THE EARTH

ITS ORIGIN
HISTORY AND PHYSICAL CONSTITUTION

BY

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FOURTH EDITION

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TO THE MEMORY OF

SIR GEORGE HOWARD DARWIN

PREFACE TO THE FOURTH EDITION

In the fourth edition many recent developments have been mentioned. It is now probably impossible for one person to be expert over the whole of geophysics, and many important contributions may have been treated too briefly.

The principal change is in the treatment of tidal friction. Imperfection of elasticity at small stresses has long been the only apparent explanation of the rotations of the Moon, Mercury, and several satellites, but there has been no basis for an estimate of its amount. This has at last been provided by the damping of the 14-monthly variation of latitude. Various laws of imperfection have been considered, but that with most evidence in its favour is a modification of one due to C. Lomnitz; this leads to satisfactory agreement with other data. It has the consequence that the type of creep needed to account for polar wandering, convection currents, and continental drift must be smaller than has ever appeared hitherto. Some writers have suggested that the peculiarities of the variation of latitude are not due to random disturbances with damping, but to non-cumulative forced disturbances and changes of free period; I consider these less satisfactory, but even if they are accepted they would imply still less damping and less imperfection of elasticity in long-continued disturbances.

Recent investigations of the thermal history of the Earth have mostly been based on cosmogonical hypotheses, and imply a rising temperature and an all-time solid state. This would contradict the thermal contraction theory of mountain formation. New discoveries and corrections to the theory have sometimes increased and sometimes decreased the estimates of the available and required contraction. The chief increase in the available contraction, since the third edition of this book was published, comes from Hales's revision of the mechanism; on the other hand, physical theory has reduced the estimate of the coefficient of thermal expansion at great depths, while the increased estimate of the age of the Earth, which now seems well established, puts a smaller fraction of the contraction into Cambrian and later times. However, there still seems to be no satisfactory alternative mechanism. What still seems to me decisive evidence for former fluidity is the existence of the granitic layer and the concentration of radioactivity toward the surface. If rocks down to 300 km. depth were as radioactive as surface rocks, the heat would have had time to reach the surface and the rate of heat outflow would be about 20 times what it is. Goldschmidt's mechanism explains how, in a formerly fluid Earth, radioactive elements could have become concentrated in a thin surface layer, and I know of no alternative.

The section on the Moon's surface features has been somewhat expanded. I am undecided between the igneous hypotheses and some form of the meteoritic hypothesis; but I am sure that some accounts of the latter are unfair to the former.

HAROLD JEFFREYS

ST JOHN'S COLLEGE
CAMBRIDGE
1958 September 30

PREFACE TO THE THIRD EDITION

There have been great advances in all branches of geophysics since the publication of the second edition of this book in 1929, and corresponding changes have been necessary in the present edition.

The four chapters relating to the origin of the Solar System have been deleted. They contained some results that need to be taken into account in any theory, but the theory as developed in them has needed drastic alteration, and it is probable that one on totally different lines is needed. Consequently, I have been content to give a general description in Chapter IX of the main features of some recent theories, with references for further study. This shortening has made it possible to attend to recent developments in the specifically geophysical problems without greatly increasing the size of the book.

The remaining chapters have been extended and almost completely rewritten. Seismology has now yielded information in some detail about the elastic properties of the Earth all the way to the centre, and in combination with results given by the study of the Earth's gravitational field it gives also much fuller information about the distribution of density and important indications about the probable composition. The theory of the Earth's external field has been developed to the second order in the ellipticity, since this is necessary for a consistent discussion of modern observations. Isostasy and its imperfections are discussed, and the results are applied to give estimates of the strength of the Earth's outer parts. The Earth's age and thermal history are treated with regard to recent developments, and the account of the origin of the Earth's surface features has been made much fuller. Some special problems are discussed in the last chapter.

