. If from time to time I succeed in evoking the feeling in the reader that I could have written more about this or about that, then the selection has achieved its goal: it has awakened associations, and connected the presented examples with knowledge the reader has from other sources. In this way it will have reconstructed, at the individual level, the bridges that symmetry has built in the collective consciousness of academics and artists between various branches of knowledge.

The first few chapters are a bit more dry in nature. Following the generalization of concepts and a historical introduction, I present some of the most successful interdisciplinary uses of the concept of symmetry. Then, tracing the path of the organization of matter from its physical structure, through the chemical, and molecules, which play a biologically important role, to living matter, the human brain, and finally to the products of human cognition and consciousness, I inspect the interdependence of art and science through the unusual lens of a series of symmetry breakings. The conceptual framework laid out at the start of the book will make it increasingly easy to become acquainted with the increasingly expanding world of this group of phenomena. Rather like a kaleidoscope, a lens, through which, on the following pages, we will observe the world, giving us a new way of seeing. I would like to emphasize this attitude as one of the book's important attributes. I would only be too pleased if its readers decide to adopt it, and put the approach they have learned and understood in these pages to use in their own respective disciplines.

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Introductory chapters

Chapter 1

Symmetry, invariance, harmony

The interpretation of the concept of symmetry in everyday life, science and art

The concept of symmetry

"How nice and symmetrical," we often say or think to ourselves. The associations that the word awakens in us depend on the experiences in our past that have established its meaning for us. The term 'symmetry' can have three separate types of meaning, as a phenomenon, a concept, or an operation. The *phenomenon* is what we consider to be symmetrical on the basis of our experience or of knowledge we have learned. The *concept* is what circumscribes all such phenomena. The *operation* is what gives rise to the phenomenon or makes it possible.

The etymology of the word 'symmetry'

The word 'symmetry is a combined word of Greek origin. Its two components are the prefix $\sigma\upsilon\nu$ [syn] and the word $\mu\epsilon\tau\rho(\iota)o\varsigma$ [metr(i)os]: $\sigma\upsilon\nu + \mu\epsilon\tau\rho(\iota)o\varsigma$. The prefix *syn-* can appear separately as an adverb, or, in combination, as a preposition. It means together, a group, simultaneously, common, or together with. It occurs in many other words in English, like sympathy, symphony, synthesis, syntax, synonym, synod, synagogue. The 'n' at the end of 'syn' often becomes an 'm'. The adverb $\mu\epsilon\tau\rho\acute{\iota}\omega\varsigma$ [metrios] means in (good) measure, suitably; as the adjective $\mu\tau\rho\iota o\varsigma$ it means measured, moderate, middling, average. $\mathrm{M}\epsilon\tau\rho\iota\tau\eta\varsigma$ [metriotes] means the correct measure, moderation, the right proportion. In Greek texts these are used in a number of figurative senses (for example, suitable, worthy, just, honourable, collected, modest, decent), and these figurative meanings were all to enrich the later content of the term symmetry that was made up of them.

The two-part verb $\sigma\upsilon\mu\mu\epsilon\tau\rho\omega$ [simmetreo] means measure by the same standard, compare; the adjective $\sigma\mu\mu\epsilon\tau\rho o\varsigma$ [simmetros] means of the same size, of the same content, commensurate, proportionate with something, suitable, moderated; in its adverbial form, $\sigma\upsilon\mu\mu\tau\rho\omega\varsigma$ [simmetros] means proportionately, in the right proportion, at the right time. Its meaning as the compound noun $\sigma\upsilon\mu\mu\epsilon\tau\rho\alpha$ [simmetria] is combined measurement, measurement through comparison, good proportions, and in a figurative sense agreement, harmony.

In short, as far as its content is concerned, to the Greeks two and a half thousand years ago the word symmetry meant *the common measure of things*.

As we think of it today, symmetry belongs to the great organizing concepts. It is a comprehensive concept that appears in many areas: in our everyday lives, in science, in art. We encounter it so often that we sometimes feel as if the world were only made up of symmetrical things. But before we attribute symmetry too much significance, let me emphasize that it is by no means the only concept or phenomenon we have as a comprehensive organizing principle. We have a need for organizing principles to deal with the many phenomena around us, the many experiences we acquire, and the multitude of knowledge we deduce from these. To help us, we call upon general concepts applicable in a variety of areas and with a variety of goals. Examples of concepts playing an organizing role in our thinking are order and orderliness, harmony, hierarchy, and system. In both a wider and narrower sense, the concepts of beauty, proportion and rhythm appear as characteristics with many common features; the same can be said of (logical and aesthetic) perfection (or attempts to achieve it), analogy, invariance under change (which is very close to symmetry), and, finally — as just another in this list — the concept of symmetry.

