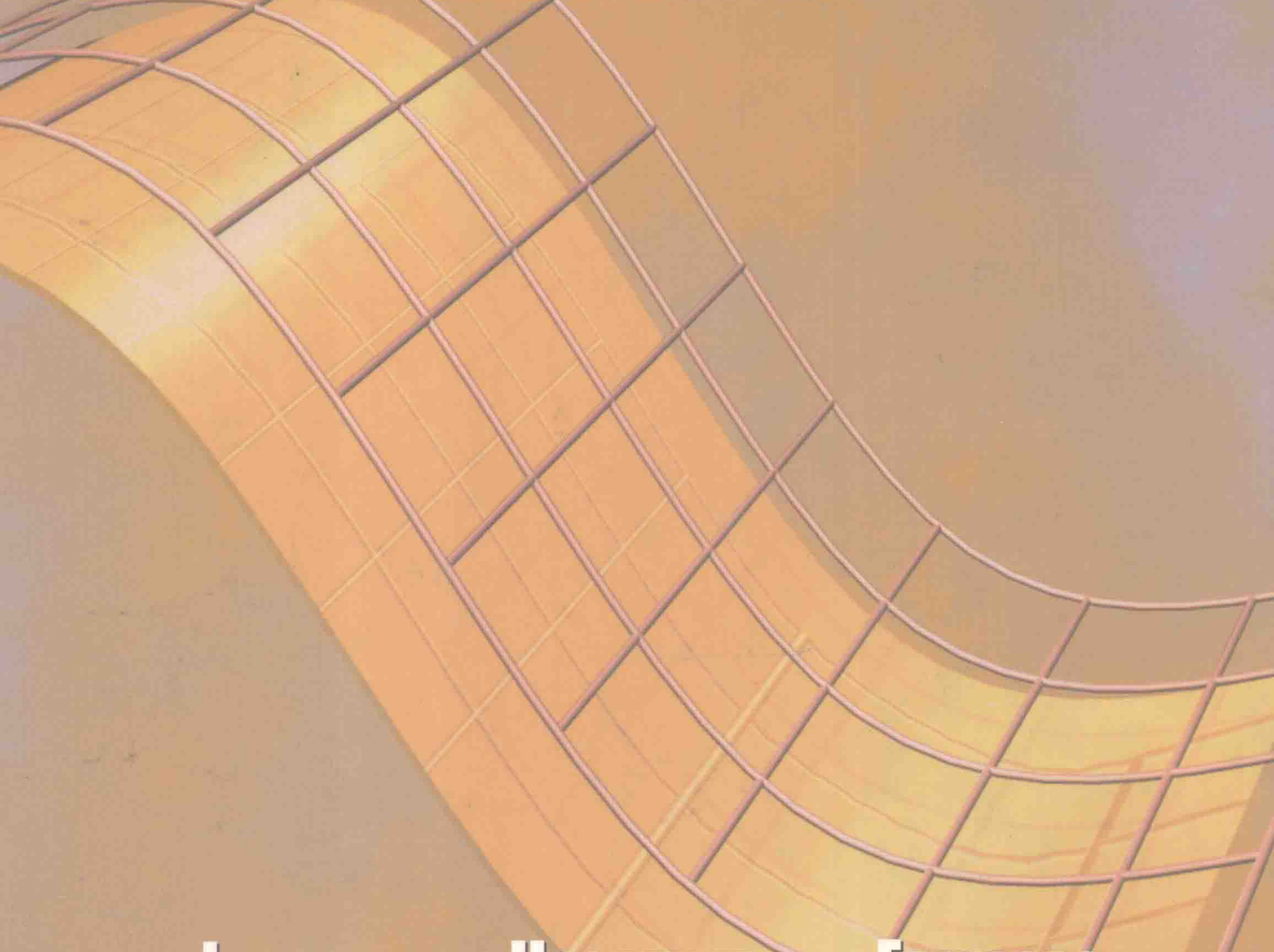
PRECALCULUS
FUNCTIONS AND GRAPHS
A GRAPHING APPROACH
Third Edition



LARSON ◆ HOSTETLER ◆ EDWARDS

Precalculus Functions and Graphs A Graphing Approach

Third Edition

Ron Larson
Robert P. Hostetler
The Pennsylvania State University
The Behrend College

Bruce H. Edwards
University of Florida

With the assistance of David C. Falvo
The Pennsylvania State University
The Behrend College

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We have included examples and exercises that use real-life data as well as technology output from a variety of software. This would not have been possible without the help of many people and organizations. Our wholehearted thanks go to all for their time and effort.

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A Word from the Authors

Welcome to *Precalculus Functions and Graphs: A Graphing Approach*, Third Edition. In this revision we have focused on student success, accessibility, and flexibility.

Accessibility: Over the years we have taken care to write a text for the student. We have paid careful attention to the presentation, using precise mathematical language and clear writing to create an effective learning tool. We believe that every student can learn mathematics and we are committed to providing a text that makes the mathematics within it accessible to all students.

In the Third Edition, we have revised and improved upon many text features designed for this purpose. Our pedagogical approach includes presenting solutions to examples from multiple perspectives—algebraic, graphic, and numeric. The side-by-side format allows students to see that a problem can be solved in more than one way, and to compare the accuracy of the solution methods.

Technology has been fully integrated into the text presentation. Also, the *Exploration* and *Study Tip* features have been expanded. *Chapter Tests*, which give students an opportunity for self-assessment, now follow every chapter in the Third Edition. The exercise sets now contain both *Synthesis* exercises, which check students' conceptual understanding, and *Review* exercises, which reinforce skills learned in previous sections and chapters. Students also have access to several media resources—videotapes, *Interactive Precalculus Functions and Graphs: A Graphing Approach* CD-ROM, and a *Precalculus Functions and Graphs: A Graphing Approach* website—that provide additional text-specific support.

Student Success: During our past 30 years of teaching and writing, we have learned many things about the teaching and learning of mathematics. We have found that students are most successful when they know what they are expected to learn and why it is important to learn it. With that in mind, we have restructured the Third Edition to include a thematic study thread in every chapter.

Each chapter begins with a study guide called *How to Study This Chapter*, which includes a comprehensive overview of the chapter concepts (*The Big Picture*), a list of *Important Vocabulary* that is integral to learning *The Big Picture* concepts, a list of study resources, and a general study tip. The study guide allows students to get organized and prepare for the chapter.

An old pedagogical recipe goes something like this, "First I'm going to tell you what I'm going to teach you, then I will teach it to you, and finally I will go over what I taught you." Following this recipe, we have included a set of learning objectives in every section that outlines what students are expected to learn, followed by an interesting real-life application that illustrates why it is important to learn the concepts in that section. Finally, the chapter summary (*What did you learn?*), which reinforces the section objectives, and the chapter *Review Exercises*, which are correlated to the chapter summary, provide additional study support at the conclusion of each chapter.

Our new *Student Success Organizer* supplement takes this study thread one step further, providing a content-based study aid.

Flexibility: From the time we first began writing in the early 1970s, we have always viewed part of our authoring role as that of providing instructors with flexible teaching programs. The optional features within the text allow instructors with different pedagogical approaches to design their courses to meet both their instructional needs and the needs of their students. In addition, we provide several print and media resources to support instructors, including a new *Instructor Success Organizer*.

We hope you enjoy the Third Edition.

Ron Larson

Robert P. Hostetler

Bruce H. Edwards

Acknowledgments

We would like to thank the many people who have helped us prepare the text and the supplements package. Their encouragement, criticisms, and suggestions have been invaluable to us.

Third Edition Reviewers

Jamie Whitehead Ashby, Texarkana College; Teresa Barton, Western New England College; Diane Burleson, Central Piedmont Community College; Alexander Burstein, University of Rhode Island; Victor M. Cornell, Mesa Community College; Marcia Drost, Texas A & M University; Kenny Fister, Murray State University; Susan C. Fleming, Virginia Highlands Community College; Nicholas E. Geller, Collin County Community College; Betty Givan, Eastern Kentucky University; John Kendall, Shelby State Community College; Donna M. Krawczyk, University of Arizona; JoAnn Lewin, Edison Community College; David E. Meel, Bowling Green University; Beverly Michael, University of Pittsburgh; Jon Odell, Richland Community College; Laura Reger, Milwaukee Area Technical College; Craig M. Steenberg, Lewis-Clark State College; Mary Jane Sterling, Bradley University; Ellen Vilas, York Technical College. In addition, we would like to thank all the algebra & trigonometry instructors who took the time to respond to our survey.

We would like to extend a special thanks to Ellen Vilas for her contributions to this revision.

We would like to thank the staff of Larson Texts, Inc., and the staff of Meridian Creative Group, who assisted in proofreading the manuscript, preparing and proofreading the art package, and typesetting the supplements.

