CONTEMPORARY PRECALCULUS

A GRAPHING APPROACH

HUNGERFORE

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This book was printed on paper made from waste paper, containing 10% post-consumer waste and 40% pre-consumer waste. measured as a percentage of total fiber weight content.

Dedicated to the Parks sisters,
whose presence in my life has greatly enriched it:
 To my aunts,
 Florence M. Parks
 Irene Parks Mills
 And to the memory of my mother,
 Grace Parks Hungerford

Preface

Like most books with similar titles, Contemporary Precalculus: A Graphing Approach is intended to provide the essential mathematical background needed in calculus for students who have had two or three years of high school mathematics. Unlike traditional texts, however, this one takes full advantage of the availability of relatively inexpensive graphing technology and effectively integrates it into the course, without losing sight of the fact that the underlying mathematics is the crucial issue. The book enables students to become active participants instead of passive observers, but avoids the very real danger of making technology an end in itself rather than a means of developing mathematical understanding.

Graphing calculators and software enable students to explore mathematics on their own and deal with realistic applications in ways that were not previously feasible. Students can develop a more thorough intuitive understanding of the essential ideas and be better prepared to deal with a solid calculus course. Instructors can spend less time on "mechanics" and more on the underlying concepts. They can focus on the essentials, without getting bogged down in calculations that obscure the key ideas.

The emphasis in the book, therefore, is on developing the concepts that play a central role in calculus by exploring these ideas from graphical, numerical, and algebraic perspectives. With the assistance of technology, the interplay between these viewpoints can be fully exploited to give students insight into what is going on and the confidence to work with it on their own.

We have done our best to present sound mathematics in an informal manner that stresses meaningful motivation, careful explanations, and numerous examples, with an ongoing emphasis on real-world problem solving. In some instances, technology plays no role at all, but in others it provides a different emphasis or point of view that significantly changes the traditional treatment, as noted below.

Mathematical Features

Functions and Functional Notation Technology does not remove the need for students to understand and be comfortable with functions and functional no-

tation. So more time is devoted to this than is often the case, beginning in Chapter 2 and continuing throughout the text.

Equation Solving By interpreting the solutions of the equation f(x) = 0 as the *x*-intercepts of the graph of the function f, the power of graphing technology can be brought into play to determine approximate solutions to a high degree of accuracy. The crucial connection between algebraic and graphical information is emphatically brought home to the student. More importantly, problems and applications are no longer artificially restricted to situations involving relatively simple equations.

Parametric Graphing With the traditional graphing difficulties eliminated by technology, the basic facts about parametric equations can be introduced early and used thereafter to illustrate such concepts as inverse functions, the definition of trigonometric functions, and the graphs of conic sections.

Polynomial and Rational Functions Graphing technology makes it possible to provide a thorough intuitive treatment of these functions, including maxima and minima problems, that lays the groundwork for a formal consideration in calculus.

Trigonometry Trigonometric functions are introduced as functions of real numbers—the way they are most widely used in calculus—with traditional triangle trigonometry treated later. A much more complete account of the graphs of periodic functions is possible with graphing technology than was previously the case.

Logarithms The exponential function $f(x) = e^x$ and natural logarithms are emphasized because of their central role in calculus. However, the text is arranged so that logarithms to an arbitrary base may be covered first by instructors who prefer that approach.

Systems of Equations Systems of nonlinear equations that may have formerly been intractable can now be readily solved by means of graphing technology, further expanding the types of practical problems that beginning students can handle. Since most graphing calculators have significant matrix capabilities, the algebraic solution of systems of linear equations is also much easier than previously. The treatment here reflects these facts.

Rates of Change In addition to their difficulties with functional notation, many students have had very limited experience with the idea of average rate of change and thus are unable to fully comprehend the concept of the derivative and instantaneous rates of change. Consequently, this book provides a full discussion of rates of change and uses the calculator to explore the intuitive connection between average and instantaneous rates of change.

Introduction to Limits In the optional final chapter, graphing technology is used to provide an intuitive introduction to limits that makes the idea "come alive." This approach also makes the discussion of continuity more meaningful to students.