As there are many instances in this book where several references are made to the same book or paper, they are made to a bibliography at the end to reduce repetition. The system is approximately that used by the Royal Society, which itself differs slightly from the 'Harvard system'. The outstanding recommendation of these systems is that, by making the date second in importance only to the author's name in identifying a paper, they help the reader to see whether the results of one paper can be discussed in another—a paper of 1927 is unlikely to make use of results published in 1938. On the other hand it is often useful to know the length of a paper, and these systems make no explicit provision for giving it; I give both initial and final pages. For instance, the reader may wish to know whether a paper is an extended discussion or a brief note, possibly an abstract. When a reference is to a particular page in a paper, the page is indicated in the text; it will not be necessary to scan a 40-page paper for something discussed in one paragraph in the middle. In the absence of such an indication of page it may be taken that the reference is to the paper as a whole.

I do not enclose the date in brackets. In many years' experience the British Association Mathematical Tables Committee and its successor, the corresponding committee of the Royal Society, have found that legibility is improved by having white spaces around the entry that is meant to catch the eye. Brackets about the date delay the eye in finding it.

Where there are several references to the same author, the author's name is not repeated. To identify a paper it is necessary to find first the name and then the date; and the second step is easier if the reader looks down a series of entries, in each of which the date is the first item, than if he has to look down the interior of a block of type and make sure on the way that the name has not changed. Nobody would think of repeating the name of a substance in every determination of its properties quoted in a table of physical constants.

As both these departures from the practice recommended tend to reduce the cost of printing there may be other reasons for recommending them for wider adoption.

The method of numbered references is unsatisfactory, because insertion of a new one entails renumbering, and therefore unnecessary labour and risk of mistakes.

Additional matter added at a late stage of printing is given in notes at the end. Such additions are indicated by a Clarendon index letter in the text; for instance, an entry[a] in section **10·05** would refer to a note **10·05a** at the end.

The main sections are numbered decimally at intervals of 0·01; subsections are indicated by further decimals. When the argument of a section or subsection continues that of the previous one, the numbering of the equations also continues.

I must express my thanks again to Dr R. Stoneley, who has read the whole in manuscript and made valuable suggestions.

In response to appeals from the hard-worked administrative staff of the University, I should like to point out to possible correspondents that communications addressed to The University, Cambridge, are delivered at the University Offices and have to be re-addressed and forwarded by post. My address is that given below.

HAROLD JEFFREYS

ST JOHN'S COLLEGE
CAMBRIDGE

1950 August 2

PREFACE TO THE FIRST EDITION

During the years 1920–23 I delivered three times in this college a course of eight lectures on the Physics of the Earth's Interior. The course was intended to give an outline of present knowledge concerning what may be called the major problems of geology, namely the physical constitution of the earth, the causes of mountain formation, and the nature of isostasy. It was, however, impossible to give in the lectures full accounts of the arguments employed, partly because the course was too short, and partly because the mathematical knowledge of the listeners was extremely varied. Accordingly this book has been written; the argument has been made as connected as appeared possible, and various geophysical topics that could not be discussed in the lectures, such as the variation of latitude, have been introduced. The aim has been to discuss the theories of the main problems of geophysics, and to exhibit their interrelations. Several large branches have, however, been almost entirely omitted; terrestrial magnetism, atmospheric electricity, tidal theory (apart from tidal friction) and meteorology have received little or no attention, because to give anything approaching an adequate account of any of them would have required a longer discussion than their connexion with the original topics of the book seemed to warrant.

I have attempted to describe the present position of the subject rather than its history. For this reason several pieces of work of capital importance in the development of geophysics have escaped mention. Lord Kelvin's estimate of the rigidity of the earth from its tidal yielding is a case in point; it has not been discussed because more detailed and definite information can now be derived from seismology. Sir G. H. Darwin's pioneer work on tidal friction, again, has been only outlined, because it was mainly a discussion of bodily tides in a homogeneous earth, which now appear to be comparatively unimportant in influencing the evolution of the earth and moon. Nevertheless if the contents of the second volume of Darwin's collected papers had not been published, it is improbable that Chapters III and XIV of this book would have been written.

