LECTURE NOTES IN PHYSICS

Arthur D. Yaghjian

Relativistic Dynamics of a Charged Sphere

Updating the Lorentz-Abraham Model

Second Edition

Springer

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To

LUCRETIA

Foreword

This is a remarkable book. Arthur Yaghjian is by training and profession an electrical engineer; but he has a deep interest in fundamental questions usually reserved for physicists. He has studied the relevant papers of an enormous literature that accumulated for longer than a century. The result is a fresh and novel approach to old problems providing better solutions and contributing to their understanding.

Physicists since Lorentz in the late nineteenth century have looked at the equations of motion of a charged object primarily as a description of a fundamental particle, typically the electron. Since the limitations of classical physics due to quantum mechanics have long been known, Yaghjian considers a macroscopic object, a spherical insulator with a surface charge. He thus avoids the pitfalls that have misguided research in the field since Dirac's famous paper of 1938.

The first edition of this book, published in 1992, was an apt tribute to the centennial of Lorentz's seminal paper of 1892 in which he first proposed the extended model of the electron. In the present second edition, attention is also paid to very recent work on the equation of motion of a classical charged particle. Mathematical approximations for specific applications are clearly distinguished from the physical validity of their solutions. It is remarkable how these results call for empirical tests yet to be performed at the necessarily extreme conditions and with sufficiently high accuracy. In these important ways, the present book thus revives interest in the classical dynamics of charged objects.

Syracuse University
2005

Fritz Rohrlich

Preface to the Second Edition

Chapters 1 through 6 and the Appendices in the Second Edition of the book remain the same as in the First Edition except for the correction of a few typographical errors, for the addition and rewording of some sentences, and for the reformatting of some of the equations to make the text and equations read more clearly. A convenient three-vector form of the equation of motion has been added to Chapter 7 that is used in expanded sections of Chapter 7 on hyperbolic and runaway motions, as well as in Chapter 8. Several references and an index have also been added to the Second Edition of the book.

The method used in Chapter 8 of the First Edition for eliminating the noncausal pre-acceleration from the equation of motion has been generalized in the Second Edition to eliminate pre-deceleration as well. The generalized method is applied to obtain the causal solution to the equation of motion of a charge accelerating in a uniform electric field for a finite time interval. Alternative derivations of the Landau-Lifshitz approximation to the Lorentz-Abraham-Dirac equation of motion are also given in Chapter 8 along with Spohn's elegant solution of this approximate equation for a charge moving in a uniform magnetic field. A necessary and sufficient condition is found for this Landau-Lifshitz approximation to be an accurate solution to the exact Lorentz-Abraham-Dirac equation of motion.

Many of the additions that have been made to the Second Edition of the book have resulted from illuminating discussions with Professor W.E. Baylis of the University of Windsor, Professor Dr. H. Spohn of the Technical University of Munich, and Professor Emeritus F. Rohrlich of Syracuse University. Dr. A. Nachman of the United States Air Force Office of Scientific Research supported and encouraged much of the research that led to the Second Edition of the book.

Concord, Massachusetts
2005

Arthur D. Yaghjian

Preface to the First Edition

This re-examination of the classical model of the electron, introduced by H. A. Lorentz 100 years ago, serves as both a review of the subject and as a context for presenting new material. The new material includes the determination and elimination of the basic cause of the pre-acceleration, and the derivation of the binding forces and total stress-momentum-energy tensor for a charged insulator moving with arbitrary velocity. Most of the work presented here was done while on sabbatical leave as a guest professor at the Electromagnetics Institute of the Technical University of Denmark.

I am indebted to Professor Jesper E. Hansen and the Danish Research Academy for encouraging the research. I am grateful to Dr. Thorkild B. Hansen for checking a number of the derivations, to Marc G. Cote for helping to prepare the final camera-ready copy of the manuscript, and to Jo-Ann M. Ducharme for typing the initial version of the manuscript.

The final version of the monograph has benefited greatly from the helpful suggestions and thoughtful review of Professor F. Rohrlich of Syracuse University, and the perceptive comments of Professor T. T. Wu of Harvard University.

