Solving Problems in

FLUID MECHANICS
Volume 2

J F Douglas

Solving Problems in Fluid Mechanics
Volume 2

J. F. Douglas MSc PhD DIC ACGI CEng MICE MIMechE MIStructE

Longman Scientific & Technical

Longman Scientific & Technical
Longman Group UK Limited
Longman House, Burnt Mill, Harlow
Essex CM20 2JE, England
and Associated Companies throughout the world

First published by Pitman Publishing Limited in 1970 under the title
Solution of Problems in Fluid Mechanics Part 2
All metric edition first published in 1975
Ninth impression 1984
This edition published by Longman Scientific & Technical in 1986 under the title
Solving Problems in Fluid Mechanics Volume 2

British Library Cataloguing in Publication Data
Douglas, J.F.
[Solution of problems in fluid mechanics]
Solving problems in fluid mechanics. – All-metric ed. – (Solving problems)
Vol. 2
1. Fluid mechanics
I. [Solution of problems in fluid mechanics]
II. Title
620.1'06 TA357

ISBN 0-582-28643-3

Produced by Longman Singapore Publishers (Pte) Ltd.
Printed in Singapore.

Solving Problems in Fluid Mechanics

Volume 2

Other titles in the Series

Solving Problems in Vibrations, *J.S. Anderson and M. Bratos-Anderson*

Solving Problems in Structures Volume 1, *P.C.L. Croxton and L.H. Martin*

Solving Problems in Fluid Mechanics Volume 1, *J.F. Douglas*

Solving Problems in Soil Mechanics, *B.H.C. Sutton*

Solving Problems in Solid Mechanics Volume 1, *S.A. Urry and P.J. Turner*

Solving Problems in Solid Mechanics Volume 2, *S.A. Urry and P.J. Turner*

Preface

The treatment adopted in this second volume is exactly the same as that employed so successfully in the first volume, the subject matter of each section being presented in the form of question and answer. The reader will find all the definitions and theory required, together with selected problems which are fully worked out, and plenty of exercise questions with numerical answers on which to practice and develop skill and understanding.

The material included in this volume covers more advanced work in Fluid Mechanics for engineering students in Universities, Polytechnics and Colleges of Higher Education. The fullness of the treatment has in some places had to be restricted owing to the limited space available. The reader seeking further information in any particular field will find it helpful to refer to "Fluid Mechanics" by Douglas, Gasiorek and Swaffield (Pitman 2nd. Edn 1985).

I would again like to express my appreciation of the assistance which I have received from my former colleagues in the teaching profession. I am particularly indebted to Dr. R.D. Matthews for his advice on the preparation of this new text and for the provision of examples and exercises with particular reference to Chapter 9.

I hope that my readers will not hesitate to let me know of any difficulties that they may experience with this text and I will be glad to receive any constructive criticism.

John Douglas — September 1985

Contents

1

Dimensional analysis

Dimensional analysis is a mathematical method which is of considerable value in problems which occur in fluid mechanics. As explained in Part One all quantities can be expressed in terms of certain primary quantities which in mechanics are Length (L), Mass (M) and Time (T). For example

$$\text{Force} = \text{mass} \times \text{acceleration}$$
$$= \text{mass} \times \text{length/time}^2$$

Thus the dimensions of Force will be MLT^{-2}.

In any equation representing a real physical event every term must contain the same powers of the primary quantities (L, M and T). In other words, like must be compared with like or else the equation is meaningless, although it may balance numerically.

This principle of homogeneity of dimensions can be used, (1) to check whether an equation has been correctly formed, (2) to establish the form of an equation relating a number of variables, and (3) to assist in the analysis of experimental results.

1.1 Checking equations

Show by dimensional analysis that the equation

$$p + \tfrac{1}{2}\rho v^2 + \rho g z = H$$

is a possible relationship between the pressure p, velocity v and height above datum z for frictionless flow along a streamline of a fluid of mass density ρ, and determine the dimensions of the constant H.

Solution. If the equation represents a physically possible relationship each term must have the same dimensions and therefore contain the same powers of the primary quantities L, M and T.

The procedure to be adopted is first to determine the dimensions of each of the variables in terms of L, M and T, and then to examine the dimensions of each term in the equation.