In what does the general nature of these concepts lie? What makes them suitable to act as organizing principles? Firstly, the fact that they are applicable to a wider group of phenomena, are group concepts, useable in various different areas of life, some more than others, but always to a number of phenomena. They are at once present in everyday life, academic discourse (the search for the truth), while a good number of them

Figure 1.1. *The Allegory of Symmetry.* D. Calvaert (1540–1619), Bologna. (Graphical archive of the Museum of Fine Arts, Budapest, K.66.25.)

serve as a benchmark in arts, in aesthetics (the search for beauty) or in ethics (the search for what is right or just). They help to give us a wide interdisciplinary survey of the world.

Of the above, perhaps it is symmetry which has — by conveying analogies, ideas and methods — played a role in inspiring creativity in the greatest number of fields. As it is present in all the four areas mentioned above, it is a suitable auxiliary concept for taking us right through the dangers and beauty of interdisciplinary thought. With its help, we are given an insight into how arts and sciences, different disciplines, and various different human cultures have affected and conceptually enriched one another.

What exactly is symmetry?

Let us try to begin with the ordinary meaning of the word and built up the academically-founded concept of symmetry. In the course of this journey, we first become acquainted with the everyday interpretations of the concept of symmetry. We then consider whether we essentially see the world around us symmetric. Is symmetry a natural phenomenon? It is in the light of the answers to the above questions that we can estimate the significance of symmetry violations.

On hearing the word symmetry, the majority of people think of the simplest examples of the concept.

The best-known example of geometric symmetry is *reflection*. If we reflect a (planar) shape in a linear one (the axis of symmetry), then it appears on the far side such that the respective points of the shape and its reflection are at the same distance from the axis, albeit in opposite directions. The figure retains its shape — in mirrored form — and its size and the angles between the lines connecting its various points are also unchanged, as is its colour. Reflection changes the direction of orientation, however: left and right are swapped. We have completed an operation, that of reflection, in the course of which certain characteristics of the reflected object have changed, but some have remained the same. It is these unchanged characteristics which represent the symmetry of the original figure in relation to its reflection. If we reflect the reflected image in the same axis again, we are returned to the original shape (Figure 1.2). Taking the same reflection operation one dimension further, we can produce the same results with spatial figures by using a mirror plane instead of an axis.

The second-most mentioned symmetry operation is *rotation*. If we rotate a (planar) figure around an axis perpendicular to the plane, the figure preserves its internal characteristics and the distance of its points from the axis. Its symmetry lies in keeping these properties intact. If we complete a rotation of such an angle that after a finite number of rotations — 2, 3, 4, 5, 6, ... — the figure exactly overlaps the original, then, depending on the angle of the rotation, we can talk of 2-, 3-, 4-, 5- or 6-fold symmetry. In general, if we rotate the object by an angle of $360°/n$, we term this n-fold symmetry, where n is a natural number. We can rotate spatial figures in the same way around an axis.