Quantitative comparison of theory with fact has always been the main object of the book, and practically all the theories advocated have survived the test of quantitative application to several phenomena. Accurate theories have been given where they seemed necessary for this purpose; but where an estimate of an order of magnitude was all that was required, and could be obtained by rough methods, such methods have always been used. I have been encouraged in the latter course by several facts. First, though the method of orders of magnitude is not convincing to the pure mathematician, it is a matter of experience that when a problem discussed by this method is afterwards solved by more formal methods, the answer is found to be of the correct order of magnitude, which is all that the method claims; it could be vitiated only by a fortuitous numerical coincidence. Second, though in some cases formally accurate solutions of related problems exist, or could be obtained, the problems actually so soluble differ so much from those that actually arise in geophysics that, in their actual application, they could at best be correct only as regards order of magnitude. Thus they are not inherently any more informative than the rougher methods. Third, a direct proof that a particular hypothesis will account for particular data is not very strong confirmation of the hypothesis when both the data and the consequences of the hypothesis are known only vaguely; but if it is shown that the results of the hypothesis agree with the facts as regards order of magnitude, while the results of denying it are in definite disagreement, the confirmation of the hypothesis will be almost as strong as if a close agreement had been obtained. The method of exhaustion of alternatives is specially useful in geophysics, because incorrect geophysical hypotheses usually fail by extremely large margins.

Two criticisms are certain to be made by geologists, and therefore I venture to attempt to meet them in advance. The first is that the book contains a great deal of matter not of a geophysical character. I thought at first of avoiding this objection by dividing the book into two parts, one cosmogonical and one geophysical; but I found such a course impossible, since the two were too closely interwoven, each depending in part on the results of the other. In a work mainly theoretical rather than descriptive in character it therefore seemed best to develop the implications of a hypothesis wherever they might lead to results capable of empirical test, rather than to confine my attention to one particular planet. If a theory is satisfactory, the more it is shown to explain the more reliable it is; and if it is unsound, it will be unsound whether the fact inconsistent with it happens to relate to the earth or to the satellites of Uranus.

The other objection I anticipate is that the book is too mathematical for geological readers. The answer is simple: the results aimed at are quantitative, and there is no way of obtaining quantitative results without mathematics. I have tried to keep the mathematics as elementary as possible; but some problems could not be handled by simple mathematics, and I had no alternative except to give all that was necessary. If the geologist cannot follow a part of the book, I hope he will omit it and go on to the next non-mathematical passage, trusting that someone else will point out any intervening mistakes (and the mathematically trained readers, with few exceptions, will do just the same). He will then know that at any rate some people will be able to follow the argument all through, and he will see just where he fails to follow it himself; whereas a so-called non-mathematical 'exposition' would only bewilder the mathematical physicist, while making it impossible for the geologist to see how much is hypothesis and how much is merely the systematic investigation of the consequences of hypotheses already made and data already found. In short, if geophysics requires mathematics for its treatment, it is the earth that is responsible, not the geophysicist.

The paragraphs are numbered according to the decimal system; of any two paragraphs, that with the smaller number comes earlier in the book. The integral part of a paragraph number is the number of the chapter. Equations have as a rule been numbered consecutively through each paragraph; but in some cases they have been numbered consecutively through several closely related paragraphs. In cross-references, where reference is made to another equation in the same paragraph, only the number of the equation is given; but where the reference is to a different paragraph, the numbers of that paragraph and of the equation are given; *e.g.* 14·61 (3).

I wish to express my thanks to the staff of the Cambridge University Press for their courtesy during the publication of this book; to Mr R. Stoneley, who has read the whole in proof and checked a great deal of the mathematical work, suggesting many improvements in the process; to Dr J. H. Jeans, who verified Chapter IX; to Dr A. A. Griffith, who gave me much of the information incorporated in Chapter XI, though he does not wholly approve of my terminology; to Dr Arthur Holmes, whose influence on my geophysical thought has been none the less important because I experienced it before beginning this book; and to Sir Ernest Rutherford, Prof. Eddington, Prof. Shapley, and Dr Wrinch, who have read various portions in typescript and suggested improvements.

