Turbulent Shear Flows

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Selected Papers from
the Fifth International Symposium
on Turbulent Shear Flows

Editors:
F. Durst B.E. Launder J.L. Lumley
F.W. Schmidt J.H. Whitelaw

Springer-Verlag

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With 260 Figures

Springer-Verlag Berlin Heidelberg New York
London Paris Tokyo

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ISBN 3-540-16885-0 Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-16885-0 Springer-Verlag New York Heidelberg Berlin

Library of Congress Cataloging-in-Publication Data.
International Symposium on Turbulent Shear Flows (5th : 1985 : Cornell University)
Selected papers from the Fifth International Symposium on Turbulent Shear Flows, Cornell University, Ithaca, New York, USA, August 7–9, 1985.
1. Shear flow – Congresses. 2. Turbulence – Congresses. I. Durst, F. II. Title.
TA357.I59 1985 620.1′064 86-22076

Typesetting: Daten- und Lichtsatz-Service, Würzburg; Printing: Sala-Druck, Berlin;
Bookbinding: Lüderitz & Bauer, Berlin
2153/3020-543210

Preface

The first four symposia in the series on turbulent shear flows have been held alternately in the United States and Europe with the first and third being held at universities in eastern and western States, respectively. Continuing this pattern, the Fifth Symposium on Turbulent Shear Flows was held at Cornell University, Ithaca, New York, in August 1985. The meeting brought together more than 250 participants from around the world to present the results of new research on turbulent shear flows. It also provided a forum for lively discussions on the implications (practical or academic) of some of the papers. Nearly 100 formal papers and about 20 shorter communications in open forums were presented. In all the areas covered, the meeting helped to underline the vitality of current research into turbulent shear flows whether in experimental, theoretical or numerical studies.

The present volume contains 25 of the original symposium presentations. All have been further reviewed and edited and several have been considerably extended since their first presentation. The editors believe that the selection provides papers of archival value that, at the same time, give a representative statement of current research in the four areas covered by this book:

- Homogeneous and Simple Flows
- Free Flows
- Wall Flows
- Reacting Flows

Each of these sections begins with an introductory article by a distinguished worker in the field. These articles provide both a thumbnail sketch of the contributions made by the different papers and sets new contributions against the background of the research in the field. It is the editors' hope that in this way the book provides an up-to-date collection of high quality papers for the expert and, at the same time, offers sufficient signposts to help the newcomer (in conjunction with the earlier volumes in this series) to gain an appreciation of the present preoccupations in turbulent flow research.

The papers in this volume are arranged according to their complexity. The accounts of studies of homogeneous and simple flows include descriptions of modelling of pressure terms of the scalar fluxes and considerations on inhomogeneous turbulence with applications to boundary layer flows. Interactions of turbulent scales are studied and structural considerations of homogeneous turbulence are presented. Wake shear layer interactions are described and mixing layer and jet measurements presented. Particular attention is given to wall boundary layer flows providing new experimental and numerical results. The last section concentrates on experimental and numerical studies of combustion flows.

Financial support for the Fifth Symposium was generously contributed by the Boeing Aerospace Company, and the ASME-Heat Transfer Division also kindly provided material assistance. The success of the Cornell meeting depended strongly on the efforts of many

individuals in various aspects of the pre-conference organization as well as in the highly visible contributions during the Symposium itself.

As with earlier symposia responsibility for setting the technical programme rested with a *Papers Committee* which for the Fifth Symposium was composed of J. L. Lumley (Chairman), B. E. Launder, W. C. Reynolds and J. H. Whitelaw. Each of the nearly two hundred 1000-word abstracts offered for presentation at the Symposium were reviewed by two members of the Advisory Committee. Many Advisory Committee members later served as session chairmen at the Symposium and have throughout been valuable sources of advice and helpful criticism. The Advisory Committee consisted of:

R. J. Adrian
J.-C. Andre
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The editors of this book are also very thankful to the staff of Springer-Verlag who were actively involved and helped greatly in the completion of the present book.