The dimensions of the variables are

$$\text{Pressure } p = \frac{\text{force}}{\text{area}} = \frac{\text{mass} \times \text{acceleration}}{\text{area}}$$
$$= ML^{-1}T^{-2}$$

$$\text{Mass density } \rho = \frac{\text{mass}}{\text{volume}} = ML^{-3}$$

$$\text{Velocity } v = \frac{\text{length}}{\text{time}} = LT^{-1}$$

$$\text{Gravitational acceleration } g = LT^{-2}$$

$$\text{Height above datum } z = L$$

The dimensions of each term on the left-hand side are

$$p = ML^{-1}T^{-2}, \quad \tfrac{1}{2}\rho v^2 = ML^{-3} \times L^2T^{-2} = ML^{-1}T^{-2}$$

$$\rho g z = ML^{-3} \times LT^{-2} \times L = ML^{-1}T^{-2}$$

Thus all terms have the same dimensions and the equation is physically possible if the constant H also has the dimensions $ML^{-1}T^{-2}$.

1.2 Velocity of a pressure wave

> The velocity of propagation a of a pressure wave through a liquid could be expected to depend upon the elasticity of the liquid represented by the bulk modulus K and its mass density ρ. Establish by dimensional analysis the form of a possible relationship.

Solution. Assume a simple exponential equation

$$a = CK^a\rho^b \tag{1}$$

where C is a numerical constant and a and b are unknown powers.

The dimensions of the variables are: velocity $a = LT^{-1}$, bulk modulus $K = ML^{-1}T^{-2}$, mass density $\rho = ML^{-3}$. If equation (1) is to be correct the powers a and b must be such that both sides of the equation contain the same powers of M, L and T. Rewrite equation (1) replacing each quantity by its dimensions, remembering that the constant C is a pure number.

$$LT^{-1} = M^aL^{-a}T^{-2a} \times M^bL^{-3b}$$

Equating powers of M, L and T,

$$0 = a + b$$
$$1 = -a - 3b$$
$$-1 = -2a$$

from which $\quad a = +\tfrac{1}{2} \quad \text{and} \quad b = -\tfrac{1}{2}$

Thus a possible equation is $a = C\sqrt{\dfrac{K}{\rho}}$.

Compare this result with **10.5**.

Dimensional analysis gives the form of a possible equation but the value of the constant C would have to be determined experimentally.

1.3 Pipe flow

Show that a rational formula for the loss of pressure when a fluid flows through geometrically similar pipes is

$$p = \frac{\rho l v^2}{d}\,\phi\left(\frac{v d \rho}{\mu}\right)$$

where d is the diameter of the pipe, l is the length of the pipe, ρ is the mass density and μ the dynamic viscosity of the fluid, v is the mean velocity of flow through the pipe and ϕ means "a function of".

Solution. Assume $p = C\rho^a l^b v^c d^e \mu^f$ where C is a numerical constant and a, b, c, e, f are unknown powers.

The dimensions of the quantities are: $p = ML^{-1}T^{-2}$, $\rho = ML^{-3}$, $l = L$, $v = LT^{-1}$, $d = L$ and $\mu = ML^{-1}T^{-1}$.

Substituting these dimensions for the quantities,

$$ML^{-1}T^{-2} = M^a L^{-3a} \times L^b \times L^c T^{-c} \times L^e \times M^f L^{-f} T^{-f}$$

Equating powers of M, L and T,

$$1 = a + f \qquad (1)$$

$$-1 = -3a + b + c + e - f \qquad (2)$$

$$-2 = -c - f \qquad (3)$$

There are five unknown powers and only three equations, so that it must be decided to solve for three of the unknown powers in terms of the others. In practice this decision is made from experience; in examination problems some indication is usually given in the question as to the form of the final result which depends on the choice of unknowns to be solved. In this case solve for the powers of ρ, v and d, namely a, c and e.

From equation (1) $a = 1 - f$
From equation (3) $c = 2 - f$
From equation (2) $e = -1 + 3a - c - b + f$
$= -f - b$

Substituting these values in the original equation

$$p = C\rho^{1-f} l^b v^{2-f} d^{-f-b} \mu^f$$

$$= C\rho v^2 \left(\frac{l}{d}\right)^b \left(\frac{\rho v d}{\mu}\right)^{-f}$$

$$= \frac{\rho v^2 l}{d}\, C \left(\frac{l}{d}\right)^{b-1} \left(\frac{\rho v d}{\mu}\right)^{-f}$$

For geometrically similar pipes $\frac{l}{d}$ is a constant and $\left(\frac{l}{d}\right)^{b-1}$ can be combined with C. Putting $K = C\left(\frac{l}{d}\right)^{b-1}$

$$p = \frac{\rho l v^2}{d} K \left(\frac{\rho v d}{\mu}\right)^{-f}$$

Since neither K nor f are known this is written simply as

$$p = \frac{\rho l v^2}{d} \phi \left(\frac{\rho v d}{\mu}\right) \tag{4}$$

It is interesting to compare this result with the Darcy formula

$$h_f = 4f \frac{l}{d} \frac{v^2}{2g}$$

From equation (4) $$h_f = \frac{p}{\rho g} = \frac{l v^2}{d g} \phi \left(\frac{\rho v d}{\mu}\right)$$

which indicates that the Darcy coefficient f must be a function of the pipe Reynolds number $\rho v d/\mu$. This has already been shown by more orthodox methods (*see* Volume 1).