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CHAPTER I

The Mechanical Properties of Rocks

'So far', admitted Lao Ting, 'it is more in the nature of a vision. There are, of necessity, many trials, and few can reach the ultimate end. Yet even the Yangtze-Kiang has a source.'
ERNEST BRAMAH, *Kai Lung's Golden Hours*

1·01. In studying the constitution and history of the Earth we are largely concerned with the phenomena of change of dimensions, especially of shape. Hence the physical properties that we most need to know are the elastic properties. Perfectly elastic solids are the subject of an extensive theory, which is well confirmed by experiment when the strains are small. When the strains are large enough different substances behave very differently. The information available is mainly experimental, but progress is being made towards its coordination into a general theory under the name of rheology.

1·02. Stress; equations of motion. For all types of material it is possible to define the stress at a point in terms of nine components, six of which are equal in pairs. It is convenient to use rectangular coordinates x_1, x_2, x_3, or, more shortly, x_i, where it is understood that i can take any of the values 1, 2, 3.* If we take a plane of x_i constant through the point, the force per unit area between parts of the material on opposite sides of it, tending to displace the matter with smaller x_i in the direction of increasing x_k, is the stress-component denoted by p_{ik}. It is shown in works on elasticity and hydrodynamics that $p_{ik} = p_{ki}$, and that the equations of motion are

$$\rho f_i = \rho X_i + \sum_{k=1,2,3} \frac{\partial p_{ik}}{\partial x_k}. \tag{1}$$

Here ρ is the density, f_i a component of acceleration, and X_i the external force per unit mass (usually gravity). These relations assume nothing about the properties of the material except that it is continuous and that there are no infinite accelerations near the point considered. Also we have the equation of continuity, expressing the fact that the rate of increase of the mass within an element of volume is equal to the net rate of inflow of mass into it. This equation is

$$\frac{\partial \rho}{\partial t} = -\sum_i \frac{\partial}{\partial x_i}(\rho v_i), \tag{2}$$

where v_i is the component of velocity in the direction of x_i.

Full analysis of the hypothesis that the material is continuous would require consideration of atomic theory. We may, however, remark briefly

* For a fuller discussion see H. and B. S. Jeffreys (1946, Chapters 2, 3).

that it requires the quantities in the equations to be determinate within the accuracy appropriate to the thermal agitation. The temperature is determined to an accuracy of order 0·1% if the volume contains about 10^6 atoms, and as the spacing between atoms in a solid or liquid is of order 10^{-8} cm., we should expect our results to be right as long as we do not try to apply them to regions of dimensions less than about 10^{-6} cm. Essentially the notion of stress supposes the atomic structure smoothed out in such a way as to preserve the actual mass, momentum and force when regions more than 10^{-6} cm. in linear extent are considered.

If a plane whose normal has direction cosines l_i is considered, the stress across it is obtained from the p_{ik} by the rule that the component in the direction x_i is $\sum_k l_k p_{ik}$. In particular, if a surface is free, that is, in contact with no matter on one side, there can be no stress across it, and these three sums are zero at every point of the surface.

Another property of importance is as follows. If we take an inclined set of axes x'_j, such that the cosine of the angle between x_i and x'_j is l_{ij}, the stress-components with regard to the new axes are p'_{jl}, where j and l can take the values 1, 2, 3 and

$$p'_{jl} = \sum_{i,k} l_{ij} l_{kl} p_{ik}. \tag{3}$$

(l as a suffix specifies an axis, l on the line indicates a direction cosine. There need be no confusion if this is borne in mind.) To abbreviate writing it is customary to omit the sign of summation in such equations as (1), (2) and (3). It is then understood that in an expression containing a repeated suffix, that suffix is to be given all possible values and the results are to be added. Thus the occurrence of a repeated suffix implies the summation.