As this volume was going to press, the turbulent shear flows community was saddened by the death of Stanley Corrsin after a long illness. Stan, who had contributed so much over forty years to the measurement and understanding of turbulence, served as a member of the Advisory Committee for all five TSF Symposia. To his memory, therefore, this volume is affectionately dedicated.

Erlangen, November 1986 The Editors

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Selected Papers from the First International Symposium on Turbulent Shear Flows, The Pennsylvania State University, University Park, Pennsylvania, USA, April 18-20, 1977

Editors: F. Durst, B.E. Launder, F.W. Schmidt, J.H. Whitelaw
1979. 256 figures, 4 tables. VI, 415 pages.
ISBN 3-540-09041-X

Here is a survey of the latest developments in the calculation of turbulent shear flows, emphasizing their flow and heat transfer properties. The improvement of physical understanding and related measurements is considered essential throughout. Skill-fully edited and carefully selected, a third of the papers presented at the symposium have been included in this volume.
There are five distinct themes:
- Free shear flows
- Wall flows
- Recirculating flows
- Stress transport models
- Modeling developments.

2
Selected Papers from the Second International Symposium on Turbulent Shear Flows, Imperial College London, July 2-4, 1979

Editors: L.J.S. Bradbury, F. Durst, B.E. Launder, F.W. Schmidt, J.H. Whitelaw
1980. 310 figures, 12 tables. IX, 391 pages.
ISBN 3-540-10067-9

The articles appearing in this volume were selected from contributions to the 2nd Turbulent Shear Flows Symposium held at the Imperial College London, July 2-4, 1979.
They reflect current research in five of the areas addressed by the symposium: mathematical modelling of turbulence, two-dimensional thin shear flows near walls, coherent structures, environmental flows; and complex flows involving recirculation and/or three-dimensional straining.

3
Selected Papers from the Third International Symposium on Turbulent Shear Flows, University of California, Davis, September 9-11, 1981

Editors: L. J. S. Bradbury, F. Durst, B. E. Launder, F. W. Schmidt, J. H. Whitelaw
1982. 269 figures. VIII, 321 pages. ISBN 3-540-11817-9

This volume is a collection of papers from the Third International Symposium on Turbulent Shear Flows held at the University of California, Davis, September 1981. The papers are divided into four sections: wall flows, scalar transport, recirculating flows and fundamentals. As with previous volumes, each section is preceded by a brief introductory article whose purpose is to make some general observations about the various sections and to fit the individual papers into the context of the general topic.

4
Selected Papers from the Fourth International Symposium on Turbulent Shear Flows, University of Karlsruhe, Karlsruhe, FRG, September 12-14, 1983

Editors: L. J. S. Bradbury, F. Durst, B. E. Launder, F. W. Schmidt, J. H. Whitelaw
1985. 286 figures. VIII, 397 pages. ISBN 3-540-13744-0

Contents: Fundamentals. - Free Flows. - Boundary Layers. - Reacting Flows. - Index of Contributors.

This is the fourth volume in a series which is destined to become a standard reference in the study of turbulent shear flows. It contains selected papers of the Symposium held at Karlsruhe, Germany, all carefully edited and reviewed. Each section is introduced by an authoritative article which fits the individual contributions into the larger context of current research. Compared with former volumes, greater emphasis is placed on experimental work and on the examination of complex flows. Three-dimensional, recirculating and reacting flows feature strongly in the programme and are complemented by considerations of two-phase flows and discussions of both numerical and experimental techniques.

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A Computational Method in Plasma Physics

1978. 22 figures. VIII, 144 pages. ISBN 3-540-08833-4

Springer-Verlag
Berlin Heidelberg New York
London Paris Tokyo

Springer

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Part III **Wall Flows**

Part IV **Reacting Flows**

Part I

Homogeneous and Simple Flows

Introductory Remarks

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The papers selected for this chapter are all concerned either directly or indirectly with the central problem of turbulence theory, which is to obtain realistic statistical solutions of the Navier-Stokes and scalar transport equations and apply them to practical situations. Before dealing with the papers in detail, it is important to remind ourselves of why this extremely complex topic developed the way it did, take stock of its achievements and review the direction it is heading.