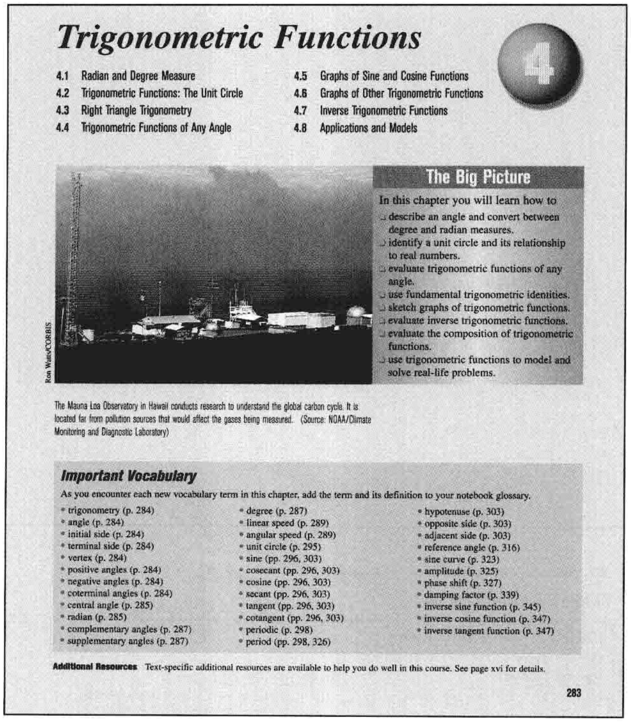
On a personal level, we are grateful to our wives, Deanna Gilbert Larson, Eloise Hostetler, and Consuelo Edwards for their love, patience, and support. Also, a special thanks goes to R. Scott O'Neil.

If you have suggestions for improving this text, please feel free to write to us. Over the past two decades we have received many useful comments from both instructors and students, and we value these very much.

Ron Larson
Robert P. Hostetler
Bruce H. Edwards

Features Highlights

Student Success Tools



Trigonometric Functions

4

The Big Picture

In this chapter you will learn how to
- describe an angle and convert between degree and radian measures.
- identify a unit circle and its relationship to real numbers.
- evaluate trigonometric functions of any angle.
- use fundamental trigonometric identities.
- sketch graphs of trigonometric functions.
- evaluate inverse trigonometric functions.
- evaluate the composition of trigonometric functions.
- use trigonometric functions to model and solve real-life problems.

Ron Watts/CORBIS

The Mauna Loa Observatory in Hawaii conducts research to understand the global carbon cycle. It is located far from pollution sources that would affect the gases being measured. (Source: NOAA/Climate Monitoring and Diagnostic Laboratory)

Important Vocabulary

As you encounter each new vocabulary term in this chapter, add the term and its definition to your notebook glossary.

- trigonometry (p. 284)
- angle (p. 284)
- initial side (p. 284)
- terminal side (p. 284)
- vertex (p. 284)
- positive angles (p. 284)
- negative angles (p. 284)
- coterminal angles (p. 284)
- central angle (p. 285)
- radian (p. 285)
- complementary angles (p. 287)
- supplementary angles (p. 287)
- degree (p. 287)
- linear speed (p. 289)
- angular speed (p. 289)
- unit circle (p. 295)
- sine (pp. 296, 303)
- cosecant (pp. 296, 303)
- cosine (pp. 296, 303)
- secant (pp. 296, 303)
- tangent (pp. 296, 303)
- cotangent (pp. 296, 303)
- periodic (p. 298)
- period (pp. 298, 326)
- hypotenuse (p. 303)
- opposite side (p. 303)
- adjacent side (p. 303)
- reference angle (p. 316)
- sine curve (p. 323)
- amplitude (p. 325)
- phase shift (p. 327)
- damping factor (p. 339)
- inverse sine function (p. 345)
- inverse cosine function (p. 347)
- inverse tangent function (p. 347)

Additional Resources Text-specific additional resources are available to help you do well in this course. See page xvi for details.

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New Section Openers include:

▶ "What you should learn"

Objectives outline the main concepts and help keep students focused on *The Big Picture*.

▶ "Why you should learn it"

A real-life application or a reference to other branches of mathematics illustrates the relevance of the section's content.

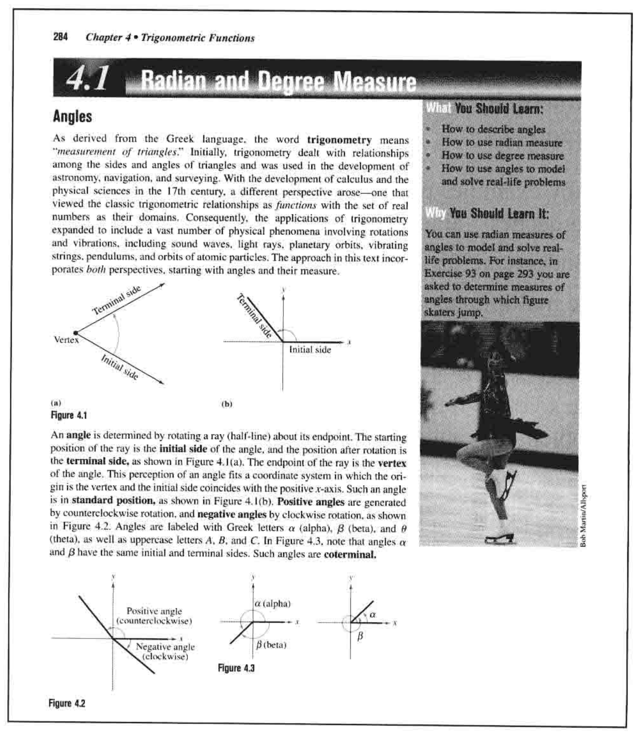
New Chapter Openers include:

▶ The Big Picture

An objective-based overview of the main concepts of the chapter.

▶ Important Vocabulary

Mathematical terms integral to learning *The Big Picture* concepts.



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4.1 Radian and Degree Measure

Angles

As derived from the Greek language, the word **trigonometry** means *"measurement of triangles."* Initially, trigonometry dealt with relationships among the sides and angles of triangles and was used in the development of astronomy, navigation, and surveying. With the development of calculus and the physical sciences in the 17th century, a different perspective arose—one that viewed the classic trigonometric relationships as *functions* with the set of real numbers as their domains. Consequently, the applications of trigonometry expanded to include a vast number of physical phenomena involving rotations and vibrations, including sound waves, light rays, planetary orbits, vibrating strings, pendulums, and orbits of atomic particles. The approach in this text incorporates *both* perspectives, starting with angles and their measure.

(a) (b)
Figure 4.1

An **angle** is determined by rotating a ray (half-line) about its endpoint. The starting position of the ray is the **initial side** of the angle, and the position after rotation is the **terminal side,** as shown in Figure 4.1(a). The endpoint of the ray is the **vertex** of the angle. This perception of an angle fits a coordinate system in which the origin is the vertex and the initial side coincides with the positive x-axis. Such an angle is in **standard position,** as shown in Figure 4.1(b). **Positive angles** are generated by counterclockwise rotation, and **negative angles** by clockwise rotation, as shown in Figure 4.2. Angles are labeled with Greek letters α (alpha), β (beta), and θ (theta), as well as uppercase letters A, B, and C. In Figure 4.3, note that angles α and β have the same initial and terminal sides. Such angles are **coterminal.**

Figure 4.3

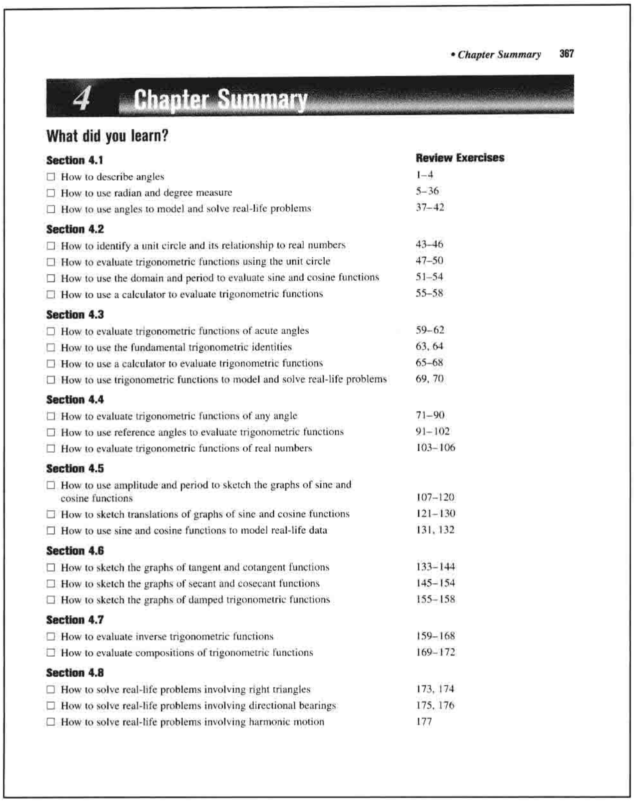
Figure 4.2

What You Should Learn:
- How to describe angles
- How to use radian measure
- How to use degree measure
- How to use angles to model and solve real-life problems

Why You Should Learn It:

You can use radian measures of angles to model and solve real-life problems. For instance, in Exercise 93 on page 293 you are asked to determine measures of angles through which figure skaters jump.