1.4 Pipe flow

A rational formula for loss of pressure when fluid flows through geometrically similar pipes is

$$p = \frac{\rho l v^2}{d} \phi \left(\frac{\rho v d}{\mu}\right)$$

The measured loss of head in a 50 mm diam pipe conveying water at 0·6 m/s is 800 mm of water per 100 m length. Calculate the loss of head in millimetres of water per 400 m length when air flows through a 200 mm diam pipe at the "corresponding speed". Assume that the pipes have geometrically similar roughness and take the densities of air and water as 1·23 and 1000 kg/m³ and the absolute viscosities as $1{\cdot}8 \times 10^{-4}$ and $1{\cdot}2 \times 10^{-2}$ poise respectively.

Solution. The formula $p = \frac{\rho l v^2}{d} \phi \left(\frac{\rho v d}{\mu}\right)$, derived by dimensional analysis in example **1.3**, might appear to be of little use since the nature of the function $\phi(\rho v d/\mu)$ is unknown, but it can be used for comparison of the pressure drops in two geometrically similar pipes provided that the value of the Reynolds number $\rho v d/\mu$ is the same in both cases. Then

$$\phi \left(\frac{\rho_1 v_1 d_1}{\mu_1}\right) = \phi \left(\frac{\rho_2 v_2 d_2}{\mu_2}\right)$$

and the ratio of pressure drops simplifies to

$$\frac{p_1}{p_2} = \frac{\rho_1}{\rho_2} \cdot \frac{l_1}{l_2} \cdot \frac{v_1^2}{v_2^2} \cdot \frac{d_2}{d_1}$$

The velocity of flow in the second pipe required to make the Reynolds number the same in both is known as the *corresponding speed*. Using the suffix w for the pipe containing water and a for that containing air, for equality of Reynolds numbers,

$$\frac{\rho_w v_w d_w}{\mu_w} = \frac{\rho_a v_a d_a}{\mu_a}$$

Corresponding speed for air $= v_a = v_w \dfrac{\rho_w}{\rho_a} \dfrac{d_w}{d_a} \dfrac{\mu_a}{\mu_w}$

$$= 0{\cdot}6 \frac{1000}{1{\cdot}23} \times \frac{50}{200} \times \frac{1{\cdot}8 \times 10^{-4}}{1{\cdot}2 \times 10^{-2}} = 1{\cdot}83\,\text{m/s}$$

Ratio of pressure drops $\dfrac{p_a}{p_w} = \dfrac{\rho_a}{\rho_w} \dfrac{l_a}{l_w} \dfrac{d_w}{d_a} \dfrac{v_a^2}{v_w^2}$

$$= \frac{1{\cdot}23}{1000} \times \frac{400}{100} \times \frac{50}{200} \times \frac{1{\cdot}83^2}{0{\cdot}6^2} = 0{\cdot}01144$$

If loss of head per 100 m in 50 mm pipe is 800 mm of water

Loss of head per 400 m in 200 mm pipe $= 0{\cdot}01144 \times 800$

$= $ **9·15 mm of water**

1.5 Resistance to a partially-submerged body

Find by dimensional analysis a rational formula for the resistance to motion R of geometrically similar bodies moving partially submerged through a viscous, compressible fluid of density ρ and coefficient of dynamic viscosity μ with a uniform velocity V.