Most efforts of the past have focused on homogeneous turbulence. Because of its relative simplicity, isotropic homogeneous turbulence has been studied the most. Between the years of 1930 and 1960 most of the basic formulations of the problem were made and the state of knowledge for that period is well summarized in books by Batchelor (1971), Townsend (1976) and Hinze (1975). It has not been possible even for the simple isotropic case to obtain general complete solutions for the differential equations involved except for the final period of decay of grid turbulence. The essential difficulty is that the Navier-Stokes equations when time or ensemble averaged do not yield a closed set of differential equations for the velocity covariance. To quote Kraichnan (1959) "the equation of motion for this covariance contain third-order moments of the velocity field, the equations of motion for the third-order moments contain fourth-order moments and so fourth *ad infinitum*. A central goal of turbulence theory is the closing of this infinite chain of coupled equations into a determinate set containing only moments below some finite order". During the 70's and up to the present day a world wide industry has sprung up devoted to this closure problem. The practical enigneer may well ask why we torture ourselves attempting what has proved to be a most difficult if not impossible task and why concentrate on homogeneous turbulence. Surely with modern large computers we could solve the complete time dependent Navier-Stokes equations by specifying the appropriate initial and boundary conditions and allow the solution to run its course.

As Leslie (1973) states (and it is still true today) "The short answer is that large though they are, present day computers are not large enough". A mesh must be sufficiently fine to resolve the dissipating eddies. The so called "Full Direct Simulations" or "Full Turbulence Simulations" mentioned in some of the papers here, attempt to do this. However, they are limited to low Reynolds numbers. For channel flow, Leslie quotes 10^8 mesh points are required for (Re) $= 10^4$ and 10^{13} for (Re) $= 10^6$ where (Re) is the channel flow Reynolds number. Also the number of time steps required for numerical stability rises rapidly with (Re). See also Corrsin (1961). For one of the papers here, the numbers of mesh points used is of order 10^6. Then there is the question of running time and cost. A simulation needs to be run many times to obtain ensemble averages with stable statistics although if homogeneous turbulence is considered we could average along the homogeneous direction (Lumley and Bejan Khajeh-Nouri, 1974). It has been the experience of the writer that using "nature's own computer" – namely, the wind tunnel, 40,000 data samples are needed for convergence of quantities like Reynolds shear stress at a point with 1% repeatability.

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The reason why there has been such an interest in homogeneous isotropic turbulence is that this is the case we know most about. In more general flow situations, perturbations about this "basic" flow are made and most papers considered in this chapter are concerned with various types of perturbations of this basic state. Many early schemes were based on the so called two-point closure methods where the equations for the two-point correlations were often solved in wavenumber space. Even when truncated to fourth order moments, the computational power required for such simulations can often rival that needed for the Full-Direct Simulations as is pointed out in the paper by Bertoglio and Jeandal given later in this chapter. Unless someone can come up with a better idea, it appears that the only hope we have at present of producing practical computational methods is to develop closure models for what Lumley (1978) calls "second order modelling" or "invariant modelling" or what the French call "one-point closure". The $k - \varepsilon$ model is an example of this. Inspired by some early ideas of Donaldson (1968), workers developed a general tensor notation and expressed everything as far as possible in an invariant form. All unknown correlations are expressed in terms of the second order correlations. With this invariant form, models developed for one flow geometry, with all of the associated empirically determined constants, have the greatest likelihood of being applicable in another flow geometry. However, since we need to truncate to a finite probability moment (usually the fourth) and because of the necessary over simplications made in the modelling details, there will probably never be a single "law of turbulence" based on this formulation for all cases. The method is basically an expansion about the homogeneous stationary case and Lumley (1978) sums up the underlying philosophy when explaining why it can be successfully used for flows with appreciable inhomogeneity; "by following a rational procedure we have created a physically possible phenomenon, not quite real turbulence perhaps, but one which conserves momentum and energy; transports the right amount of everything budgeted (momentum, energy, Reynolds stress, heat flux, etc.) although not by quite the right mechanism; satisfies realizability (so that non-negative quantities are never negative, Schwarz' inequality is always satisfied, etc.); behaves correctly for both large and small Reynolds numbers; and reduces to real turbulence in one limit (weak inhomogeneity and unsteadiness.) Probably any mechanism that satisfied all of these restrictions would behave about the same".