Pedagogical Features

Algebra Review The book is designed to get to the heart of the subject as quickly as possible, so the usual review of basic algebra is in an appendix rather than at the beginning of the book. It can be omitted by well-prepared classes or covered as an introductory chapter if necessary.

Geometry Review Frequently used facts from plane geometry are summarized, with examples, in an appendix at the end of the book.

Graphing Explorations Students are expected to participate actively in the development of many examples. The "Graphing Exploration" heading indicates that they are to use a graphing calculator (or suitable software) as directed to complete the discussion in the text. Typically, this involves graphing one or more functions and answering questions about the graphs.

Artwork There are two types of graphical illustrations: color figures prepared by computer-equipped artists and black-and-white representations of graphing calculator screens prepared with special software that precisely emulates a typical calculator. The latter assist students to interpret the images on their own calculator screens and make clear the difference between an "ideal" graph and the sometimes less than ideal one produced by calculators or computers.

Warnings Students are alerted to common errors and misconceptions (both mathematical and technological) by clearly marked "warning boxes."

Exercises Each exercise set begins with routine calculation, graphing, or drill problems, then proceeds to exercises that are less mechanical and require some thought. Some sets include "Unusual Problems," a few of which are quite challenging. More often, however, these "unusual problems" are not difficult, but simply different from what students may have seen before. Answers for all odd-numbered problems are given at the back of the book, and solutions for all odd-numbered problems are provided in the Student Solutions Manual.

Chapter Reviews Each chapter concludes with a list of important concepts (referenced by section and page number), a summary of important facts and formulas, and a set of review questions.

Supplements

The following chart summarizes the print and software supplements available to users of this text. A full description of each is given in the "To the Instructor" section on page xx or the "To the Student" section on page xxiii.

	Print Ancillaries	Software
Instructors	Instructor's Manual	FCP Graph
	Lecture Guide and Student Notes	
	Test Bank	EXAMaster+ Computerized Test Bank
	Transparency Masters	
Students	Student Solutions Manual	FCP Graph
	Lecture Guide and Student Notes	

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Thomas W. Hungerford CLEVELAND STATE UNIVERSITY

December 1993

Text Features

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Review and Technology Preview

Roadmap*



The first half of this chapter reviews the essential facts about real numbers, equations, and the coordinate plane that are needed in this course and in calculus. The Algebra Review Appendix at the end of the book is a prerequisite for this material.

The graphing calculator is introduced in the second half of the chapter. Graphing techniques are presented that enable you to solve complicated equations easily and to deal effectively with real-world problems. However, a graphing calculator cannot be used with maximum efficiency makes are also the problems.

have a sound knowledge of a variety of different types of funct functions are studied in later chapters, at which time we shall re material and explore it in greater depth.

1.1 The Real Number System

You have been using real numbers most of your life. They include:

Natural numbers (or positive integers): 1, 2, 3, 4,

Integers: 0, 1, -1, 2, -2, 3, -3, 4, -4,

Rational numbers:† every number that can be expressed as a r/s, with r and s integers and $s \neq 0$; for instance, $\frac{1}{2}$, $-.983 = \frac{1}{2}$

*The Roadmap at the beginning of each chapter shows the interdependence of section An arrow, such as 1 → 2, means that Section 1 is a prerequisite for Section 2.

*The word "rational" here has no psychological implications; it refers to the "ratio" of integers.

Important Facts

Key definitions, theorems, and procedures are highlighted in color boxes and titled for easy reference.

Integrated Technology

Graphing technology, which is an essential component of the text rather than an optional add-on, is used effectively to introduce, develop, and explore fundamental concepts in ways not previously feasible.

Chapter Opener

Each chapter begins with a short introduction and a roadmap showing the interdependence of chapter sections to facilitate maximum flexibility in course design.

CHAPTER 1 REVIEW AND TECHNOLOGY PREVIEW

-Solutions and Intercepts

The real solutions of the one-variable equation e(x)=0 are the x-intercepts of the graph of the two-variable equation y=e(x).

Consequently, the approximate solutions of the equation e(x) = 0 can be found by using a graphing calculator to approximate the x-intercepts of the graph of y = e(x).

Approximating the x-intercepts can be done automatically on some calculators. If your calculator has a root-finding feature, learn to use it.* If your calculator does not have such a feature, or if the root finder fails with a particular equation (see Exercise 45), use the "manual" method described in the next example.