Bob Martin/Allsport



• *Chapter Summary* 367

4 Chapter Summary

What did you learn?

	Review Exercises
Section 4.1	
☐ How to describe angles	1–4
☐ How to use radian and degree measure	5–36
☐ How to use angles to model and solve real-life problems	37–42
Section 4.2	
☐ How to identify a unit circle and its relationship to real numbers	43–46
☐ How to evaluate trigonometric functions using the unit circle	47–50
☐ How to use the domain and period to evaluate sine and cosine functions	51–54
☐ How to use a calculator to evaluate trigonometric functions	55–58
Section 4.3	
☐ How to evaluate trigonometric functions of acute angles	59–62
☐ How to use the fundamental trigonometric identities	63, 64
☐ How to use a calculator to evaluate trigonometric functions	65–68
☐ How to use trigonometric functions to model and solve real-life problems	69, 70
Section 4.4	
☐ How to evaluate trigonometric functions of any angle	71–90
☐ How to use reference angles to evaluate trigonometric functions	91–102
☐ How to evaluate trigonometric functions of real numbers	103–106
Section 4.5	
☐ How to use amplitude and period to sketch the graphs of sine and cosine functions	107–120
☐ How to sketch translations of graphs of sine and cosine functions	121–130
☐ How to use sine and cosine functions to model real-life data	131, 132
Section 4.6	
☐ How to sketch the graphs of tangent and cotangent functions	133–144
☐ How to sketch the graphs of secant and cosecant functions	145–154
☐ How to sketch the graphs of damped trigonometric functions	155–158
Section 4.7	
☐ How to evaluate inverse trigonometric functions	159–168
☐ How to evaluate compositions of trigonometric functions	169–172
Section 4.8	
☐ How to solve real-life problems involving right triangles	173, 174
☐ How to solve real-life problems involving directional bearings	175, 176
☐ How to solve real-life problems involving harmonic motion	177

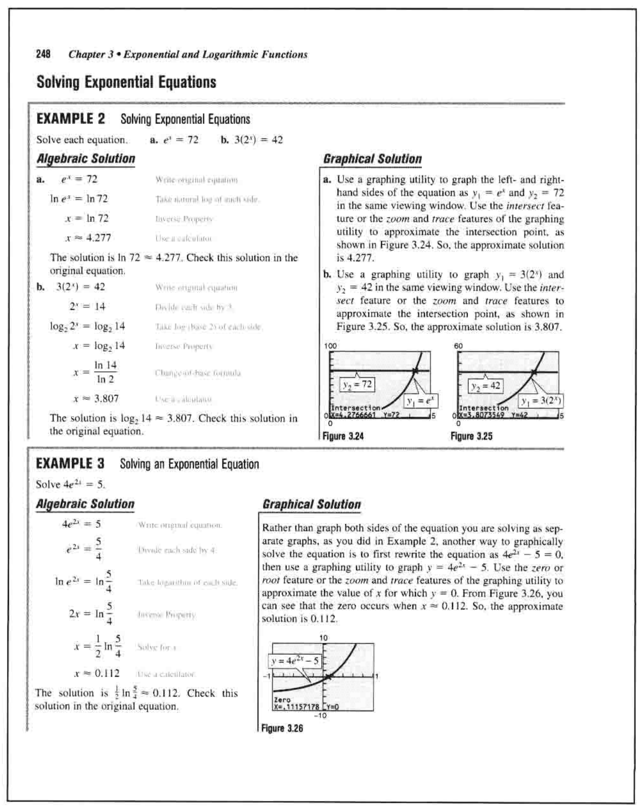
▶ "What did you learn?" Chapter Summary

This chapter summary provides a concise, section-by-section review of the section objectives. These objectives are correlated to the chapter Review Exercises.

Flexibility and Accessibility

▶ Algebraic, Graphical, and Numerical Approach

- Many examples present solutions from multiple approaches—algebraic, graphical, and numerical.
- Solutions are displayed side-by-side.
- The multiple-approach format shows students different solution methods that can be used to reach the same answer.
- The solution format helps students expand their problem-solving abilities.



248 *Chapter 3 • Exponential and Logarithmic Functions*

Solving Exponential Equations

EXAMPLE 2 Solving Exponential Equations

Solve each equation. **a.** $e^x = 72$ **b.** $3(2^x) = 42$

Algebraic Solution

a. $e^x = 72$ Write original equation.

$\ln e^x = \ln 72$ Take natural log of each side.

$x = \ln 72$ Inverse Property

$x \approx 4.277$ Use a calculator.

The solution is $\ln 72 \approx 4.277$. Check this solution in the original equation.

b. $3(2^x) = 42$ Write original equation.

$2^x = 14$ Divide each side by 3.

$\log_2 2^x = \log_2 14$ Take log (base 2) of each side.

$x = \log_2 14$ Inverse Property

$x = \dfrac{\ln 14}{\ln 2}$ Change-of-base formula

$x \approx 3.807$ Use a calculator.

The solution is $\log_2 14 \approx 3.807$. Check this solution in the original equation.

Graphical Solution

a. Use a graphing utility to graph the left- and right-hand sides of the equation as $y_1 = e^x$ and $y_2 = 72$ in the same viewing window. Use the *intersect* feature or the *zoom* and *trace* features of the graphing utility to approximate the intersection point, as shown in Figure 3.24. So, the approximate solution is 4.277.

b. Use a graphing utility to graph $y_1 = 3(2^x)$ and $y_2 = 42$ in the same viewing window. Use the *intersect* feature or the *zoom* and *trace* features to approximate the intersection point, as shown in Figure 3.25. So, the approximate solution is 3.807.

Figure 3.24

Figure 3.25

EXAMPLE 3 Solving an Exponential Equation

Solve $4e^{2x} = 5$.

Algebraic Solution

$4e^{2x} = 5$ Write original equation.

$e^{2x} = \dfrac{5}{4}$ Divide each side by 4.

$\ln e^{2x} = \ln \dfrac{5}{4}$ Take logarithm of each side.

$2x = \ln \dfrac{5}{4}$ Inverse Property

$x = \dfrac{1}{2} \ln \dfrac{5}{4}$ Solve for x.

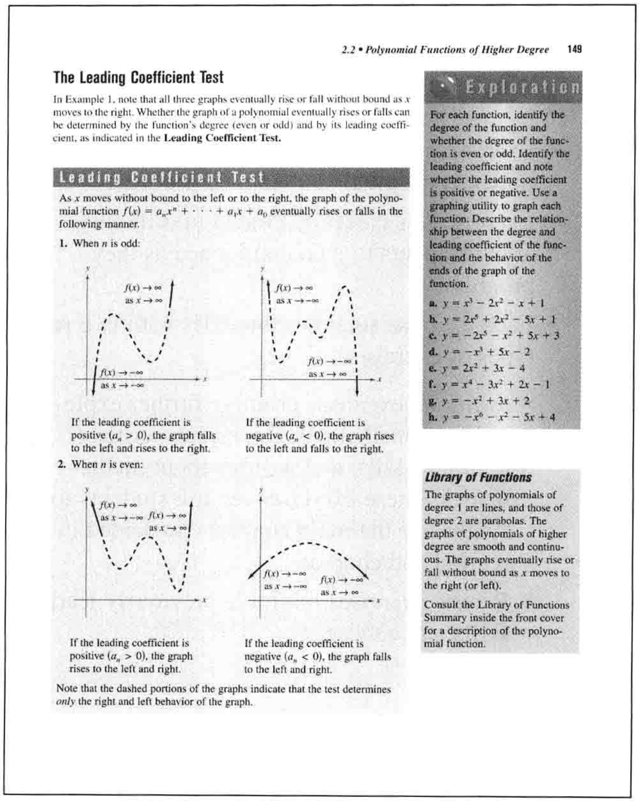
$x \approx 0.112$ Use a calculator.

The solution is $\frac{1}{2} \ln \frac{5}{4} \approx 0.112$. Check this solution in the original equation.

