Nonlinear Spatio-Temporal Dynamics and Chaos in Semiconductors

Eckehard Schöll
Technische Universität Berlin

CAMBRIDGE
UNIVERSITY PRESS

PUBLISHED BY THE PRESS SYNDICATE OF THE UNIVERSITY OF CAMBRIDGE
The Pitt Building, Trumpington Street, Cambridge, United Kingdom

CAMBRIDGE UNIVERSITY PRESS
The Edinburgh Building, Cambridge CB2 2RU, UK
40 West 20th Street, New York, NY 10011-4211, USA
10 Stamford Road, Oakleigh, VIC 3166, Australia
Ruiz de Alarcón 13, 28014, Madrid, Spain
Dock House, The Waterfront, Cape Town 8001, South Africa

http://www.cambridge.org

First published 2001

Printed in the United Kingdom at the University Press, Cambridge

Typeface Times 10.25/13.25pt. *System* LaTeX 2_ε [DBD]

A catalogue record of this book is available from the British Library

ISBN 0 521 45186 8 hardback

Nonlinear Spatio-Temporal Dynamics and Chaos in Semiconductors

Nonlinear transport phenomena are an increasingly important aspect of modern semiconductor research. *Nonlinear Spatio-Temporal Dynamics and Chaos in Semiconductors* deals with complex nonlinear dynamics, pattern formation, and chaotic behavior in such systems. In doing so it bridges the gap between two well-established fields: the theory of dynamic systems, and nonlinear charge transport in semiconductors. This unified approach is used to consider important electronic transport instabilities. The initial chapters lay a general framework for the theoretical description of nonlinear self-organized spatio-temporal patterns, such as current filaments, field domains, and fronts, and for analysis of their stability. Later chapters consider important model systems in detail: impact-ionization-induced impurity breakdown, Hall instabilities, superlattices, and low-dimensional structures. State-of-the-art results include chaos control, spatio-temporal chaos, multistability, pattern selection, activator–inhibitor kinetics, and global coupling, linking fundamental issues to electronic-device applications. This book will be of great value to semiconductor physicists and nonlinear scientists alike.

ECKEHARD SCHÖLL was born on 6 February 1951 in Stuttgart, Germany. He received his Diplom degree in physics (MSc) from the University of Tübingen, Germany, in 1976, his PhD in applied mathematics from the University of Southampton, UK, in 1978, and the Dr rer. nat. degree and the *venia legendi* from Aachen University of Technology (RWTH Aachen), Germany, in 1981 and 1986 respectively. During 1983–1984 he was a visiting assistant professor in the Department of Electrical and Computer Engineering, Wayne State University Detroit, Michigan, USA. Since 1989 he has been a Professor of Theoretical Physics at the Technical University of Berlin, Germany. His research interests include the theory of nonlinear charge transport and current instabilities in semiconductors, in particular, low-dimensional structures; nonlinear spatio-temporal dynamics, chaos, and pattern formation; and electro-optical nonlinearities. Dr Schöll is the author of the book *Nonequilibrium Phase Transitions in Semiconductors* (1987), translated into Russian in 1991, the coauthor of *The Physics of Instabilities in Solid State Electron Devices* (1992), and has recently edited a monograph on the *Theory of Transport Properties of Semiconductor Nanostructures* (1998). He has published almost 200 articles in international journals, including *Physical Review Letters*, *Physical Review*, *Semiconductor Science and Technology*, *Applied Physics Letters*, the *Journal of Applied Physics*, and many others. In 1997 he was awarded a prize as a Champion of Teaching by the Technical University of Berlin.