EXAMPLE 1 Solve the equation $x^4 - x^3 + x^2 - x - 5 = 0$ graphically.

SOLUTION We begin by graphing the equation $y = x^4 - x^3 + x^2 - x - 5$ in the standard viewing window:

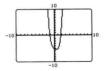


Figure 1-40

Since the scale marks on the x-axis are one unit apart, we see that there is one x-intercept (solution of the equation) between -2 and -1 and another one between 1 and 2. Assuming that Figure 1-40 includes all x-intercepts of the graph, this means that the original equation has exactly two real solutions.†

this means that the original equation has exactly two real solutions. To get a better approximation of the solution between 1 and 2, we change the viewing window to show only the portion of the graph with $1 \le x \le 2$ (Figure

*Depending on the calculator, you select 'root' on the ''calc'' menu or on the 'math' submenu of the 'graph' menu or select ''x-intercept' on the ''jump' menu. Check your instruction manual.

†Unless stated otherwise in the examples and exercises of this section, you may assume that the standard viewing window includes all x-intercepts of a graph.

Foreshadowing of Calculus

Key concepts, such as average rates of change and maxima and minima, are introduced graphically and algebraically to build the intuitive foundation needed to deal with these topics in depth in calculus. An optional chapter on limits is included.

The Difference Quotient

If f is a function, then the difference quotient of f is the quantity

f(x+h)-f(x)

The difference quotient is the average rate of change of f over the interval

For instance, Example 5 shows that the difference quotient of the function $f(x) = 16x^2$ is 32x + h.

EXAMPLE 6 Find the difference quotient of $f(x) = x^2 - x + 2$.

SOLUTION Use the definition of difference quotient and algebra:

$$\frac{f(x+h) - f(x)}{h} = \underbrace{\frac{f(x+h)^2 - (x+h) + 2}{h} - \underbrace{\frac{f(x^2 - x + 2)}{h^2}}_{h}}_{= \underbrace{\frac{(x^2 + 2xh + h^2) - (x+h) + 2 - (x^2 - x + 2)}{h}}_{= \underbrace{\frac{x^2 + 2xh + h^2 - x - h + 2 - x^2 + x - 2}{h}}_{= \underbrace{\frac{2xh + h^2 - h}{h}}_{= \underbrace{\frac{h^2(2x+h - 1)}{h}}_{= \underbrace{\frac{2xh + h^2 - h}{h}}_{= \underbrace{\frac{2xh + h^2 - h}_{h}}_{= \underbrace{\frac{2xh + h}_{h$$

EXAMPLE 7 What is the average rate of change of the function f(x) = $x^2 - x + 2$ from 5 to 5.01?

SOLUTION This rate of change can be computed directly, as in Examples 1-3, but it's easier to use the results of Example 6, where we saw that the difference quotient of f is the quantity 2x - 1 + h. The interval from 5 to 5.01 is the case when x = 5 and h = .01. Hence the average rate of change is 2(5) - 1 + .01 = 9.01.

Instantaneous Rate of Change

Rates of change are a major theme in calculus-not just the average rate of change discussed above, but also the instantaneous rate of change of a function (that is, its rate of change at a particular instant). Even without calculus, however, we can obtain quite accurate approximations of instantaneous rates of change by using average rates appropriately.

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CHAPTER 4 POLYNOMIAL AND RATIONAL FUNCTIO

EXAMPLE 6 To graph $f(x) = \frac{x^3 + 3x^2 + x + 1}{x^2 + 2x - 1}$, we begin by finding the vertical asymptotes and the x- and y-intercepts. The quadratic formula can be used to find the roots of the denominator:

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1(-1)}}{2 \cdot 1} = \frac{-2 \pm \sqrt{8}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}.$$

It is easy to verify that neither of these numbers is a root of the numerator, so the graph has vertical asymptotes at $x=-1-\sqrt{2}$ and $x=-1+\sqrt{2}$. The y-intercept is f(0)=-1. The x-intercepts are the roots of the numerator.

➤ GRAPHING EXPLORATION Use a calculator to verify that $x^3 + 3x^2 + x + 1$ has exactly one real root, located between -3 and -2.