A set of nine quantities w_{ik} that transform to new axes according to the rule (3) is called a tensor of the second order. If it also has the property $w_{ik} = w_{ki}$ it is called *symmetrical*. Such a tensor has the remarkable property that it is possible to choose a set of axes such that all components with j, l unequal are zero; then the components with $j = l$ are the roots of the cubic equation in λ (taking p_{ik} as the typical case):

$$\begin{vmatrix} p_{11}-\lambda & p_{12} & p_{13} \\ p_{21} & p_{22}-\lambda & p_{23} \\ p_{31} & p_{32} & p_{33}-\lambda \end{vmatrix} = 0. \tag{4}$$

These roots are all real and are called the *principal stresses*. Since p_{ik} in general vary from place to place, the principal stresses do so too. We denote them by P_1, P_2, P_3. Thus any system of stresses at a point can be represented as the resultant of three simple tensions or pressures at right angles. The mean of the roots P_1, P_2, P_3 is also the mean of p_{11}, p_{22}, p_{33}. We denote it by $-p$ (because it is nearly always negative in our problems). If $P_1 = P_2 = P_3$ at a point the stress there is called *hydrostatic*, because this condition is satisfied in a fluid at rest. Accordingly, it is often useful to regard the stress as composed of a hydrostatic stress equal to the mean of the principal stresses together with a part known as the *deviatoric stress*, which measures the departure of the stress from symmetry. The reason is that any substance, even a gas, behaves almost as perfectly elastic under hydrostatic stress alone; the density depends only on the pressure and the temperature at the actual time, not on their rates of change or on the past history, and the relation is the same whatever the deviatoric stress may be. For gases it is expressed by the laws of Boyle and Charles. There is actually a slight dependence on the rate of change (Liebermann, 1949). This gives extra damping of sound, and is important in shock waves. It would lead to a modification of the relation between dilatation and mean pressure analogous to that given for shear in (30)

$$k\frac{d\Delta}{dt} = \left(\frac{d}{dt}+\frac{1}{\tau_1}\right)(\tfrac{1}{3}p_{ii}),$$

and $k\tau_1$ would behave as a second coefficient of viscosity. For some liquids it is about 100 times the ordinary viscosity, and in both liquids and gases leads to a much greater damping of sound waves as they travel than is given by the ordinary viscosity (Rosenhead *et al.* 1954). A full theoretical discussion of its effects, especially on the formation of shock waves, is given by M. J. Lighthill (in Batchelor and Davies, eds., 1956). On the other hand, changes of shape are related to the deviatoric stress and only to a subordinate extent to the mean stress. Hence it is convenient to introduce the set of quantities δ_{ik}, known as the *substitution tensor*, and defined by the equations

$$\delta_{ik} = 1 \quad (i = k); \qquad \delta_{ik} = 0 \quad (i \neq k). \tag{5}$$

Then the deviatoric stress-components are given by

$$p'_{ik} = p_{ik} + p\delta_{ik}. \tag{6}$$

1·03. Rotation and strain; stress-strain relations for liquid and perfectly elastic solids. The motion in the neighbourhood of a point is specified if we have the velocity at the point and all the nine derivatives of the velocity with respect to the coordinates. That is, we need the velocity components v_i and the derivatives $\partial v_i/\partial x_k$. It is best to use these in the combinations

$$e_{ik} = \frac{1}{2}\left(\frac{\partial v_k}{\partial x_i}+\frac{\partial v_i}{\partial x_k}\right), \quad \xi_{ik} = \frac{1}{2}\left(\frac{\partial v_k}{\partial x_i}-\frac{\partial v_i}{\partial x_k}\right). \tag{7}$$

This is because if the e_{ik} are zero at all points it can be shown that distances between given particles are not changing and therefore the whole is moving as a rigid body; and then the rate of rotation is completely determined by the ξ_{ik}, which are constant through the body. If we consider two neighbouring particles, the rate of change of the distance between them depends on the velocities only through the e_{ik}, which are therefore called the *rates of strain*. They form a symmetrical tensor of the second order, like the p_{ik}.