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Preface

More than a dozen years after my book on *Nonequilibrium Phase Transitions in Semiconductors – Self-organization Induced by Generation and Recombination Processes* appeared, the subject of nonlinear dynamics and pattern formation in semiconductors has become a mature field. The aim of that book had been to link two hitherto separate disciplines, semiconductor physics and nonlinear dynamics, and advance the view of a semiconductor driven far from thermodynamic equilibrium as a nonlinear dynamic system. It focussed on one particular class of instabilities related to nonlinear processes of generation and recombination of carriers in bulk semiconductors, and was essentially restricted to either purely *temporal* nonlinear dynamics, or nonlinear *stationary spatial* patterns. Within the past decade extensive research, both theoretical and experimental, has elaborated a great wealth of complex self-organized spatio-temporal patterns in various semiconductor structures and material systems. Thus semiconductors have been established as a model system with several advantages over the classical systems in which self-organization and nonlinear dynamics have been studied, viz. hydrodynamic, optical, and chemically reacting systems. First, semiconductor structures nowadays can be designed and fabricated by modern epitaxial growth technologies with almost unlimited flexibility. By controlling the vertical and lateral dimensions of those structures on an atomic length scale, systems with specific electric and optical properties can be tailored. Second, the dynamic variables describing nonlinear charge-transport properties are directly and easily accessible to measurement as electric quantities. Elaborate experimental techniques for detecting spatio-temporal patterns with high spatial and temporal resolution have recently been developed. Third, semiconductors may provide particularly useful applications of fundamental concepts and results of nonlinear dynamics. Many modern microelectronic devices operate in the regime of controlled electric instabilities. Semiconductor nanostructures, due to the occurrence of large electric fields, inherently possess strongly nonlinear transport properties that may be applied in electronic oscillating or switching devices.

This book provides a theoretical framework for complex nonlinear spatio-temporal dynamics, pattern formation, and chaotic behavior in semiconductors, illustrated by computer simulations and experimental results. It expands the scope of my earlier book substantially in a three-fold way. First, the emphasis is on nonlinear *spatially and temporally* modulated self-organized patterns. While purely temporal or spatial structures generally occur as first bifurcations of a dynamic system on going away from thermodynamic equilibrium, within the past decade research in the field of nonlinear dynamics has focussed more and more on higher bifurcations leading to complex structures such as breathing or spiking modes and spatio-temporal chaos. Second, the analysis of semiconductor instabilities is extended to a wide variety of transport mechanisms. Since the interest in semiconductor physics has generally shifted away from bulk materials, toward low-dimensional structures, this book features in particular modulated semiconductor structures, including heterojunctions between layers of different materials, such as quantum wells, superlattices, and resonant tunneling structures. Third, the book aims to introduce modern concepts and methods of dynamic systems into the field of semiconductors and thereby provide a unified and coherent approach to diverse models and results of electronic instabilities. Therefore the systematic methodology is stressed and illustrated by simple models offering insight, rather than by analyzing sophisticated, highly specialized transport models, even though those might provide a better quantitative description of specific cases.

The aim of this book is to build a bridge between two well-established fields: the theory of dynamic systems, and nonlinear charge transport in semiconductors. A unified approach toward various scattered results on electronic transport instabilities is adopted. The first three chapters lay the general framework for the theoretical description of nonlinear self-organized spatio-temporal patterns, such as current filaments, field domains, and fronts, and the analysis of their stability, while in the last four chapters some important model systems are treated in detail: impact-ionization-induced low-temperature impurity breakdown; Hall instabilities in crossed electric and magnetic fields; vertical high-field transport in superlattices; and spatio-temporal chaos in layered and low-dimensional semiconductor structures. These model systems are not exhaustive; rather they should be viewed as examples and give guidance for the application of the general concepts to other semiconductor structures. Particular emphasis is placed on state-of-the-art results such as chaos control, spatio-temporal chaos, multistability, pattern selection, activator–inhibitor kinetics, and global couplings due to the circuit. Wherever appropriate, I have tried to link fundamental issues with aspects of applications in electronic devices.

The book is aimed at physicists, electronic engineers, applied mathematicians, and materials scientists. It should be of interest to graduate students and researchers who want to familiarize themselves with this new field. A brief summary of nonlinear dynamics and chaos theory, and of semiconductor transport theory is provided in Chapters 1 and 2, respectively, with further references to more detailed treatises. Although this book can not substitute for an introduction to dynamic systems or semiconductors on a textbook level, it nevertheless introduces all of the basic notions and concepts needed. It has been written with the intention of providing the reader with the tools to apply the methods of nonlinear dynamics to further models of his specific interest.

This book would not have been possible without the interaction with many of my colleagues and students. I am grateful for valuable discussions with K. Aoki, M. Asche, N. Balkan, L. L. Bonilla, L. Eaves, H. Engel, H. Gajewski, H. P. Herzel, W. Just, H. Kostial, K. Kunihiro, A. S. Mikhailov, F. J. Niedernostheide, V. Novak, J. Parisi, J. Peinke, W. Prettl, H. G. Purwins, B. K. Ridley, P. Rodin, L. Schimansky-Geier, J. Socolar, S. W. Teitsworth, P. Vogl, and S. M. Zoldi. I am indebted to my collaborators and students who have over many years contributed substantially to the results presented in this book. In particular, special thanks are due to A. Amann, S. Bose, M. Meixner, P. Rodin, G. Schwarz, and A. Wacker for helpful comments and for critically reading parts of the manuscript. I am deeply indebted to my academic teachers P. T. Landsberg and F. Schlögl, who introduced me to this field. Last but not least, I want to thank my family for their patience and encouragement.