Therefore the graph of f(x) has one x-intercept. Using this information and the calculator graph in Figure 4–35 (which erroneously shows some vertical segments), we conclude that the graph looks approximately like Figure 4-36.



Figure 4-35

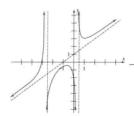


Figure 4-36

At the left and right ends, the graph moves away from the x-axis. To understand the behavior of the graph when |x| is large, divide the numerator of f(x) by its

Graphics

Both "ideal" graphs drawn by artists and exact replicas of calculator screens are an integral part of the presentation. The latter assist students in understanding and intelligently interpreting the graphs they obtain on their calculators or computers.

Graphs and Transformations

There are a number of algebraic operations that can be applied to the rule of a -x + 2, then a constant can be added or subtracted from the rule of f or the rule can be multiplied by a constant to obtain new functions, such as:

$$g(x) = f(x) + 5 = (x^2 - x + 2) + 5 = x^2 - x + 7$$

$$h(x) = -4f(x) = -4(x^2 - x + 2) = -4x^2 + 4x - 8.$$

Similarly, replacing the variable x by x + 3 produces the new function

$$k(x) = f(x + 3) = (x + 3)^2 - (x + 3) + 2 = x^2 + 6x + 9 - x - 3 + 2$$

= $x^2 + 5x + 8$

In this section we shall see that when the rule of a function is changed algebraically to produce a new function, then the graph of the new function can be obtained from the graph of the original function by a geometric transformation such as a vertical or horizontal shift, a reflection in the x-axis, or by stretching or shrinking.

Vertical and Horizontal Shifts

➤ GRAPHING EXPLORATION Using the standard viewing window, graph these three functions on the same screen:

 $f(x) = x^2$ $g(x) = x^2 + 5$ $h(x) = x^2 - 7$ nd answer these questions:

Do the graphs of g and h look very similar to the graph of f in

How do their vertical positions differ?

Where would you predict that the graph of $k(x) = x^2 - 9$ is located relative to the graph of $f(x) = x^2$ and what is its shape.

Confirm your prediction by graphing k.on the same screen as f, g,

The results of this Exploration should make the following statement plausible:

Vertical Shifts

If c > 0, then the graph of g(x) = f(x) + c is the graph of f shifted

If c > 0, then the graph of h(x) = f(x) - c is the graph of f shifted

Problem Solving

Students are encouraged to use appropriate tools and strategies, rather than applying rote procedures. The advantages and disadvantages of algebraic, graphical, and numerical approaches are considered in a variety of situations.

Graphing Explorations

These interactive examples involve students in the development of key ideas and encourage them to explore mathematics on their own.

CHAPTER 1 REVIEW AND TECHNOLOGY PREVIEW

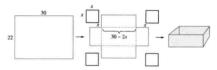


Figure 1-53

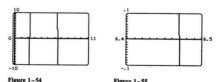
SOLUTION Let x denote the length of the side of the square to be cut from each corner. The dashed rectangle in Figure 1-53 is the bottom of the box. Its length is 30 - 2x as shown in the figure. Similarly, the width of the box will be 22 - 2xand its height will be x inches. Therefore

length × width × height = volume of box

$$(30-2x) \cdot (22-2x) \cdot x = 1000$$

 $(660-104x+4x^2)x = 1000$
 $4x^3-104x^2+660x-1000=0$

Since the cardboard is 22 inches wide, x must be less than 11 (otherwise you can't Since the cardboard is 22 inches whoe, x must be less than 11 (outer wise you can cut out two squares of length x). Since x is a length, it is positive. So we need only find solutions of the equation between 0 and 11, which we shall do graphically. The viewing window in Figure 1-54 shows that the graph of $y = 4x^3 - 104x^2 + 660x - 1000$ has x-intercepts between 2 and 3 and between 6 and 7, roughly at 2.2 and 6.4.



When 2 < x < 3, however, the length 30 - 2x of the box will be more than 18 inches (for instance, when x = 2.2, the length is 30 - 2(2.2) = 25.6). Verify that the solution between 6 and 7 will result in a box with each dimension less than 18

Figure 1-55

Warnings

Students are alerted to common mistakes with algebra, functions, and graphing calculators as well as misconceptions that can lead to erroneous conclusions.