The sum e_{ii} is called the rate of *dilatation*, because it gives the relative rate of increase of volume $-d\rho/\rho dt$ by the equation of continuity.

Both for solids and fluids there is a relation between p_{ii} and e_{ii}, because for a material of given composition ρ is determined by the pressure and temperature. If the variations are small, as they are in our problems, the relation can be written

$$e_{ii} = \frac{1}{3k}\frac{\partial p_{ii}}{\partial t} + 3\alpha\frac{\partial \vartheta}{\partial t}, \tag{8}$$

where k is a constant of the material called the *bulk-modulus* or *incompressibility* ($1/k$ being the compressibility), α is the coefficient of linear thermal expansion, and ϑ the temperature. For the present we can neglect changes of temperature.

It is convenient to separate e_{ik} into dilatational and deviatoric parts, representing changes of volume and shape respectively;

$$e_{ik} = \tfrac{1}{3}e_{mm}\delta_{ik} + e'_{ik}. \tag{9}$$

In a normal fluid (that is, not a 'liquid crystal') there is a simple relation between p'_{ik} and e'_{ik}, namely,

$$p'_{ik} = 2\eta e'_{ik}, \tag{10}$$

where η is the coefficient of viscosity. Then we have relations connecting all the components of stress with those of rate of strain and therefore with the velocity; and substituting these in the equations of motion we get differential equations for the velocities.

An isotropic, perfectly elastic solid is a substance such that the rates of change of the p'_{ik} are proportional to the e'_{ik}; that is, there is a constant μ, called the *rigidity*, such that

$$\frac{\partial p'_{ik}}{\partial t} = 2\mu e'_{ik}. \tag{11}$$

This is not true for crystals. It is true with considerable accuracy for most of the Earth. This is probably because rocks are usually aggregates of crystals oriented in all directions, and the differences of properties in different directions average out when we consider specimens of the sizes that usually concern us in geophysics. It may be expected to fail in stratified rocks, but little is known about their elastic properties, and what is known has so far defied summary.

On account of the similarity between (10) and (11) it is convenient in the theory of elasticity to regard the particle at x_i as having originally been at a point with coordinates $x_i - u_i$, where the u_i are components of *displacement* and are regarded as small, so that we can neglect their squares and products. Then to this accuracy $v_i = \dfrac{\partial u_i}{\partial t}$. We can then define a set of components of strain

$$\epsilon_{ik} = \frac{1}{2}\left(\frac{\partial u_k}{\partial x_i} + \frac{\partial u_i}{\partial x_k}\right), \tag{12}$$

and
$$e_{ik} = \frac{\partial \epsilon_{ik}}{\partial t}. \tag{13}$$

At present we take the material to have been originally in a state of zero stress and under no external forces. Then

$$p_{ii} = 3k\epsilon_{ii}, \quad p'_{ik} = 2\mu\epsilon'_{ik}, \tag{14}$$

$$p_{ik} = \tfrac{1}{3}p_{mm}\delta_{ik} + p'_{ik}$$

$$= k\epsilon_{mm}\delta_{ik} + 2\mu(\epsilon_{ik} - \tfrac{1}{3}e_{mm}\delta_{ik}) \tag{15}$$

$$= (k - \tfrac{2}{3}\mu)\,\epsilon_{mm}\delta_{ik} + 2\mu\epsilon_{ik}. \tag{16}$$

We write
$$k - \tfrac{2}{3}\mu = \lambda. \tag{17}$$

λ has no particular name, but its introduction simplifies the writing. λ and μ together specify the elastic properties of the material and are called Lamé's constants.