Many of the papers here are concerned with this modelling although there are also hybrid models. Let us consider the papers in detail.

The first paper is by T. Dakos and M. M. Gibson "*On Modelling the Pressure Terms of the Scalar Flux Equations*". The emphasis is placed on modelling the pressure and scalar gradient correlation $\left\langle \frac{p}{\varrho} \frac{\partial \theta}{\partial x_i} \right\rangle$ which appears in the equation for the scalar flux $\langle u_i \theta \rangle$. The method assumes nearly homogeneous undirectional flow with a weak mean scalar gradient. Part of the expression for $\left\langle \frac{p}{\varrho} \frac{\partial \theta}{\partial x_i} \right\rangle$ has been derived from theoretical considerations using formal solutions in wavenumber space. The remaining part is modelled using dimensional considerations and conventional ideas about the dependence on second moments and the anisotropy tensor. Various coefficients have been evaluated from the data of Sirivat and Warhaft (1982), Dakos (1985) and Tavoularis and Corrsin (1981). The authors hope in the near future to see if the model will correctly predict inhomogeneous flow data.

The second paper is by J. P. Bertoglio and D. Jeandel "*Simplified Spectral Closure for Homogeneous Turbulence: Application to the Boundary Layer*". The authors consider this as an "attempt to show that two-point closures can provide practical tools for computing complex flows". They point out that the application of the two-point closure scheme, particularly the Eddy Damped Quasi-Normal Markovian theory by Orszag (1970) to inhomogeneous turbulence in its full form, would probably require as much computational power as a Large Eddy Simulation or even a Fully Direct Simulation. However, the authors suggest that by means of certain crude assumptions perhaps some aspects of the two-point modelling (for the energy transfer between wavenumbers) could be combined with single point modelling (eg. the $k - \varepsilon$ model) for the inhomogeneous transport terms. Based on this, the authors claim to have produced a method of calculation which is "one step further in sophistication than the standard $k - \varepsilon$ model" and does not suffer from many of the disadvantages of that model. Comparison with the data of Klebanoff (1955) look promising, although these days there is an abundance of more extensive and detailed data which the authors could have chosen. Perhaps this data will be used in future work.

The third paper to be considered here is "*The Interaction of Two Distinct Turbulent Velocity Scales in the Absence of Mean Shear*" by S. Veeravalli and Z. Warhaft. Here we have a novel perturbation to homogeneous turbulence which is sufficiently simple to give insights into many aspects of scale interactions and transport phenomena. A *shearless* mixing layer bounded by two homogeneous fields of different properties is considered. The first experiment along these lines was carried out by Gilbert (1980). However, here a greater scale ratio of the two streams is considered and this reveals vital effects unobserved by Gilbert. The authors point out that the important point about this work is that this "second order" mixing layer focuses particular attention on the third and fourth moments and has a distinctly different character to the "first order" mixing layer where production of kinetic energy and shear stress play an important role. Furthermore with the shearless layer, simple gradient diffusion models are quite inadequate. The experimental results are preliminary but the facility could readily be used for investigating the transport of scalar quantities in the near future.