Applications

Numerous real-world examples and exercises enable the student to see the relevance of precalculus mathematics and lay the groundwork for the more powerful techniques of calculus.

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CHAPTER 4 POLYNOMIAL AND RATIONAL FUNCTIONS

The remaining roots of f(x) are the roots of the factor $3x^2 + x + 1$. The quadratic formula shows that this polynomial has no real roots. Therefore the repeated root 2 is the only real root of f(x).

N O T E A calculator can be used to check the accuracy of factoring. For instance, if you have factored $f(x)=x^3-8x^4+16x^3-5x^2+4x-20$ as $(x-2)^2(x-5)(x^2+x+1)$, then graph both of the equations

$$y = x^5 - 8x^4 + 16x^3 - 5x^2 + 4x - 20$$
$$y = (x - 2)^2(x - 5)(x^2 + x + 1)$$

on the same screen. If the two graphs coincide, then the factorization is correct. If the graphs differ, then the factorization is in error and should be recalculated.

Irrational Roots

The Rational Root Test enables us to find all the rational roots of a polynomial. Sometimes, we can also find the roots that are irrational numbers, as in Example 4. Even when all the roots of a polynomial cannot be found exactly, they can always be approximated by the graphical techniques of Section 1.5.



Figure 4-5

Therefore f(x) has at least three real roots. Using zoom-in as we did in Section 1.5, we find that these roots are -.336, 1.427, and 2.101 (with a maximum error of

How can we be sure that the three roots found in Example 6 are the only real roots of $f(x) = x^5 - 2x^4 - x^3 + 3x + 1$? One way might be to graph f(x) in very large viewing windows. If no additional x-intercepts were found, we might

highest or lowest point over a particular range of z values.* When the trace feature and manual zoom-in are used, as in Example 6, it is important to choose viewing windows that have a sufficient degree of accuracy, but do not make the graph too flat to read easily (as in Figure 1–58 above). The general rule for avoiding flat graphs is

Choose a viewing window that is much wider than it is high.

This may result in a higher degree of accuracy in the y-coordinate than in the

x-coordinate, as happened in Figure 1-59.

Most calculators with a graphical root-finding feature also have a "max/min" feature that will automatically find a graph's highest and lowest points in any given viewing window.† If yours has such a feature, learn to use it.

WARNING

On some calculators with a "trace" key, the cursor can also be moved around the screen without using the trace key. If this is done, the cursor may appear to be on the graph when it is only near a point on the graph. In such cases, the displayed coordinates may not be the coordinates of a point on the graph, as they would be if the trace key were used.

EXAMPLE 7 A cylindrical can of volume 58 cubic inches (approximately 1 quart) is to be designed. For convenient handling, it must be at least 1 inch high and 2 inches in diameter. What dimensions will use the least amount of material?

SOLUTION We can construct a can by rolling a rectangular sheet of metal into a tube and then attaching the top and bottom, as shown in Figure 1-60.

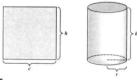


Figure 1-6

*In Example 6, for instance, we were interested only in points with $0 \le x \le 11$, even though there were other points on the graph that were higher than the one we found.

†Depending on the calculator, you select "max" or "min" on the "calc" menu or the "math' submenu of the "graph" menu or on the "jump" menu. Check your instruction manual.

Notes

The notes throughout the text provide useful background information or helpful advice on the use of graphing technology.

In other words, the graph of f has a peak at (c, f(c)) provided that

 $f(x) \le f(c)$ for all x in some open interval around c.

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In this case we say that the function f has a **local maximum** (or **relative maximum**) at x = c and that the number f(c) is the local maximum value of the function.

Similarly, a valley is the lowest point in neighborhood (though not necessarily the lowest point on the graph), so that all nearby points have larger y-coordinates. The graph of a function f has a valley at (d, f(d)) provided that

$$f(x) \ge f(d)$$
 for all x in some open interval around d.

In this case we say that the function f has a **local minimum** (or **relative minimum**) at x=d and that the number f(d) is the local minimum value of the function.