Much of this work was supported by various grants from the Deutsche Forschungsgemeinschaft and the Alexander-von-Humboldt-Stiftung.

Berlin, January 2000
Eckehard Schöll

Contents

Chapter 1

Semiconductors as continuous nonlinear dynamic systems

This book deals with complex nonlinear spatio-temporal dynamics, pattern formation, and chaotic behavior in semiconductors. Its aim is to build a bridge between two well-established fields: The theory of dynamic systems, and nonlinear charge transport in semiconductors. In this introductory chapter the foundations on which the theory of semiconductor instabilities can be developed in later chapters will be laid. We will thus introduce the basic notions and concepts of continuous nonlinear dynamic systems. After a brief introduction to the subject, highlighting dissipative structures and negative differential conductivity in semiconductors, the most common bifurcations in dynamic systems will be reviewed. The notion of deterministic chaos, some common scenarios, and the particularly challenging topic of chaos control are introduced. Activator–inhibitor kinetics in spatially extended dynamic systems is discussed with specific reference to semiconductors. The role of global couplings is illuminated and related to the external circuits in which semiconductor elements are operated.

1.1 Introduction

Semiconductors are complex many-body systems whose physical, e.g. electric or optical, properties are governed by a variety of nonlinear dynamic processes. In particular, modern semiconductor structures whose structural and electronic properties vary on a nanometer scale provide an abundance of examples of nonlinear transport processes. In these structures nonlinear transport mechanisms are given, for instance, by quantum mechanical tunneling through potential barriers, or by thermionic emission of *hot electrons* that have enough kinetic energy to overcome the barrier. A further important feature connected with potential barriers and quantum wells in such semiconductor structures is the ubiquitous presence of space charge. This, according to Poisson's equation, induces a further feedback between the charge-carrier distribution and the electric potential distribution governing the transport. This mutual nonlinear interdependence is particularly pronounced in the cases of semiconductor heterostructures and low-dimensional structures, in which abrupt junctions between different materials on an atomic length scale cause conduction band discontinuities resulting in potential barriers and wells. The local charge accumulation in these potential wells and nonlinear processes for transport of charge across the barriers have been found to provide a number of nonlinearities.

Another important class of nonlinear processes that influences, in particular, the electric transport properties, but also optical phenomena, is generation and recombination processes of nonequilibrium charge carriers (Schöll 1987). These are generally described by rate equations for the change in time of the carrier concentrations, which are nonlinear functions of these concentrations and of the electric field. Other nonlinearities are exhibited by nonlinear scattering processes of hot carriers, which may lead to a field-dependent mobility, and thus to a current density that is a nonlinear function of the local electric field.

1.1.1 Dissipative structures

Although the features described above have been known for a long time, the view of a semiconductor as a nonlinear dynamic system is a fairly recent development. Such nonlinear dynamic systems can exhibit a variety of complex behaviors such as bifurcations, phase transitions, spatio-temporal pattern formation, self-sustained oscillations, and deterministic chaos. Semiconductors are *dissipative* dynamic systems, i.e. a steady state can be maintained only by a continuous flux of energy, and possibly matter, through them. Mathematically, this is described by the feature that volume elements in a suitable space of dynamic variables – the *phase space* – shrink with increasing time. In the language of thermodynamics, this represents an *open* system that is driven by external fluxes and forces so far from thermodynamic equilibrium that linear dynamic laws no longer hold. Owing to the driving forces and the inherent nonlinearities of these systems, they may spontaneously evolve into a state of highly ordered spatial or temporal structures, so-called *dissipative structures*. Unlike an isolated, closed system, which after a perturbation always returns to a thermodynamic equilibrium state characterized by maximum entropy, an open dissipative nonlinear system may exhibit a process of *self-organization*, in which the entropy is locally decreased. Such processes usually involve qualitative changes in the state of the system, similar to phase transitions. Nonequilibrium phase transitions and dissipative structures have been noted in a great number of very different dissipative systems occurring in physics, chemistry, biology, ecology