Graphical Solution

Rather than graph both sides of the equation you are solving as separate graphs, as you did in Example 2, another way to graphically solve the equation is to first rewrite the equation as $4e^{2x} - 5 = 0$, then use a graphing utility to graph $y = 4e^{2x} - 5$. Use the *zero* or *root* feature or the *zoom* and *trace* features of the graphing utility to approximate the value of x for which $y = 0$. From Figure 3.26, you can see that the zero occurs when $x \approx 0.112$. So, the approximate solution is 0.112.

Figure 3.26



The Leading Coefficient Test

In Example 1, note that all three graphs eventually rise or fall without bound as x moves to the right. Whether the graph of a polynomial eventually rises or falls can be determined by the function's degree (even or odd) and by its leading coefficient, as indicated in the **Leading Coefficient Test.**

Leading Coefficient Test

As x moves without bound to the left or to the right, the graph of the polynomial function $f(x) = a_n x^n + \cdots + a_1 x + a_0$ eventually rises or falls in the following manner.

1. When n is odd:

If the leading coefficient is positive ($a_n > 0$), the graph falls to the left and rises to the right.

If the leading coefficient is negative ($a_n < 0$), the graph rises to the left and falls to the right.

2. When n is even:

If the leading coefficient is positive ($a_n > 0$), the graph rises to the left and right.

If the leading coefficient is negative ($a_n < 0$), the graph falls to the left and right.

Note that the dashed portions of the graphs indicate that the test determines *only* the right and left behavior of the graph.

Exploration

For each function, identify the degree of the function and whether the degree of the function is even or odd. Identify the leading coefficient and note whether the leading coefficient is positive or negative. Use a graphing utility to graph each function. Describe the relationship between the degree and leading coefficient of the function and the behavior of the ends of the graph of the function.

a. $y = x^3 - 2x^2 - x + 1$
b. $y = 2x^5 + 2x^2 - 5x + 1$
c. $y = -2x^5 - x^2 + 5x + 3$
d. $y = -x^3 + 5x - 2$
e. $y = 2x^2 + 3x - 4$
f. $y = x^4 - 3x^2 + 2x - 1$
g. $y = -x^2 + 3x + 2$
h. $y = -x^6 - x^2 - 5x + 4$

Library of Functions

The graphs of polynomials of degree 1 are lines, and those of degree 2 are parabolas. The graphs of polynomials of higher degree are smooth and continuous. The graphs eventually rise or fall without bound as x moves to the right (or left).

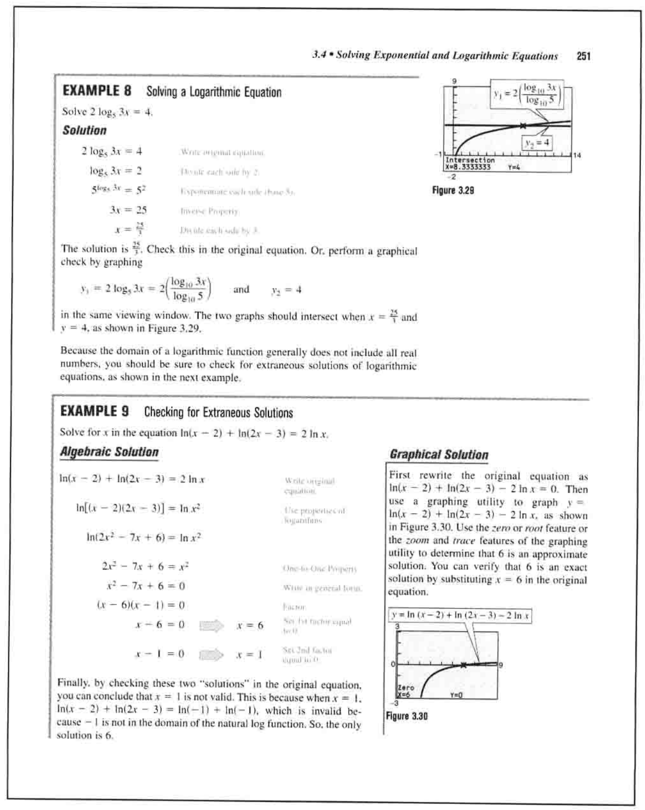
Consult the Library of Functions Summary inside the front cover for a description of the polynomial function.

Exploration

- Before introducing selected topics, *Exploration* engages students in active discovery of mathematical concepts and relationships, often through the power of technology.
- *Exploration* strengthens students' critical thinking skills and helps them develop an intuitive understanding of theoretical concepts.
- *Exploration* is an optional feature and can be omitted without loss of continuity in coverage.

▶ Examples

- Each example was carefully chosen to illustrate a particular mathematical concept or problem-solving skill.
- Every example contains step-by-step solutions, most with side-by-side explanations that lead students through the solution process.
- Many examples provide side-by-side solutions utilizing two separate approaches.



EXAMPLE 8 Solving a Logarithmic Equation

Solve $2 \log_5 3x = 4$.

Solution

$2 \log_5 3x = 4$	Write original equation.
$\log_5 3x = 2$	Divide each side by 2.
$5^{\log_5 3x} = 5^2$	Exponentiate each side (base 5).
$3x = 25$	Inverse Property
$x = \frac{25}{3}$	Divide each side by 3.

The solution is $\frac{25}{3}$. Check this in the original equation. Or, perform a graphical check by graphing

$$y_1 = 2 \log_5 3x = 2\left(\frac{\log_{10} 3x}{\log_{10} 5}\right) \quad \text{and} \quad y_2 = 4$$

in the same viewing window. The two graphs should intersect when $x = \frac{25}{3}$ and $y = 4$, as shown in Figure 3.29.

Figure 3.29

Because the domain of a logarithmic function generally does not include all real numbers, you should be sure to check for extraneous solutions of logarithmic equations, as shown in the next example.

EXAMPLE 9 Checking for Extraneous Solutions

Solve for x in the equation $\ln(x - 2) + \ln(2x - 3) = 2 \ln x$.

Algebraic Solution

$\ln(x - 2) + \ln(2x - 3) = 2 \ln x$	Write original equation.
$\ln[(x - 2)(2x - 3)] = \ln x^2$	Use properties of logarithms.
$\ln(2x^2 - 7x + 6) = \ln x^2$	
$2x^2 - 7x + 6 = x^2$	One-to-One Property
$x^2 - 7x + 6 = 0$	Write in general form.
$(x - 6)(x - 1) = 0$	Factor.
$x - 6 = 0 \Rightarrow x = 6$	Set 1st factor equal to 0.
$x - 1 = 0 \Rightarrow x = 1$	Set 2nd factor equal to 0.

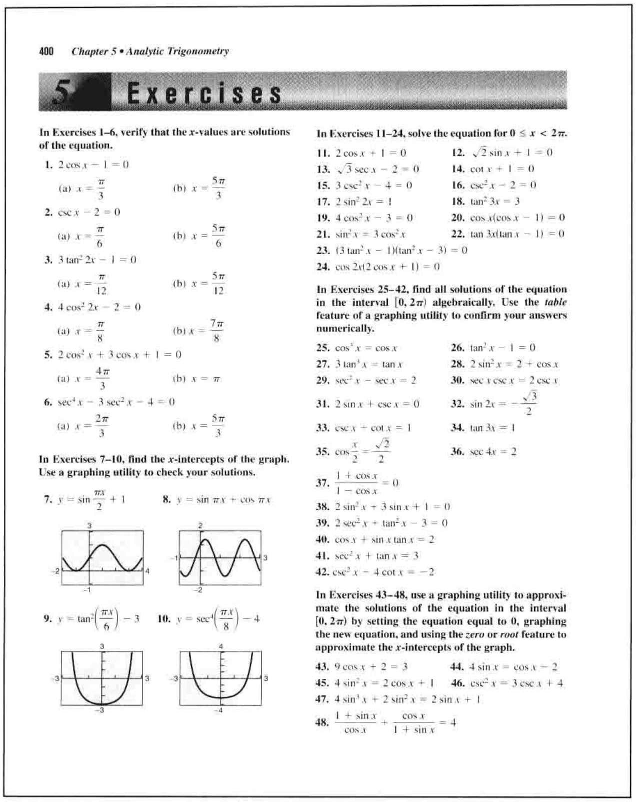
Finally, by checking these two "solutions" in the original equation, you can conclude that $x = 1$ is not valid. This is because when $x = 1$, $\ln(x - 2) + \ln(2x - 3) = \ln(-1) + \ln(-1)$, which is invalid because -1 is not in the domain of the natural log function. So, the only solution is 6.