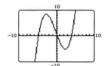
The term **local extremum** (plural extrema) refers to either a local maximum (plural maxima) or a local minimum (plural minima). In this terminology, the fact in the box on page 204 can be restated as follows:

A polynomial function of degree n has at most n-1 local extrema

Calculus is usually needed to determine the exact location of local extrema. As we saw in Section 1.5, however, a calculator's trace feature and zoom-in can be used to obtain very accurate approximations.

EXAMPLE 2 Find the local extrema of $f(x) = .2x^3 + .1x^2 - 4x + 1$.

SOLUTION The graph of f(x) in the standard viewing window (Figure 4–14) shows two local extrema, the maximum possible for a third-degree polynomial. The trace feature shows that the peak is approximately at the point (-2.842, 8.584). Zooming in to the viewing window with $-2.9 \le x \le -2.7$ (x-sci. 0.1) and $8.58 \le y \le 8.6$ (y-sci. 0.01) and using the trace feature (Figure 4–15) shows that the peak is approximately (-2.755, 8.97) with a maximum error of at most .01 in the x-coordinate and .00 in the y-coordinate.



-2.9 N=-2.754732 V=B.59685

Figure 4-15

Figure 4-14

CHAPTER 4 POLYNOMIAL AND RATIONAL FUNCTIONS

Detailed examples are used both to introduce and

clarify basic concepts. Examples of solved prob-

lems allow students to gain confidence and deal

48. Find a polynomial function g of degree 4 such that the roots of g are 0, -1, 2, -3 and g(3) = 288.

Examples

with new ideas.

In Exercises 49-52, find a number k satisfying the given condition.

49. x + 2 is a factor of $x^3 + 3x^2 + kx - 2$.

50. x - 3 is a factor of $x^4 - 5x^3 - kx^2 + 18x + 18$.

51. x - 1 is a factor of $k^2x^4 - 2kx^2 + 1$.

52. x + 2 is a factor of $x^3 - kx^2 + 3x + 7k$. **53.** Use the Factor Theorem to show that for every real number x = -2

53. Use the Factor Theorem to show that for every real number c, x - c is not a factor of x⁴ + x² + 1.
 54. Let c be a real number and n a positive integer.

(a) Show that x − c is a factor of xⁿ − cⁿ.
 (b) If n is even, show that x + c is a factor of xⁿ − cⁿ.
 [Remember: x + c = x − (−c).]

55. (a) If c is a real number and n an odd positive integer, give an example to show that x + c may not be a factor of xⁿ - cⁿ.

(b) If c and n are as in part (a), show that x + c is a factor of $x^n + c^n$.

Unusual Problems

56. For what value of k is the difference quotient of $g(x) = kx^2 + 2x + 1$ equal to 7x + 2 - (3.5)h?

57. For what value of k is the difference quotient of $f(x) = x^2 + kx$ equal to 2x + 5 + h?

4.1.A

Excursion: Synthetic Division

Synthetic division is a fast method of doing polynomial division when the divisor is a first-degree polynomial of the form x - c for some real number c. To see how it works, we first consider an example of ordinary long division:

$$\begin{array}{c} 3x^3 + 6x^2 + 4x - 3 & \longleftarrow \textit{Quotient} \\ \textit{Divisor} \longrightarrow x - 2 \frac{1}{3}x^4 - 8x^2 - 11x + 1 & \longleftarrow \textit{Dividend} \\ \frac{3x^4 - 6x^3}{6x^3 - 8x^2} \\ \frac{6x^3 - 8x^2}{4x^2 - 11x} \\ \frac{4x^2 - 8x}{3x + 1} \\ - 3x + 6 \end{array}$$

This calculation obviously involves a lot of repetitions. If we insert 0 coefficients for terms that don't appear above and keep the various coefficients in the proper columns, we can eliminate the repetitions and all the x's:



Exercises and Unusual Problems

Exercise sets proceed from the routine to the thought-provoking and cover the entire spectrum of computational and conceptual problems, as well as applications. Many ask students to explore mathematical concepts in new and unusual ways.

Excursions

These optional sections explore a variety of topics related to the key concepts developed in the text.