(Glansdorff and Prigogine 1971, Nicolis and Prigogine 1977, Haken 1983, Bergé *et al.* 1987, Feistel and Ebeling 1989, Manneville 1990, Murray 1993, Cross and Hohenberg 1993, Mikhailov 1994, Walgraef 1997, Mori and Kuramoto 1998, Busse and Müller 1998), and even economics and social sciences (Weidlich and Haag 1984, Mantegna and Stanley 1999, Moss de Oliveira *et al.* 1999), but the phenomena observed are similar. Famous examples are the laser (Graham and Haken 1968), the Bénard and Taylor instabilities in hydrodynamics (Swinney and Gollub 1984), and chemical reaction systems (Kuramoto 1988). In the field of semiconductor physics, nonlinear generation–recombination processes (Schöll 1987) and nonlinear optical effects (Haug 1988) may give rise to nonequilibrium phase transitions. In the first case they manifest themselves as electric instabilities such as current runaway, threshold switching between a nonconducting and a conducting state, spontaneous oscillations of the current or voltage, and nucleation and growth of current filaments or high-field domains, if sufficiently high electric or magnetic fields, injected currents, or optical or microwave irradiations are applied. The study of such nonlinear effects in semiconductors is now established as a mature part of the interdisciplinary field of *synergetics* which was pioneered by Haken (1983, 1987).

Though these matters have become an active field of semiconductor research only recently, there was some singular early work, for example on the bifurcation of current filaments in connection with dielectric breakdown of solids (Lueder *et al.* 1936), and phase-portrait analysis of field domains in CdS crystals (Böer and Quinn 1966). The analogy of an overheating instability of the electron gas with an equilibrium phase transition was pointed out by Volkov and Kogan (1969), and Pytte and Thomas (1969) drew this analogy in the case of the Gunn instability of the electron drift velocity at about the same time. The early theory of domain instabilities in semiconductors was reviewed by Bonch-Bruevich *et al.* (1975). Generation–recombination-induced phase transitions in semiconductors were first noted by Landsberg and Pimpale (1976), stimulated by the similarity with Schlögl's chemical reaction models for nonequilibrium phase transitions (Schlögl 1972). Impact ionization of electrons or holes by hot carriers across the bandgap or from localized levels was recognized as the main autocatalytic process which is necessary for phase transitions, and low-temperature impurity breakdown was studied as a prominent example (Landsberg *et al.* 1978, Schöll and Landsberg 1979, Robbins *et al.* 1981). Current filamentation was treated as a process of self-organization in a system far from equilibrium (Schöll 1982*b*). The interest in this field was greatly increased by the experimental discovery of deterministic chaos in semiconductors (Aoki *et al.* 1981, Teitsworth *et al.* 1983). The introduction of concepts and methods from nonlinear dynamics subsequently stimulated a large amount of experimental and theoretical work on chaos and spatio-temporal self-organized pattern formation in a variety of semiconducting materials, e.g. p-Ge and n-GaAs in the regime of low-temperature impurity breakdown, and in layered semiconductor structures such as p–i–n and p–n–p–n diodes, double-barrier resonant-tunneling structures, heterostructure hot-electron diodes, and semiconductor superlattices (Schöll 1987, 1998*b*, Shaw *et al.* 1992, Thomas 1992, Peinke *et al.* 1992, Kerner and Osipov 1994, Niedernostheide 1995, Aoki 2000).

Progress has recently been made by using elaborate experimental techniques such as scanning electron microscopy (Mayer *et al.* 1988, Wierschem *et al.* 1995), scanning laser microscopy (Brandl *et al.* 1989, Spangler *et al.* 1994, Kukuk *et al.* 1996), potential probe measurements (Baumann *et al.* 1987, Niedernostheide *et al.* 1992*a*), and quenched photoluminescence (Eberle *et al.* 1996, Belkov *et al.* 1999) to detect spatially and temporally resolved structures. Detailed computer simulations in one and two spatial dimensions have allowed a quantitative comparison between theory and experiment. Whereas in earlier work irregular and chaotic *temporal* behavior was the center of interest (Abe 1989, Schöll 1992), the focus has now shifted toward more complex *spatio-temporal* dynamics including the dynamics of solitary filaments and multifilamentary states, spatio-temporal chaos, higher bifurcations of the elementary dissipative structures, interaction of defects and structural imperfections with pattern formation, and complex two-dimensional sample and contact geometries.