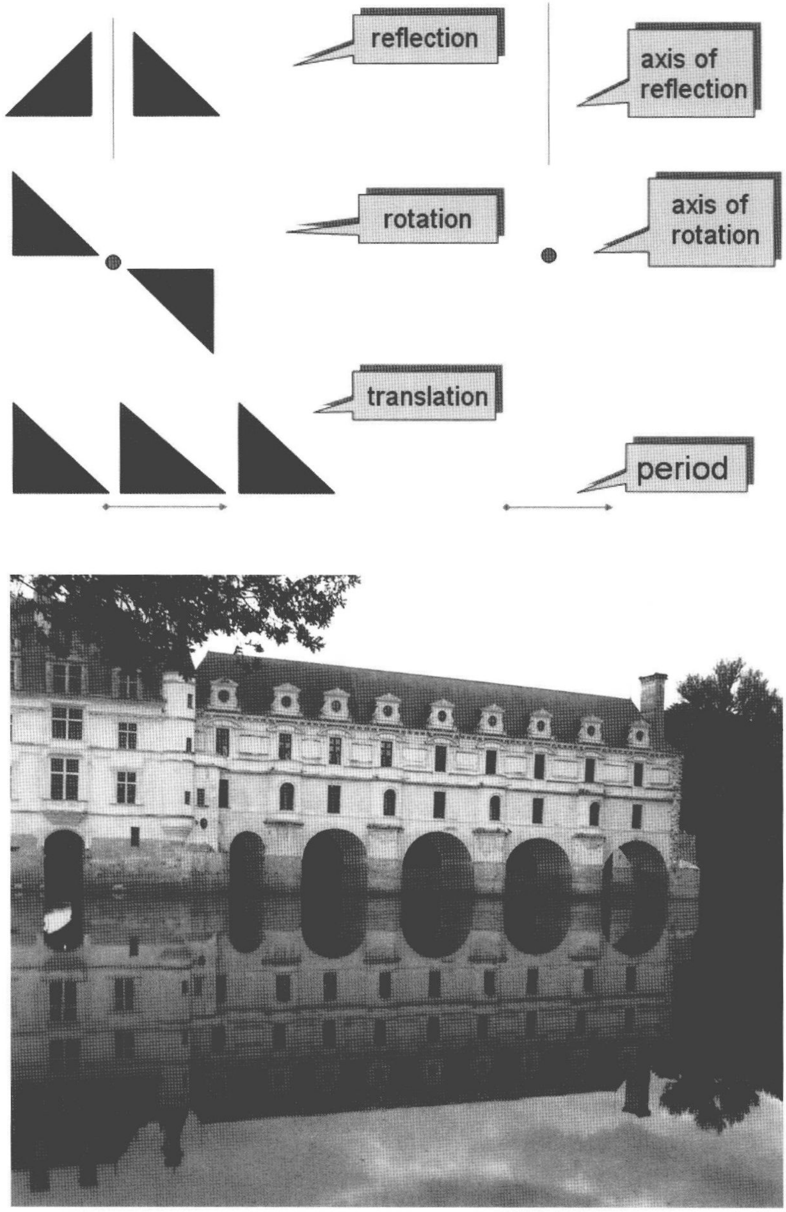

Figure 1.2. *Basic symmetry transformations, as well as the reflection and translation symmetry represented in a photo of Chenonceau Castle, France*

Figure 1.3. *M. C. Escher: Reptiles*

A good example of rotational symmetry is M. C. Escher's (1898–1972) mosaic of reptiles (Figure 1.3). For example, we can place an axis of rotation at the points where the reptiles' tails or right front legs touch. Rotating the reptile around these through 180° gives us another reptile of similar colour (twofold symmetry). If we turn the reptile through 60° around its left front leg, however, we overlap a reptile of a different colour. Let me draw the reader's attention to the fact that this design is also a beautiful artistic example of continuous planar layout — with figures that are congruent — for which Escher uses three reptiles of different colour but the same shape. If we are to mark the points where the possible axes of rotation would be, it becomes clear that they form a web of congruent regular triangles filling the plane of the drawing. The artist formed the congruent lizards by joining these points with similar broken lines, three by three. As seen in Figure 1.4, in a similar design, he even marked the lines of the web of triangles which assisted him.

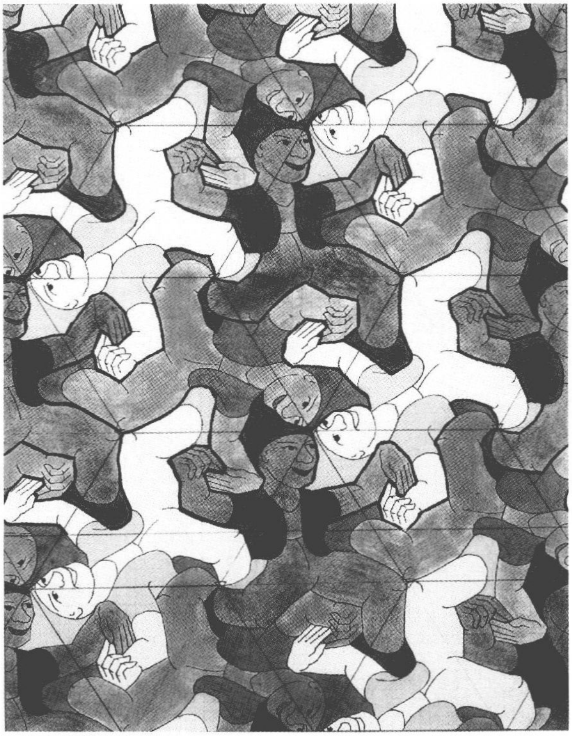

Figure 1.4. *M. C. Escher: Symmetrical Drawing 21 (1938)*(page)

The operation we have completed is a rotation around an axis. The object on which we completed it is a geometric figure. With the exception of its spatial position, almost all of its properties have remained intact, whatever we take our frame of reference to be.

Less common than the above two, but still regularly mentioned, is *translational* symmetry. If we translate a (planar) figure along a straight line with a uniform period, always in the same direction, we are given a repeated series of the given figure, in which the repeated elements are alike in every respect, and the distance between which is fixed on a periodic basis. Their symmetry lies in this uniformity. Translational symmetry is characterized by the direction of the translation and the length of the period. The symmetry operation in this instance is the straight linear shift (translation) we perform on a geometric object, as in the previous instances; in the process of this operation all of its characteristics are preserved, save its spatial location. (Figures 1.2 and 1.5).