Graphical Solution

First rewrite the original equation as $\ln(x - 2) + \ln(2x - 3) - 2 \ln x = 0$. Then use a graphing utility to graph $y = \ln(x - 2) + \ln(2x - 3) - 2 \ln x$, as shown in Figure 3.30. Use the *zero* or *root* feature or the *zoom* and *trace* features of the graphing utility to determine that 6 is an approximate solution. You can verify that 6 is an exact solution by substituting $x = 6$ in the original equation.

Figure 3.30

Revised Exercises and Applications



400 *Chapter 5 • Analytic Trigonometry*

5 Exercises

In Exercises 1–6, verify that the x-values are solutions of the equation.

1. $2\cos x - 1 = 0$
   (a) $x = \frac{\pi}{3}$ (b) $x = \frac{5\pi}{3}$
2. $\csc x - 2 = 0$
   (a) $x = \frac{\pi}{6}$ (b) $x = \frac{5\pi}{6}$
3. $3\tan^2 2x - 1 = 0$
   (a) $x = \frac{\pi}{12}$ (b) $x = \frac{5\pi}{12}$
4. $4\cos^2 2x - 2 = 0$
   (a) $x = \frac{\pi}{8}$ (b) $x = \frac{7\pi}{8}$
5. $2\cos^2 x + 3\cos x + 1 = 0$
   (a) $x = \frac{4\pi}{3}$ (b) $x = \pi$
6. $\sec^4 x - 3\sec^2 x - 4 = 0$
   (a) $x = \frac{2\pi}{3}$ (b) $x = \frac{5\pi}{3}$

In Exercises 7–10, find the x-intercepts of the graph. Use a graphing utility to check your solutions.

7. $y = \sin\frac{\pi x}{2} + 1$
8. $y = \sin \pi x + \cos \pi x$
9. $y = \tan^2\left(\frac{\pi x}{6}\right) - 3$
10. $y = \sec^4\left(\frac{\pi x}{8}\right) - 4$

In Exercises 11–24, solve the equation for $0 \le x < 2\pi$.

11. $2\cos x + 1 = 0$
12. $\sqrt{2}\sin x + 1 = 0$
13. $\sqrt{3}\sec x - 2 = 0$
14. $\cot x + 1 = 0$
15. $3\csc^2 x - 4 = 0$
16. $\csc^2 x - 2 = 0$
17. $2\sin^2 2x = 1$
18. $\tan^2 3x = 3$
19. $4\cos^2 x - 3 = 0$
20. $\cos x(\cos x - 1) = 0$
21. $\sin^2 x = 3\cos^2 x$
22. $\tan 3x(\tan x - 1) = 0$
23. $(3\tan^2 x - 1)(\tan^2 x - 3) = 0$
24. $\cos 2x(2\cos x + 1) = 0$

In Exercises 25–42, find all solutions of the equation in the interval $[0, 2\pi)$ algebraically. Use the *table* feature of a graphing utility to confirm your answers numerically.

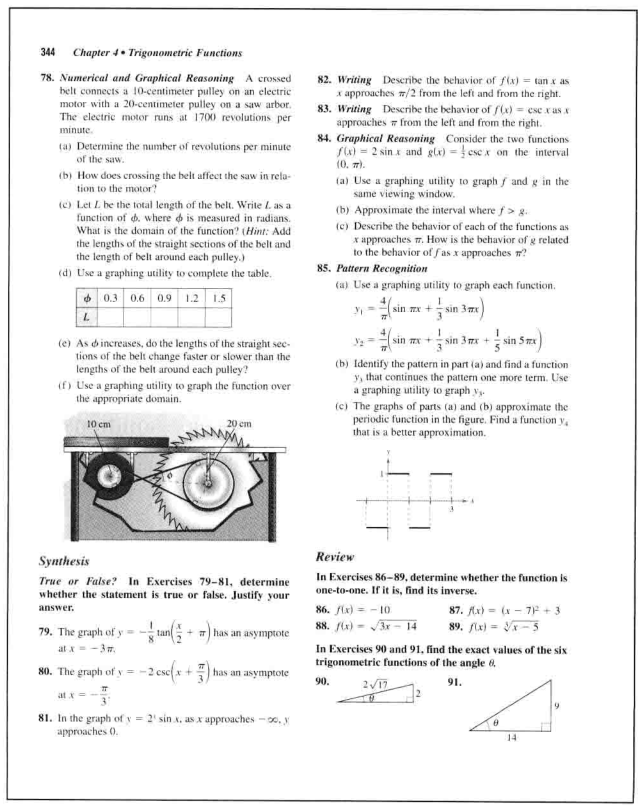
25. $\cos^3 x = \cos x$
26. $\tan^2 x - 1 = 0$
27. $3\tan^3 x = \tan x$
28. $2\sin^2 x = 2 + \cos x$
29. $\sec^2 x - \sec x = 2$
30. $\sec x \csc x = 2\csc x$
31. $2\sin x + \csc x = 0$
32. $\sin 2x = -\frac{\sqrt{3}}{2}$
33. $\csc x + \cot x = 1$
34. $\tan 3x = 1$
35. $\cos\frac{x}{2} = \frac{\sqrt{2}}{2}$
36. $\sec 4x = 2$
37. $\frac{1 + \cos x}{1 - \cos x} = 0$
38. $2\sin^2 x + 3\sin x + 1 = 0$
39. $2\sec^2 x + \tan^2 x - 3 = 0$
40. $\cos x + \sin x \tan x = 2$
41. $\sec^2 x + \tan x = 3$
42. $\csc^2 x - 4\cot x = -2$

In Exercises 43–48, use a graphing utility to approximate the solutions of the equation in the interval $[0, 2\pi)$ by setting the equation equal to 0, graphing the new equation, and using the *zero* or *root* feature to approximate the x-intercepts of the graph.

43. $9\cos x + 2 = 3$
44. $4\sin x = \cos x - 2$
45. $4\sin^2 x = 2\cos x + 1$
46. $\csc^2 x = 3\csc x + 4$
47. $4\sin^3 x + 2\sin^2 x = 2\sin x + 1$
48. $\frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} = 4$

▶ Exercises

- Exercise sets consist of a variety of computational, conceptual, and applied problems.
- Exercise sets carefully graded in difficulty allow students to gain confidence as they progress.
- Each exercise set now concludes with two new types of exercises:
  - *Synthesis* exercises promote further exploration of mathematical concepts, critical thinking skills, and writing about mathematics. These exercises require students to synthesize the main concepts presented in the section and chapter.
  - *Review* exercises reinforce previously learned skills and concepts.



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78. ***Numerical and Graphical Reasoning*** A crossed belt connects a 10-centimeter pulley on an electric motor with a 20-centimeter pulley on a saw arbor. The electric motor runs at 1700 revolutions per minute.
   (a) Determine the number of revolutions per minute of the saw.
   (b) How does crossing the belt affect the saw in relation to the motor?
   (c) Let L be the total length of the belt. Write L as a function of ϕ, where ϕ is measured in radians. What is the domain of the function? (*Hint:* Add the lengths of the straight sections of the belt and the length of belt around each pulley.)
   (d) Use a graphing utility to complete the table.

ϕ	0.3	0.6	0.9	1.2	1.5
L					

   (e) As ϕ increases, do the lengths of the straight sections of the belt change faster or slower than the lengths of the belt around each pulley?
   (f) Use a graphing utility to graph the function over the appropriate domain.

10 cm 20 cm

Synthesis

True or False? **In Exercises 79–81, determine whether the statement is true or false. Justify your answer.**

79. The graph of $y = -\frac{1}{8}\tan\left(\frac{x}{2} + \pi\right)$ has an asymptote at $x = -3\pi$.
80. The graph of $y = -2\csc\left(x + \frac{\pi}{3}\right)$ has an asymptote at $x = -\frac{\pi}{3}$.
81. In the graph of $y = 2^x \sin x$, as x approaches $-\infty$, y approaches 0.
82. ***Writing*** Describe the behavior of $f(x) = \tan x$ as x approaches $\pi/2$ from the left and from the right.
83. ***Writing*** Describe the behavior of $f(x) = \csc x$ as x approaches π from the left and from the right.
84. ***Graphical Reasoning*** Consider the two functions $f(x) = 2\sin x$ and $g(x) = \frac{1}{2}\csc x$ on the interval $(0, \pi)$.
   (a) Use a graphing utility to graph f and g in the same viewing window.
   (b) Approximate the interval where $f > g$.
   (c) Describe the behavior of each of the functions as x approaches π. How is the behavior of g related to the behavior of f as x approaches π?
85. ***Pattern Recognition***
   (a) Use a graphing utility to graph each function.
   $y_1 = \frac{4}{\pi}\left(\sin \pi x + \frac{1}{3}\sin 3\pi x\right)$
   $y_2 = \frac{4}{\pi}\left(\sin \pi x + \frac{1}{3}\sin 3\pi x + \frac{1}{5}\sin 5\pi x\right)$
   (b) Identify the pattern in part (a) and find a function y_3 that continues the pattern one more term. Use a graphing utility to graph y_3.
   (c) The graphs of parts (a) and (b) approximate the periodic function in the figure. Find a function y_4 that is a better approximation.