Concord, Massachusetts
April, 1992

Arthur D. Yaghjian

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Contents

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Introduction and Summary of Results

The primary purpose of this work is to determine an equation of motion for the classical Lorentz model of the electron that is consistent with causal solutions to the Maxwell-Lorentz equations, the relativistic generalization of Newton's second law of motion, and Einstein's mass-energy relation. (The latter two laws of physics were not discovered until after the original works of Lorentz, Abraham, and Poincaré. The hope of Lorentz and Abraham for deriving the equation of motion of an electron from the self force determined by the Maxwell-Lorentz equations alone was not fully realized.) The work begins by reviewing the contributions of Lorentz, Abraham, Poincaré, and Schott to this century-old problem of finding the equation of motion of an extended electron. Their original derivations, which were based on the Maxwell-Lorentz equations and assumed a zero bare mass, are modified and generalized to obtain a nonzero bare mass and consistent force and power equations of motion. By looking at the Lorentz model of the electron as a charged insulator, general expressions are *derived* for the binding forces that Poincaré *postulated* to hold the charge distribution together. A careful examination of the classic Lorentz-Abraham derivation reveals that the self electromagnetic force must be modified during a short time interval after the external force is first applied and after all other nonanalytic points in time of the external force. The resulting modification to the equation of motion, although slight, eliminates the noncausal pre-acceleration (and pre-deceleration) that has plagued the solution to the Lorentz-Abraham equation of motion. As part of the analysis, general momentum and energy relations are derived and interpreted physically for the solutions to the equation of motion, including "hyperbolic" and "runaway" solutions. Also, a stress-momentum-energy tensor that includes the binding, bare-mass, and electromagnetic momentum-energy densities is derived for the charged insulator model of the electron, and an assessment is made of the redefinitions of electromagnetic momentum-energy that have been proposed in the past to obtain a consistent equation of motion.

Many fine articles have been written on the classical theories of the electron, such as [7], [32], [41], [42], [52], [71], and [72], to complement the original works by Lorentz [4], Abraham [3], Poincaré [19], and Schott [16]. However, in returning to the original derivations of Lorentz, Abraham, Poincaré, and Schott, re-examining them in detail, modifying them when necessary, and supplementing them with the results of special relativity not contained explicitly in the Maxwell-Lorentz equations, it is possible to clarify and resolve a number of the subtle problems that have remained with the classical theory of the Lorentz model of the extended electron.

An underlying motivation to the present analysis is the idea that one can separate the problem of deriving the equation of motion of the extended model of the electron from the question of whether the model approximates an actual electron. Hypothetically, could not one enter the classical laboratory, distribute a charge e uniformly on the surface of an insulating sphere of radius a, apply an external electromagnetic field to the charged insulator and observe a causal motion predictable from the relativistically invariant equations of classical physics? Moreover, the short-range polarization forces binding the excess charge to the surface of the insulator need not be postulated, but should be derivable from the relativistic generalization of Newton's second law of motion applied to both the charge and insulator, and from the requirement that the charge remain uniformly distributed on the spherical insulator in its proper inertial frame of reference. A summary of the results in each of the succeeding chapters follows.

Chapter 2 introduces the original Lorentz-Abraham force and power equations of motion for Lorentz's relativistically rigid model of the electron moving without rotation[1] with arbitrary velocity. Lorentz and Abraham derived their force equation of motion by determining the self electromagnetic force induced by the moving charge distribution upon itself, and setting the sum of the externally applied and self electromagnetic force equal to zero, that is, they assumed a zero "bare mass." Similarly, they derived their power equation of motion by setting the sum of the externally applied and self electromagnetic power (work done per unit time by the forces on the charge distribution) equal to zero.

To the consternation of Abraham and Lorentz, these two equations of motion were not consistent. In particular, the scalar product of the velocity of the charge center with the self electromagnetic force (force equation of motion) did not equal the self electromagnetic power (power equation of motion). Merely introducing a nonzero bare mass into the equations of motion does not remove this inconsistency between the force and power equations of motion. Moreover, it is shown that the apparent inconsistency between self electromagnetic force and power is not a result of the electromagnetic mass in

[1] The work of Nodvik [8, eq. (7.28)] shows that the effect of a finite angular velocity of rotation on the self force and power of the Lorentz model approaches zero to the order of the radius of the charge as it approaches zero and thus classical rotational effects are of the same order as the higher order terms neglected in the Lorentz-Abraham equations of motion.

the equations of motion equaling 4/3 the electrostatic mass, nor a necessary consequence of the electromagnetic momentum-energy not transforming like a four-vector. The 4/3 factor occurs in both the force and power equations of motion, (2.1) and (2.4), and it was of no concern to Abraham, Lorentz, or Poincaré in their original works which, as mentioned above, appeared before Einstein proposed the mass-energy relationship.