"*The Mixing Layer Between Turbulent Fields of Different Scales*" by S. B. Pope and C. D. Howarth follows on from the previous paper. Here a model is proposed which fits the data of Veeravalli and Warhaft rather well. The model is based on a transport equation for the joint probability density function of the three components of velocity as formulated by Pope (1981). A novel feature of the modelling is the use of a Lagrangian formulation for the dissipation and a simple relaxation equation is used to account for the fact that packets of fluid can traverse the layer from the large scale side (say) to the small scale side more rapidly than the turbulence in the packet can respond to its changed surroundings. As discussed by Veeravalli and Warhaft, the most revealing statistics are in the third and fourth moments.

In the next paper "*on the Structure of Homogeneous Turbulence*" by J. Lee and William C. Reynolds a Full Turbulence Simulation is being used. Here homogeneous turbulence is subjected to irrotational strains. A code developed by Rogallo (1981) is used to provide "data" which might be useful for the formulation of closure hypotheses in the "single point" or "second order" modelling methods as summarized by Lumley (1978). If the computations can be believed, considerable insights are provided. Anisotropy invariant maps are produced which have the boundaries identified by Lumley.

Turbulence which is initially isotropic is subjected to various irrotational strain rates. It is found that the vorticity field is more anisotropic than the velocity field, likely due to definition and Reynolds number. Most models including those of Launder et al. (1975) and Lumley (1978) assume that the "return to isotropy" tensor ϕ_{ij} is correlated with the anisotropy tensor b_{ij} whereas these simulations show that ϕ_{ij} correlates better with the dissipation rate anisotropy d_{ij}. A simple relationship is given which not only agrees with the runs given in this paper but also with the shear runs of Rogallo (1981). As he points out, these simulations are limited to a small range of eddy scales. Nevertheless, "data" can be provided concerning those aspects of turbulence which are most difficult to resolve in wind tunnel experiments. Hence such simulations might well compliment experiments for the formulation of more efficient closure schemes.

The final paper of this selection is "*Turbulence in a stably stratified shear Flow: A Progress Report*" by J. J. Rohr, K. N. Helland, E. C. Itsweire and W. C. Van Atta. This describes work of an experimental nature where sheared homogeneous turbulence is studied in a salt statified water channel. It is found that the results exhibit trends similar to ocean data in regions where the turbulence is growing under the combined influence of shear and buoyancy but not in regions close to the grid where buoyancy is relatively unimportant. It appears that the facility developed will be most relevant to the interpretation of oceanic microstructure data and for the modelling of turbulence in stratified fluids.

References

Batchelor, K. G. (1971): *The Theory of Homogeneous Turbulence* (2nd ed). (Cambridge University Press, London)

Corrsin, S. (1961): Turbulent Flow. Am. Sci. 49, 300

Dakos, T. (1985): Fundamental Heat-Transfer Studies in Grid Generated Homogeneous Turbulence. Ph.D. Thesis Imperial College, London

Donaldson, C. (1968): see Kline, S. J., Morkovin, M. J., Sovran, G., Cockrell, J. J. (eds). Proc. Comput. Turbulent Boundary Layers 1968 AFOSR-IFP-Stanford. Thermosci. Div. Stanford University, Stanford, California, 114–118

Gilbert, B. (1980): Diffusion mixing in grid turbulence without mean shear. J. Fluid Mech. **100**, 349–365

Hinze, J. O. (1975): *Turbulence* 2nd ed. (McGraw-Hill, New York)

Klebanoff, P. S. (1955): Characteristics of turbulence in a boundary layer with zero pressure gradient. NACA Report 1247

Kraichnan, R. H. (1959): The structure of isotropic turbulence at very high Reynolds numbers. J. Fluid Mech. **15**, 497–543

Launder, B. E., Reece, G. J., and Rodi, W. (1975): Progress in the development of a Reynolds stress turbulent closure: J. Fluid Mech. **68**, 537–566

Leslie, D. C. (1973): *Developments in the Theory of Turbulence* (Clarendon Press, Oxford)

Lumley, J. L., Bejan Khajeh-Nouri (1974): Computational modeling of turbulent transport. Adv. Geophys. **18A**, 169–192