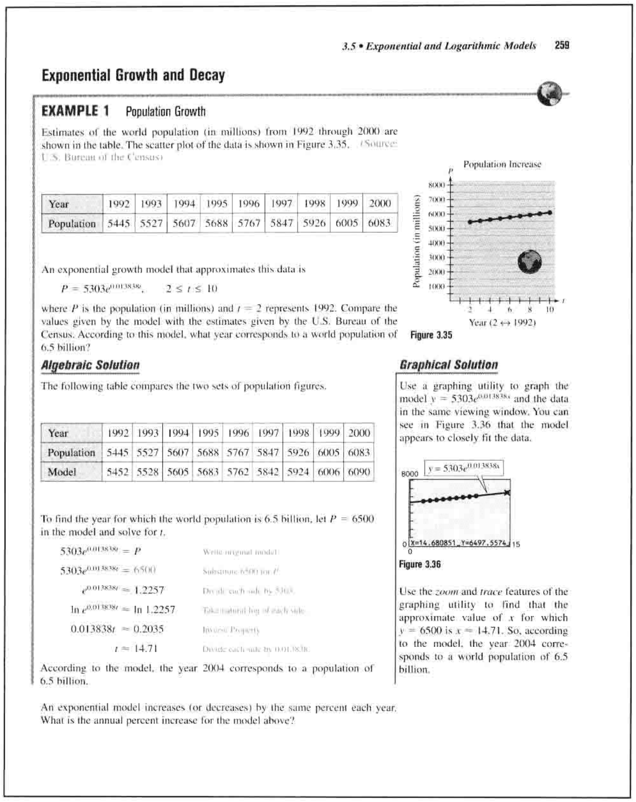
Review

In Exercises 86–89, determine whether the function is one-to-one. If it is, find its inverse.

86. $f(x) = -10$
87. $f(x) = (x - 7)^2 + 3$
88. $f(x) = \sqrt{3x - 14}$
89. $f(x) = \sqrt[3]{x - 5}$

In Exercises 90 and 91, find the exact values of the six trigonometric functions of the angle θ.

90. $2\sqrt{17}$, 2, θ
91. 9, 14, θ



Exponential Growth and Decay

EXAMPLE 1 Population Growth

Estimates of the world population (in millions) from 1992 through 2000 are shown in the table. The scatter plot of the data is shown in Figure 3.35. (Source: U.S. Bureau of the Census)

Year	1992	1993	1994	1995	1996	1997	1998	1999	2000
Population	5445	5527	5607	5688	5767	5847	5926	6005	6083

Figure 3.35

An exponential growth model that approximates this data is

$$P = 5303e^{0.013838t}, \quad 2 \le t \le 10$$

where P is the population (in millions) and $t = 2$ represents 1992. Compare the values given by the model with the estimates given by the U.S. Bureau of the Census. According to this model, what year corresponds to a world population of 6.5 billion?

Algebraic Solution

The following table compares the two sets of population figures.

Year	1992	1993	1994	1995	1996	1997	1998	1999	2000
Population	5445	5527	5607	5688	5767	5847	5926	6005	6083
Model	5452	5528	5605	5683	5762	5842	5924	6006	6090

To find the year for which the world population is 6.5 billion, let $P = 6500$ in the model and solve for t.

$5303e^{0.013838t} = P$ Write original model.

$5303e^{0.013838t} = 6500$ Substitute 6500 for P.

$e^{0.013838t} \approx 1.2257$ Divide each side by 5303.

$\ln e^{0.013838t} \approx \ln 1.2257$ Take natural log of each side.

$0.013838t \approx 0.2035$ Inverse Property

$t \approx 14.71$ Divide each side by 0.013838.

According to the model, the year 2004 corresponds to a population of 6.5 billion.

Graphical Solution

Use a graphing utility to graph the model $y = 5303e^{0.013838x}$ and the data in the same viewing window. You can see in Figure 3.36 that the model appears to closely fit the data.

Figure 3.36

Use the *zoom* and *trace* features of the graphing utility to find that the approximate value of x for which $y = 6500$ is $x \approx 14.71$. So, according to the model, the year 2004 corresponds to a world population of 6.5 billion.

An exponential model increases (or decreases) by the same percent each year. What is the annual percent increase for the model above?

▶ Real-Life Applications

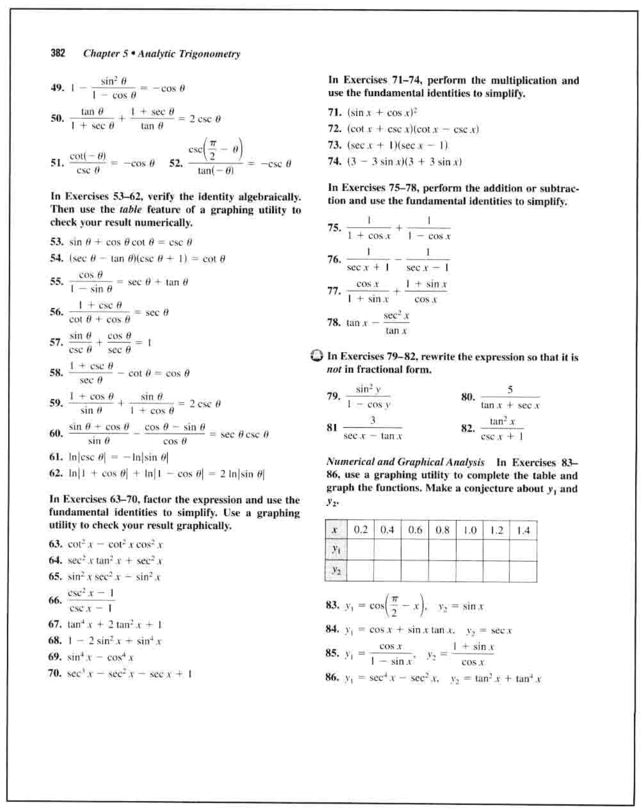
- A wide variety of real-life applications, many using current, real data, are integrated throughout the examples and exercises.
- The icon indicates an example that involves a real-life application.

▶ Algebra of Calculus

- Special emphasis is given to the algebraic techniques used in calculus.
- Algebra of Calculus examples and exercises are integrated throughout the text.
- The symbol indicates an example or exercise in which the Algebra of Calculus is featured.

▶ Additional Features

Carefully crafted learning tools designed to create a rich learning environment can be found throughout the text. These learning tools include a Library of Functions, Study Tips, Historical Notes, Writing About Math, Chapter Projects, Chapter Review Exercises, Chapter Tests, Cumulative Tests, and an extensive art program.



49. $1 - \dfrac{\sin^2\theta}{1-\cos\theta} = -\cos\theta$

50. $\dfrac{\tan\theta}{1+\sec\theta} + \dfrac{1+\sec\theta}{\tan\theta} = 2\csc\theta$

51. $\dfrac{\cot(-\theta)}{\csc\theta} = -\cos\theta$ **52.** $\dfrac{\csc\left(\frac{\pi}{2}-\theta\right)}{\tan(-\theta)} = -\csc\theta$

In Exercises 53–62, verify the identity algebraically. Then use the *table* feature of a graphing utility to check your result numerically.

53. $\sin\theta + \cos\theta\cot\theta = \csc\theta$

54. $(\sec\theta - \tan\theta)(\csc\theta + 1) = \cot\theta$

55. $\dfrac{\cos\theta}{1-\sin\theta} = \sec\theta + \tan\theta$

56. $\dfrac{1+\csc\theta}{\cot\theta+\cos\theta} = \sec\theta$

57. $\dfrac{\sin\theta}{\csc\theta} + \dfrac{\cos\theta}{\sec\theta} = 1$

58. $\dfrac{1+\csc\theta}{\sec\theta} - \cot\theta = \cos\theta$

59. $\dfrac{1+\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{1+\cos\theta} = 2\csc\theta$

60. $\dfrac{\sin\theta+\cos\theta}{\sin\theta} - \dfrac{\cos\theta-\sin\theta}{\cos\theta} = \sec\theta\csc\theta$

61. $\ln|\csc\theta| = -\ln|\sin\theta|$

62. $\ln|1+\cos\theta| + \ln|1-\cos\theta| = 2\ln|\sin\theta|$

In Exercises 63–70, factor the expression and use the fundamental identities to simplify. Use a graphing utility to check your result graphically.