Neither the self electromagnetic force-power nor the momentum-energy transforms as a four-vector. (For this reason, they are referred to herein as force-power and momentum-energy rather than four-force and four-momentum.) However, there are any number of force and power functions that could be added to the electromagnetic momentum and energy that would make the total momentum-energy (call it G^i) transform like a four-vector, and yet not satisfy $\mathrm{d}G^i/\mathrm{d}s\ u_i = 0$, so that the inconsistency between the force and power equations of motion would remain. Conversely, it is possible for the proper time derivatives of momentum and energy (force-power) to transform as a four-vector and satisfy $\mathrm{d}G^i/\mathrm{d}s\ u_i = 0$ without the momentum-energy G^i itself transforming like a four-vector. In fact, Poincaré introduced binding forces that removed the inconsistency between the force and power equations of motion, and restored the force-power to a four-vector, without affecting the 4/3 factor in these equations or requiring the momentum and energy of the charged sphere to transform as a four-vector.

The apparent inconsistency between the self electromagnetic force and power is investigated in detail in Chapter 3 by reviewing the Abraham-Lorentz derivation and rigorously rederiving the electromagnetic force and power for a charge moving with arbitrary velocity. For the Lorentz model of the electron moving with *arbitrary* velocity, one finds that the Abraham-Lorentz derivation depends in part on differentiating with respect to time the velocity in the electromagnetic momentum and energy determined for a charge distribution moving with *constant* velocity. Although Lorentz and Abraham give a plausible argument for the validity of this procedure, the first rigorous derivation of the self electromagnetic force and power for the Lorentz electron moving with arbitrary velocity was given by Schott in 1912, several years after the original derivations of Lorentz and Abraham. Because Schott's rigorous derivation of the electromagnetic force and power, obtained directly from the Liénard-Wiechert potentials for an arbitrarily moving charge, is extremely involved and difficult to repeat, a much simpler, yet rigorous derivation is provided in Appendix B.

It is emphasized in Section 3.1 that the self electromagnetic force and power are equal to the internal Lorentz force and power densities integrated over the charge-current distribution of the extended electron, and thus one has no a priori guarantee that they will obey the same relativistic transformations as an external force and power applied to a point mass. An important consequence of the rigorous derivations of the electromagnetic force and power of the extended electron, with arbitrary velocity, is that the integrated self electromagnetic force, and thus the Lorentz-Abraham force equation of motion of the extended electron, is shown to transform as an external force applied to a point mass. However, the rigorous derivations also reveal that the integrated self electromagnetic power, and thus the Lorentz-Abraham power equation of motion, for the relativistically rigid model of the extended electron do not transform as the power delivered to a moving point mass. This turns out to be true even when the radius of the charged sphere approaches zero, because the internal fields become singular as the radius approaches zero and the velocity of the charge distribution is not the same at each point on a moving, relativistically rigid shell. Thus, it is not permissible to use the simple point-mass relativistic transformation of power to find the integrated self electromagnetic power of the extended electron in an arbitrarily moving inertial reference frame from its small-velocity value. (This is unfortunate because the proper-frame and small-velocity values of self electromagnetic force and power, respectively, are much easier to derive than their arbitrary-frame values from a series expansion of the Liénard-Wiechert electric fields; see Appendix A.)

The rigorous derivations of self electromagnetic force and power in Chapter 3 critically confirm the discrepancy between the Lorentz-Abraham force and power equations of motion. Chapter 4 introduces a more detailed picture of the Lorentz model of the electron as a charge uniformly distributed on the surface of a nonrotating insulator that remains spherical with radius a in its proper inertial reference frame. (The values of the permittivity and permeability inside the insulating sphere are assumed to equal those of free space.) Applying the relativistic version of Newton's second law of motion to the surface charge and insulator separately, we prove the remarkable conclusion of Poincaré that the discrepancy between the Lorentz-Abraham force and power equations of motion is caused by the neglect of the short-range polarization forces binding the charge to the surface of the insulator. Even though these short-range polarization forces need not contribute to the total self force or rest energy of formation, they add to the total self power an amount that exactly cancels the discrepancy between the Lorentz-Abraham force and power equations of motion. Moreover, the power equation of motion modified by the addition of the power delivered by the binding forces now transforms relativistically like power delivered to a point mass. With the addition of Poincaré binding forces, the power equation of motion of the Lorentz model of the electron derives from the Lorentz-Abraham force equation of motion, and no longer needs separate consideration.

Of course, Poincaré did not know what we do today about the nature of these surface forces when he published his results in 1906, so he simply assumed the necessity of "other forces or bonds" that transformed like the electromagnetic forces. Also, Poincaré drew his conclusions from the analysis of the fields and forces of a charged sphere moving with constant velocity; see Section 4.1. The derivation in Section 4.2 from the relativistic version of Newton's second law of motion reveals, in addition to the original Poincaré stress, both "inhomogeneous" and "homogeneous" surface stresses that are