Chapter Reviews

Each chapter ends with a three-level review:

A list of important concepts, keyed to specific pages in the text;

A summary of important facts and formulas;

A full set of conceptual, computational, graphical, and applied exercises.

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Important Facts

and Formulas

CHAPTER 3 LINEAR AND QUADRATIC FUNCTIONS

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* The slope of the line through (x_1, y_1) and (x_2, y_2) (where $x_1 \neq x_2$) is

$$\frac{y_2 - y_1}{r_2 - r_2}$$

· Nonvertical parallel lines have the same slope.

. Two lines (neither vertical) are perpendicular exactly when the product of their slopes is -1.

• The equation of the line through (x_1, y_1) with slope m is

$$y-y_1=m(x-x_1).$$

• The equation of the line with slope m and y-intercept b is

$$y = mx + b$$
.

• The average rate of change of a function f as x changes from a to b is the

$$\frac{f(b) - f(a)}{b - a}$$

• The difference quotient of the function f is the quantity

$$\frac{f(x+h)-f(x)}{h}.$$

• The average rate of change of a function f as x changes from a to b is the slope of the secant line joining the points (a, f(a)) and (b, f(b)).

• The graph of $f(x) = ax^2 + bx + c$ is a parabola whose vertex has x-coor-

1. (a) What is the v-intercept of the graph of the linear function $f(x) = x - \frac{x-2}{5} + \frac{3}{5}$?

(b) What is the slope of the graph of f?

2. Find the equation of the line passing through (1, 3) and (2, 5).

Review Questions

APPFNDIX

Geometry Review

An **angle** consists of two half-lines that begin at the same point P, as in Figure A-1. The point P is called the **vertex** of the angle and the half-lines the sides of the angle



An angle may be labeled by a Greek letter, such as angle θ in Figure A-1(a), or An angle had be an accretely a order teach scale as angle of in Figure $A-\{a\}$, of by listing three points (a point on one side, the vertex, a point on the other side), such as angle QPM in Figure A-1(b).

In order to measure the size of an angle, we must assign a number to each

angle. Here is the classical method for doing this:

1. Construct a circle whose center is the vertex of the angle

2. Divide the circumference of the circle into 360 equal parts (called degrees) by marking 360 points on the circumference, beginning with the point where one side of the angle intersects the circle. Label these points 0° , 1° , 2° , 3° , and so on.

3. The label of the point where the second side of the angle intersects the circle is the degree measure of the angle.

For example, Figure A-2 shows an angle θ of measure 25 degrees (in symbols, 25°) and an angle β of measure 135°

An acute angle is an angle whose measure is strictly between 0° and 90° , such as angle θ in Figure A-2. A **right angle** is an angle that measures 90° . An

Geometry and Algebra Reviews The important geometric facts used in the text are summarized in an Appendix. Another appendix reviews the basic concepts of algebra; it can be used as an opening chapter by less well-pre-

pared classes.

To the Instructor

Every effort has been made to make this text as flexible as possible. With minor exceptions (usually exercises or occasional examples), the interdependence of chapters is given by the chart on the facing page. Each chapter begins with a **Roadmap** that indicates the interdependence of sections within the chapter.

As noted in the preface, the standard review of basic algebra is in the Algebra Review Appendix. This material, which is a prerequisite for the entire book, may be covered as Chapter 0 if necessary, or omitted by well-prepared classes.

A few sections are labeled as **Excursions.** Each Excursion is closely related to the section that precedes it and usually has that section as a prerequisite. No Excursion is a prerequisite for any other section of the text. With rare exceptions, each Excursion is a complete discussion with a full set of exercises. The "Excursion" label is designed solely to make syllabus planning easier and is *not* intended as any kind of value judgment on the topic in question.

The conventions used in the text with respect to (graphing) calculators, computers, and the use of the **Graphing Explorations** in the text are discussed in the "To the Student" section on page xxiii.

Supplements

Instructors who adopt this text may receive, free of charge, the following items:

Instructor's Manual with Transparency Masters Written by Ronald R. Dodson of New River Community College, this manual contains detailed solutions to all the exercises and end-of-chapter Review Questions to assist the instructor in the classroom and in grading assignments. Additionally, more than 200 transparency masters of important figures, theorems, and charts from the text are provided.

Interdependence of Chapters