63. $\cot^2 x - \cot^2 x\cos^2 x$

64. $\sec^2 x\tan^2 x + \sec^2 x$

65. $\sin^2 x\sec^2 x - \sin^2 x$

66. $\dfrac{\csc^2 x - 1}{\csc x - 1}$

67. $\tan^4 x + 2\tan^2 x + 1$

68. $1 - 2\sin^2 x + \sin^4 x$

69. $\sin^4 x - \cos^4 x$

70. $\sec^3 x - \sec^2 x - \sec x + 1$

In Exercises 71–74, perform the multiplication and use the fundamental identities to simplify.

71. $(\sin x + \cos x)^2$

72. $(\cot x + \csc x)(\cot x - \csc x)$

73. $(\sec x + 1)(\sec x - 1)$

74. $(3 - 3\sin x)(3 + 3\sin x)$

In Exercises 75–78, perform the addition or subtraction and use the fundamental identities to simplify.

75. $\dfrac{1}{1+\cos x} + \dfrac{1}{1-\cos x}$

76. $\dfrac{1}{\sec x + 1} - \dfrac{1}{\sec x - 1}$

77. $\dfrac{\cos x}{1+\sin x} + \dfrac{1+\sin x}{\cos x}$

78. $\tan x - \dfrac{\sec^2 x}{\tan x}$

In Exercises 79–82, rewrite the expression so that it is *not* in fractional form.

79. $\dfrac{\sin^2 y}{1-\cos y}$ **80.** $\dfrac{5}{\tan x + \sec x}$

81 $\dfrac{3}{\sec x - \tan x}$ **82.** $\dfrac{\tan^2 x}{\csc x + 1}$

Numerical and Graphical Analysis **In Exercises 83–86, use a graphing utility to complete the table and graph the functions. Make a conjecture about y_1 and y_2.**

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_1							
y_2							

83. $y_1 = \cos\left(\frac{\pi}{2} - x\right), \quad y_2 = \sin x$

84. $y_1 = \cos x + \sin x\tan x, \quad y_2 = \sec x$

85. $y_1 = \dfrac{\cos x}{1-\sin x}, \quad y_2 = \dfrac{1+\sin x}{\cos x}$

86. $y_1 = \sec^4 x - \sec^2 x, \quad y_2 = \tan^2 x + \tan^4 x$

Supplements

Resources

Website (*college.hmco.com*)
Many additional text-specific study and interactive features for students and instructors can be found at the Houghton Mifflin website.

For the Student

Student Success Organizer

Study and Solutions Guide by Bruce H. Edwards (University of Florida)

Graphing Technology Guide by Benjamin N. Levy and Laurel Technical Services

Instructional Videotapes by Dana Mosely

Instructional Videotapes for Graphing Calculators by Dana Mosely

For the Instructor

Instructor's Annotated Edition

Instructor Success Organizer

Complete Solutions Guide by Bruce H. Edwards (University of Florida)

Test Item File

Problem Solving, Modeling, and Data Analysis Labs by Wendy Metzger (Palomar College)

Computerized Testing (Windows, Macintosh)

HMClassPrep Instructor's CD-ROM

An Introduction to Graphing Utilities

Graphing utilities such as graphing calculators and computers with graphing software are very valuable tools for visualizing mathematical principles, verifying solutions to equations, exploring mathematical ideas, and developing mathematical models. Although graphing utilities are extremely helpful in learning mathematics, their use does not mean that learning algebra is any less important. In fact, the combination of knowledge of mathematics and the use of graphing utilities allows you to explore mathematics more easily and to a greater depth. If you are using a graphing utility in this course, it is up to you to learn its capabilities and to practice using this tool to enhance your mathematical learning.

In this text there are many opportunities to use a graphing utility, some of which are described below.

Some Uses of a Graphing Utility

A graphing utility can be used to

- check or validate answers to problems obtained using algebraic methods.
- discover and explore algebraic properties, rules, and concepts.
- graph functions, and approximate solutions to equations involving functions.
- efficiently perform complicated mathematical procedures such as those found in many real-life applications.
- find mathematical models for sets of data.

In this introduction, the features of graphing utilities are discussed from a generic perspective. To learn how to use the features of a specific graphing utility, consult your user's manual or the website for this text found at *college.hmco.com*. Additionally, keystroke guides are available for most graphing utilities, and your college library may have a videotape on how to use your graphing utility.

The Equation Editor

Many graphing utilities are designed to act as "function graphers." In this course, you will study functions and their graphs in detail. You may recall from previous courses that a function can be thought of as a rule that describes the relationship between two variables. These rules are frequently written in terms of x and y. For example, the equation $y = 3x + 5$ represents y as a function of x.

```
 Plot1 Plot2 Plot3
\Y1=3x+5
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

Figure 1

Many graphing utilities have an equation editor that requires an equation to be written in "$y =$" form in order to be entered, as shown in Figure 1. (You should note that your equation editor screen may not look like the screen shown in Figure 1.) To determine exactly how to enter an equation into your graphing utility, consult your user's manual.

The Table Feature

Most graphing utilities are capable of displaying a table of values with x-values and one or more corresponding y-values. These tables can be used to check solutions of an equation and to generate ordered pairs to assist in graphing an equation.

To use the *table* feature, enter an equation into the equation editor in "$y =$" form. The table may have a setup screen, which allows you to select the starting x-value and the table step or x-increment. You may then have the option of automatically generating values for x and y or building your own table using the *ask* mode. In the *ask* mode, you enter a value for x and the graphing utility displays the y-value.

For example, enter the equation

$$y = \frac{3x}{x + 2}$$

into the equation editor, as shown in Figure 2. In the table setup screen, set the table to start at $x = -4$ and set the table step to 1. When you view the table, notice that the first x-value is -4 and each value after it increases by 1. Also notice that the Y_1 column gives the resulting y-value for each x-value, as shown in Figure 3. The table shows that the y-value when $x = -2$ is ERROR. This means that the variable x may not take on the value -2 in this equation.

```
 Plot1 Plot2 Plot3
\Y1■3x/(X+2)■
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

Figure 2

X	Y1	
-4	6	
-3	9	
-2	ERROR	
-1	-3	
0	0	
1	1	
2	1.5	

X=-4

Figure 3

With the same equation in the equation editor, set the table to *ask* mode. In this mode you do not need to set the starting x-value or the table step, because you are entering any value you choose for x. You may enter any real value for x—integers, fractions, decimals, irrational numbers, and so forth. If you enter $x = 1 + \sqrt{3}$, the graphing utility may rewrite the number as a decimal approximation, as shown in Figure 4. You can continue to build your own table by entering additional x-values in order to generate y-values.

X	Y1	
2.7321	1.7321	

X=

Figure 4

If you have several equations in the equation editor, the table may generate y-values for each equation.

Creating a Viewing Window

A **viewing window** for a graph is a rectangular portion of the coordinate plane. A viewing window is determined by the following six values.

Xmin = the smallest value of x

Xmax = the largest value of x

Xscl = the number of units per tick mark on the x-axis

Ymin = the smallest value of y

Ymax = the largest value of y

Yscl = the number of units per tick mark on the y-axis

When you enter these six values into a graphing utility, you are setting the viewing window. Some graphing utilities have a standard viewing window, as shown in Figure 5.

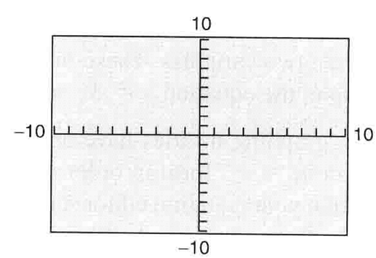
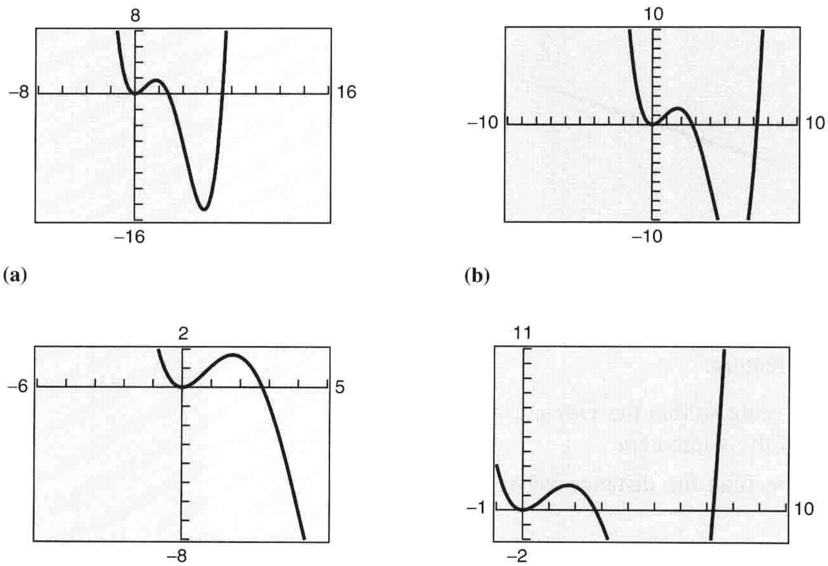


Figure 5

By choosing different viewing windows for a graph, it is possible to obtain very different impressions of the graph's shape. For instance, Figure 6 shows four different viewing windows for the graph of

$$y = 0.1x^4 - x^3 + 2x^2.$$

Of these, the view shown in part (a) is the most complete.