1.1.2 Negative differential conductivity

The electric transport properties of a semiconductor show up most directly in its current–voltage characteristic under time-independent bias conditions (dc, direct current). It is determined in a complex way by the microscopic properties of the material, which specify the current density j as a function of the local electric field $\mathcal{E}$, and by the contacts. A local, static, scalar $j(\mathcal{E})$ relation need not always exist, but in fact does in many cases.

Close to thermodynamic equilibrium, i.e. at sufficiently low bias voltage, the $j(\mathcal{E})$ relation is linear (*Ohm's Law*), but under practical operating conditions it will generally become nonlinear and may even display a regime of *negative differential conductivity*

$$\sigma_{\text{diff}} = \frac{dj}{d\mathcal{E}} < 0. \tag{1.1}$$

Thus the current density decreases with increasing field, and vice versa, which in general corresponds to an unstable situation. The actual electric response depends, for instance, upon the contact conditions and the attached circuit, which in general contains – even in the absence of external load resistors – unavoidable resistive and reactive components such as lead resistances, lead inductances, package inductances, and package capacitances.

Two important cases of negative differential conductivity (NDC) are described by an N-shaped or an S-shaped $j(\mathcal{E})$ characteristic, and denoted by NNDC and SNDC, respectively (Fig. 1.1). However, more complicated forms such as Z-shaped, loop-shaped, or disconnected characteristics are also possible (Wacker and Schöll 1995).

NNDC and SNDC are associated with voltage- and current-controlled instabilities, respectively. In the NNDC case the current density is a single-valued function of the field, but the field is multivalued: The $\mathcal{E}(j)$ relation has three branches in a certain range of j. The SNDC case is complementary in the sense that $\mathcal{E}$ and j are interchanged. This duality is in fact far-reaching, and will be elaborated upon subsequently.

The *global* current–voltage characteristic $I(U)$ of a semiconductor can in principle be calculated from the *local* $j(\mathcal{E})$ relation by integrating the current density j over the cross-section A of the current flow

$$I = \int_A j\,df \tag{1.2}$$

and the electric field $\mathcal{E}$ over the length L of the sample

$$U = \int_0^L \mathcal{E}\,dz. \tag{1.3}$$

Unlike the $j(\mathcal{E})$ relation, the $I(U)$ characteristic is not only a property of the semiconductor material, but also depends on the geometry, the boundary conditions, and the contacts of the sample. Only for the idealized case of spatially homogeneous states are the $j(\mathcal{E})$ and the $I(U)$ characteristics identical, up to re-scaling. The $I(U)$ relation is said to display *negative differential conductance* if

$$\frac{dI}{dU} < 0. \tag{1.4}$$

In case of NNDC, the NDC branch is often but not always – depending upon external circuit and boundary conditions – unstable against the formation of electric field domains, whereas in the SNDC case current filamentation generally occurs (Ridley 1963), as we shall discuss in detail in Chapter 3. These primary self-organized spatial patterns may themselves become unstable in secondary bifurcation, leading to periodically or chaotically breathing, rocking, moving, or spiking filaments or domains, or even solid-state turbulence and spatio-temporal chaos.

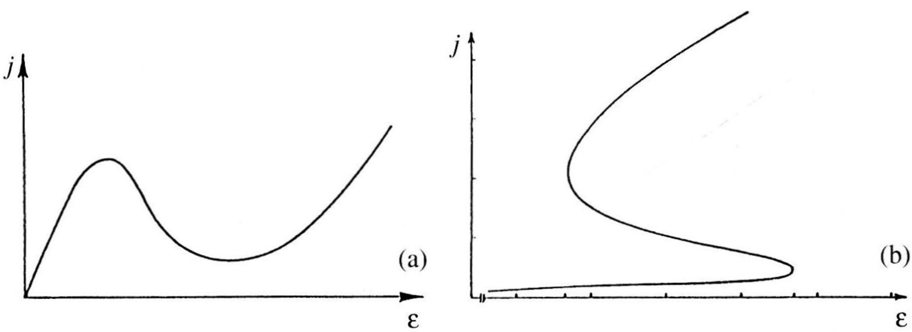

Figure 1.1. The current density j versus the electric field $\mathcal{E}$ for two types of negative differential conductivity (NDC): (a) NNDC and (b) SNDC (schematic).