Lumley, J. L. (1978): Computational modeling of turbulent flows. Adv. Appl. Mech. **18**, 23–176

Orszag, S. A. (1970): Analytical theories of turbulence. J. Fluid Mech. **41**, 363–386

Pope, S. B. (1981): Transport equation for the joint PDF of velocity and scalars in turbulent flow. Phys. Fluids **24**, 588–596

Rogallo, R. S. (1981): Numerical experiments in homogeneous turbulence. NASA T. M. 81315

Sirivat, A., Warhaft, Z. (1982): The effect of a passive cross-stream temperature gradient on the evolution of temperature variance and heat flux in grid turbulence. J. Fluid Mech. **120**, 475

Townsend, A. A. (1976): *The Structure of Turbulent Shear Flow,* 2nd ed. (Cambridge University Press)

Travoularis, S., Corrsin, S. (1981): Experiments in nearly homogeneous turbulence with a uniform mean temperature gradient. J. Fluid Mech. **104**, 311

On Modelling the Pressure Terms of the Scalar Flux Equations

T. Dakos and M. M. Gibson

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Abstract

The paper is concerned with the problem of modelling the ensemble averaged product of the pressure and scalar gradient that appears in the equations for the scalar flux $\langle u_i\theta\rangle$. The method (for nearly homogeneous unidirectional flow with weak gradients) is to derive nonlinear expressions for the fluctuating parts of the pressure and the scalar by formal solution (in wave vector space) of the Navier-Stokes and the scalar equations. Substitution in the Fourier transform of the pressure correlation shows that this quantity is the sum of four terms, one of which contains the mean scalar gradient. A fourth-order cumulant discard approximation allows this term to be expressed in terms of the single point double products and the turbulent energy and stress spectra. A numerical calculation is made when the spectra are represented by simple functions. Finally, a tentative model is proposed in which the remaining terms of the pressure correlation are evaluated by reference to experimental data.

Nomenclature

b_{ij}	Anisotropy tensor $2\langle u_i u_j\rangle/q^2 - 2\delta_{ij}/3$
$C_{\theta 1} - C_{\theta 5}$	Turbulence model constants
$I(r, t)$	Quantity defined by Eq. (7)
$G_0(\boldsymbol{r}, t; \boldsymbol{r}_1, t_1)$	Green's function, Eq. (10)
$\boldsymbol{k}$	Wave vector
k_{ij}	j component of k_i vector
$N(\boldsymbol{k}, t)$	Fourier transform defined by Eq. (5)
P	Turbulent energy production rate
p	Fluctuating part of the pressure
Pr_t	Turbulent Prandtl number
$\boldsymbol{r}$	Position vector
$S_{ij}(\boldsymbol{k}, t, t_1)$	Two-time energy spectrum tensor
$S_{ij}(\boldsymbol{k}, t)$	One-time energy spectrum tensor
T	Mean value of scalar quantity
t	Time
U	Mean velocity
$\boldsymbol{u}$	Velocity vector
$\langle u_i u_j\rangle$	Reynolds stress
$\langle u_i\theta\rangle$	Scalar flux
V	Volume
v_0^2	$\frac{1}{3}q^2$
$q^2 = \langle u_i u_i\rangle$	$2 \times$ turbulent kinetic energy
x_i $(i = 1, 3)$	Cartesian coordinates
α, β, γ	Constants in spectrum models
δ	Dirac delta function
δ_{ij}	Kronecker delta
ε	Turbulent energy dissipation rate
θ	Fluctuating part of scalar quantity
λ	Diffusivity
ϱ	Fluid density
$\Phi_{i\theta}$	Pressure-scalar-gradient correlation

Subscripts and Superscripts

i, j, k	Tensor indices
$*$	Complex conjugate

Introduction

In order to close the equations for the turbulent stresses and scalar fluxes it is necessary to model the correlations that contain the fluctuating part of the pressure. Because the pressure interactions are responsible for the distribution of turbulent kinetic energy between components, it has been convenient to rearrange the pressure term in the stress equations as the sum of a divergence (which is added to the turbulent diffusion) and a traceless distributive term (the pressure-strain correlation). Rotta [1] appears to have been the first to recognize the value of further splitting the pressure-strain correlation into a non-linear turbulence part,