(a) (b) (c) (d)

Figure 6

On most graphing utilities, the display screen is two-thirds as high as it is wide. On such screens, you can obtain a graph with a true geometric perspective by using **a square setting**—one in which

$$\frac{\text{Ymax} - \text{Ymin}}{\text{Xmax} - \text{Xmin}} = \frac{2}{3}.$$

One such setting is shown in Figure 7. Notice that the x and y tick marks are equally spaced on a square setting, but not on a standard setting.

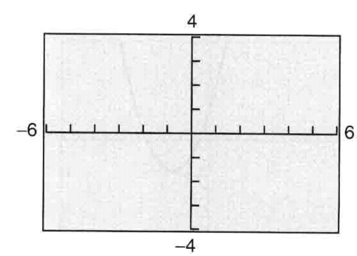


Figure 7

To see how the viewing window affects the geometric perspective, graph the semicircles $y_1 = \sqrt{9 - x^2}$ and $y_2 = -\sqrt{9 - x^2}$ in a standard viewing window. Then graph y_1 and y_2 in a square window. Note the difference in the shapes of the circles.

Zoom and Trace Features

When you graph an equation, you can move from point to point along its graph using the *trace* feature. As you trace the graph, the coordinates of each point are displayed, as shown in Figure 8. The *trace* feature combined with the *zoom* feature allows you to obtain better and better approximations of desired points on a graph. For instance, you can use the *zoom* feature of a graphing utility to approximate the x-intercept(s) of a graph [the point(s) where the graph crosses the x-axis]. Suppose you want to approximate the x-intercept(s) of the graph of $y = 2x^3 - 3x + 2$.

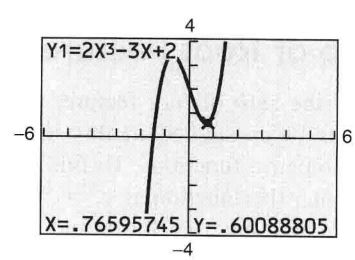
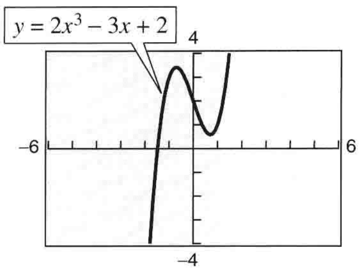
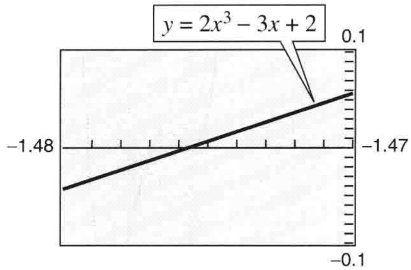


Figure 8

Begin by graphing the equation, as shown in Figure 9(a). From the viewing window shown, the graph appears to have only one x-intercept. This intercept lies between -2 and -1. By zooming in on the intercept, you can improve the approximation, as shown in Figure 9(b). To three decimal places, the solution is $x \approx -1.476$.





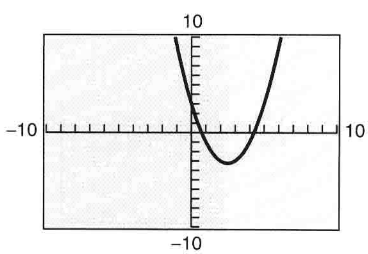
(a) **(b)**

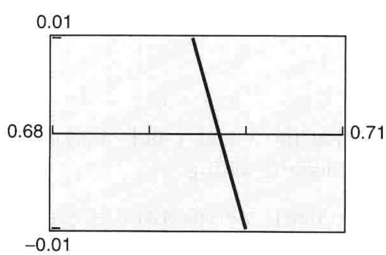
Figure 9

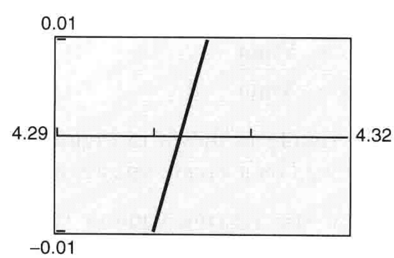
Here are some suggestions for using the *zoom* feature.

1. With each successive zoom-in, adjust the x-scale so that the viewing window shows at least one tick mark on each side of the x-intercept.
2. The error in your approximation will be less than the distance between two scale marks.
3. The *trace* feature can usually be used to add one more decimal place of accuracy without changing the viewing window.

Figure 10(a) shows the graph of $y = x^2 - 5x + 3$. Figures 10(b) and 10(c) show "zoom-in views" of the two x-intercepts. From these views, you can approximate the x-intercepts to be $x \approx 0.697$ and $x \approx 4.303$.





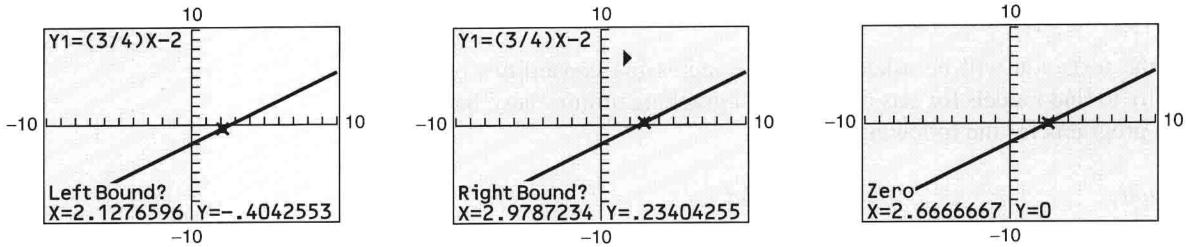


(a) **(b)** **(c)**

Figure 10

Zero or Root Feature

Using the *zero* or *root* feature, you can find the real zeros of functions of the various types studied in this text—polynomial, exponential, logarithmic, and trigonometric functions. To find the zeros of a function such as $f(x) = \frac{3}{4}x - 2$, first enter the function as $y_1 = \frac{3}{4}x - 2$. Then use the *zero* or *root* feature, which may require entering lower and upper bound estimates of the root, as shown in Figure 11.



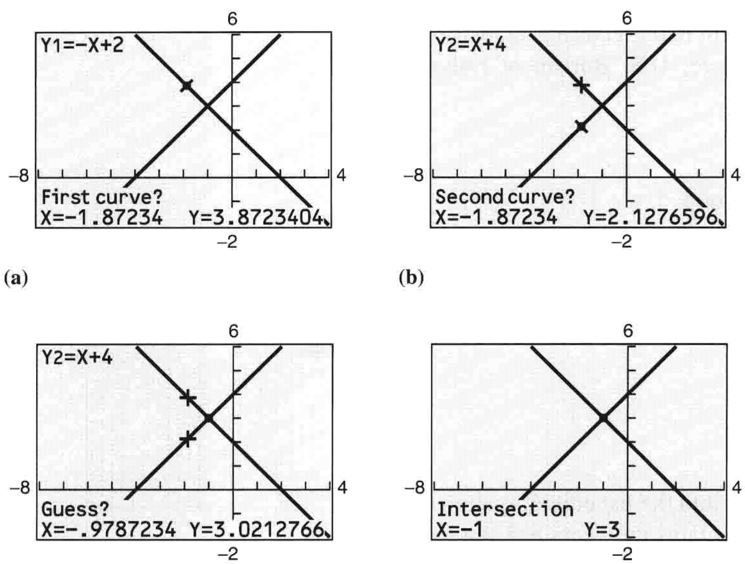
(a) **(b)** **(c)**

Figure 11

In Figure 11(c), you can see that the zero is $x = 2.6666667 \approx 2\frac{2}{3}$.

Intersect Feature

To find the points of intersection of two graphs, you can use the *intersect* feature. For instance, to find the points of intersection of the graphs of $y_1 = -x + 2$ and $y_2 = x + 4$, enter these two functions and use the *intersect* feature, as shown in Figure 12.



(c) **(d)**

Figure 12

From Figure 12(d), you can see that the point of intersection is $(-1, 3)$.