Another description of the properties is obtained by considering the case where p_{11} is given and all other stress-components are zero; this corresponds to a bar under longitudinal stress, the sides being free. The bar is stretched along x_1, but contracts in the same ratio along x_2 and x_3. Then if $\epsilon_{11} = e$, $\epsilon_{22} = \epsilon_{33} = -\sigma e$, $\epsilon_{12} = \epsilon_{23} = \epsilon_{31} = 0$, we have

$$\epsilon_{mm} = e(1-2\sigma), \tag{18}$$

$$p_{11} = \lambda e(1-2\sigma) + 2\mu e, \tag{19}$$

$$p_{22} = p_{33} = \lambda e(1-2\sigma) - 2\mu\sigma e = 0, \tag{20}$$

whence
$$\sigma = \frac{\lambda}{2(\lambda+\mu)}, \quad p_{11} = \frac{3\lambda+2\mu}{\lambda+\mu}\mu e. \tag{21}$$

The coefficient of e in (21) is *Young's modulus*, denoted by E:

$$E = \frac{3\lambda+2\mu}{\lambda+\mu}\mu. \tag{22}$$

σ is *Poisson's ratio*. Alternatively, k, λ and μ can be expressed in terms of E and σ. But σ is difficult to measure accurately. The easiest quantities to measure are E and μ. E is found by measuring the extension of a rod under given tensions, and μ by measuring the torsion under given couples applied at the ends. k can also be measured directly as follows. A cylinder of the material is placed in a slightly larger container, which is then filled with a liquid. Pressure is applied to the container by a piston, and the liquid transmits hydrostatic pressure to the solid. The pressure is measured by an instrument immersed in the liquid, and the change of volume is also measured. It is necessary to allow for the compression of the liquid. This method was introduced by Bridgman at Harvard, and has been followed by the Geophysical Laboratory at Washington.

If we substitute (22) in the equations of motion we find, again neglecting squares of displacements,

$$\rho\frac{\partial^2 u_i}{\partial t^2} = (\lambda+\mu)\frac{\partial\Delta}{\partial x_i}+\mu\nabla^2 u_i, \tag{23}$$

where

$$\Delta = \frac{\partial u_m}{\partial x_m}, \quad \nabla^2 = \frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\frac{\partial^2}{\partial x_3^2}. \tag{24}$$

1·04. Imperfections of elasticity. Most materials ordinarily regarded as solid are not quite perfectly elastic, even at small stresses. If a tension is applied to a bar and then kept constant a considerable variety of behaviour may ensue. Vibrations are set up, but these usually lose their energy by transmission to the surroundings in a much shorter time than concerns us here, and we may ignore them. If the tension is great enough there may be immediate fracture. If not, there is an initial extension, which appears to be accurately proportional to the tension. This has been verified for rocks by D. W. Phillips (1931, 1932, 1934). But in most cases the extension continues to increase slowly. The duration of experiments by Phillips and D. T. Griggs (1936) ranged from hours to months. The extension sometimes appeared to be tending to a new steady value; sometimes it showed no sign of doing so; sometimes it led to fracture before it had settled down. When the load was removed there was an immediate recovery by the amount of the original elastic displacement, but further changes followed. In cases where the extension in the first part of the experiment had appeared to be tending to a limit, the original form was almost completely recovered in time, and repetitions of the cycle led to no further residual strain. Where there had been no sign of a limit, the original form was never approximately recovered. The conditions may be classified as *simple elasticity*, *elastic hysteresis*, and *flow* or *permanent set*, *elastic afterworking*, *delayed fracture*. The property of the material associated with flow is known as *plasticity*, the essential feature being the failure of the specimen to return to its original form. They may be defined as follows.

Simple elasticity: strain is in proportion to the stress.

Elastic hysteresis: part of the strain persists when the stress is removed, even after a long time, but does not increase with duration of stress. It can be annulled only by reversing the stress. Qualitatively, it resembles the behaviour of a screw that is being screwed into wood; when the screw is released the stresses in the wood do not drive it out again but are balanced by the friction of the screw.

Flow: part of the strain increases beyond any limit if the stress is maintained long enough. The corresponding property of the material is known as *plasticity*. It is well known in such materials as clay and dough. These also bring out the point that the rate of flow is not a simple function of the stress; for the moulded body, when the stress is removed, is still under stress due