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and a "rapid" part containing the mean velocity gradient, so that the two parts could be modelled separately. The first step in the analogous treatment of the pressure terms in the scalar-flux equations is to introduce the fluctuating scalar gradient by writing

$$\left\langle \frac{\theta}{\varrho} \frac{\partial p}{\partial x_i} \right\rangle = \delta_{ik} \left\langle \frac{\partial}{\partial x_k} \left(\frac{p\theta}{\varrho} \right) \right\rangle - \left\langle \frac{p}{\varrho} \frac{\partial \theta}{\partial x_i} \right\rangle, \tag{1}$$

where the first part may be added to the transport terms as "pressure diffusion". The second step is to take the divergence of the Navier-Stokes equations and multiply the solution of the resulting Poisson equation for p by $d\theta/dx_i$. The well-known result [2]

$$\left\langle \frac{p}{\varrho} \frac{\partial \theta}{\partial x_i} \right\rangle = \frac{1}{4\pi} \int \left\{ \left\langle \left(\frac{\partial^2 u_i u_j}{\partial x_i \partial x_j} \right)' \frac{\partial \theta}{\partial x_i} \right\rangle + 2 \left(\frac{\partial U_i}{\partial x_j} \right)' \left\langle \left(\frac{\partial u_j}{\partial x_i} \right)' \frac{\partial \theta}{\partial x_j} \right\rangle \right\} \frac{dV}{r} \tag{2}$$

(where the primes mean that quantities so marked are evaluated at $\boldsymbol{x} + \boldsymbol{r}$) indicates that the pressure-scalar-gradient correlation also consists of turbulence and velocity-gradient components. The mean scalar gradient does not appear explicitly here and it is also usually omitted from models of the correlation, but not by Jones and Musonge [3]. These authors argue that, because both terms in the integral depend on the mean field, there is no point in modelling them separately. They show that the inclusion of a scalar-gradient term in the model, and the use of a scalar time scale in the turbulence-only part, produces good results for the strongly sheared, nearly-homogeneous, free shear flow realized by Tavoularis and Corrsin [4], where appreciable departures from equilibrium provide a demanding test for modelling techniques.

We now approach the problem from a different direction, following closely the lead of Weinstock [5, 6], who has used the Fourier-transform method to calculate the pressure-strain terms in the stress equations for a simple turbulent shear flow at high Reynolds numbers. We now use this method to derive nonlinear expressions for the fluctuating parts of the pressure and the scalar by formal solution (in wave-vector space) of the Navier-Stokes and the scalar equations. These expressions are substituted in the Fourier transform of the pressure-scalar-gradient correlation which is then expressed as the sum of four terms in turbulence quantities only, the mean velocity gradient, the mean scalar gradient, and the product of the two mean gradients. Although we have, for simplicity, limited the analysis to nearly homogeneous unidirectional flow with weak gradients, the results are still too complicated for immediate application to practical flow calculation. It is, however, possible to express the scalar-gradient term in terms of the single-point velocity products and the turbulent energy and stress spectrum functions. The method (like Weinstock's) is based on the neglect of the cumulant of the two-time fourth-order correlation that is basic to the direct-interaction approximation but here, as in Weinstock's analysis, the goal is not to calculate the spectra but to derive part of the pressure correlation in terms of the spectra. An approximate calculation of the term is made when the spectra are represented by simple functions. Finally, a tentative model is proposed in which the remaining terms are evaluated by reference to measurements in equilibrium free flow and to the data of Tavoularis and Corrsin [4]. The analysis is straightforward but lengthy, particularly for the second part concerning the scalar-gradient term. For some details the reader is referred to the two papers by Weinstock (as a guide to the method) and to the thesis by Dakos [7].