Solution. The resistance R will be due to skin friction, wave resistance and compressibility of the fluid and will depend on the size of the body denoted by a characteristic length l, the velocity V, the density ρ, viscosity μ and bulk modulus K of the fluid and the gravitational acceleration g (for wave resistance). Thus R is a function of l, V, ρ, μ, K and g. The form of this function may be simple as was assumed in example **1.4** or may consist of a series of terms made up of the product of the variables each raised to suitable powers

$$R = Al^x V^y \rho^z \mu^p K^q g^r + A_1 l^{x1} V^{y1} \rho^{z1} \mu^{p1} K^{q1} g^{r1} + \ldots \tag{1}$$

where A, A_1, . . . are numerical constants, x, x_1, . . ., y, y_1, . . . etc. are unknown indices. Thus

$$\frac{R}{Al^x V^y \rho^z \mu^p K^q g^r} = 1 + \frac{A_1}{A} l^{x1-x} V^{y1-y} \rho^{z1-z} \mu^{p1-p} K^{q1-q} g^{r1-r}$$

Since the first term on the right-hand side is a pure number, the equation will only be correct if dimensionally

$$R = Al^x V^y \rho^z \mu^p K^q g^r$$

The dimensions of the quantities are: $R = MLT^{-2}$, $l = L$, $V = LT^{-1}$, $\rho = ML^{-3}$, $\mu = ML^{-1}T^{-1}$, $K = ML^{-1}T^{-2}$, $g = LT^{-2}$. Substituting in equation (1)

$$MLT^{-2} = L^x \times L^y T^{-y} \times M^z L^{-3z} \times M^p L^{-p} T^{-p} \times M^q L^{-q} T^{-2q} \times L^r T^{-2r}$$

Equating powers of M, L and T

$$1 = z + p + q \tag{2}$$

$$1 = x + y - 3z - p - q + r \tag{3}$$

$$-2 = -y - p - 2q - 2r \tag{4}$$

Equations (2), (3) and (4) allow of three solutions only. A useful result is obtained by solving for x, y and z giving

$$z = 1 - p - q, \quad y = 2 - p - 2q - 2r, \quad x = 2 - p + r.$$

All the other terms on the right-hand side of equation (1) are similar to the first so that by the same dimensional reasoning

$$x_1 = 2 - p_1 + r_1, \quad y_1 = 2 - p_1 - 2q_1 - 2r_1, \quad z_1 = 1 - p_1 - q_1$$

and so on. Substituting in equation (1)

$$R = \rho V^2 l^2 \left\{ A \left(\frac{\rho V l}{\mu}\right)^{-p} \left(\frac{V}{\sqrt{(K/\rho)}}\right)^{-2q} \left(\frac{V}{\sqrt{(lg)}}\right)^{-2r} + A_1 \left(\frac{\rho V l}{\mu}\right)^{-p_1} \left(\frac{V}{\sqrt{(K/\rho)}}\right)^{-2q_1} \left(\frac{V}{\sqrt{(lg)}}\right)^{-2r_1} + \ldots \right\}$$

The series in brackets is an unknown function of $\dfrac{\rho V l}{\mu}$, $\dfrac{V}{\sqrt{(K/\rho)}}$ and $\dfrac{V}{\sqrt{(lg)}}$ and can be written

$$R = \rho V^2 l^2 \phi \left\{ \frac{\rho V l}{\mu}, \frac{V}{\sqrt{(K/\rho)}}, \frac{V}{\sqrt{(lg)}} \right\} \tag{5}$$

The terms in the function are all dimensionless groups,

$\dfrac{\rho V l}{\mu}$ is the Reynolds number,

$\dfrac{V}{\sqrt{(K/\rho)}}$ is the Mach number and

$\dfrac{V}{\sqrt{(lg)}}$ is the Froude number.

Equation (5) may also be written

$$\frac{R}{\rho V^2 l^2} = \phi \left\{ \frac{\rho V l}{\mu}, \frac{V}{\sqrt{(K/\rho)}}, \frac{V}{\sqrt{(lg)}} \right\}$$

in which case $R/\rho V^2 l^2$ will also be found to be dimensionless.

1.6 Thrust of screw propeller

Assuming that the thrust F of a screw propeller is dependent upon the diameter d, speed of advance v, fluid density ρ, revolutions per second n and coefficient of viscosity μ, show that it can be expressed by the equation

$$F = \rho d^2 v^2 \phi \left\{ \frac{\mu}{\rho d v}, \frac{dn}{v} \right\}$$

Solution. F will be a function of d, v, ρ, n and μ. Instead of expanding this function fully as in example **1.5**, since all the terms are similar we can write

$$F = \Sigma A d^m v^p \rho^q n^r \mu^s \tag{1}$$

where A is a numerical constant and m, p, q, r and s are unknown powers.

The dimensions of the variables are $F = MLT^{-2}$, $d = L$, $v = LT^{-1}$, $\rho = ML^{-3}$, $n = T^{-1}$, $\eta = ML^{-1}T^{-1}$.

Substituting the dimensions for the variables, equation (1) will be true if

$$MLT^{-2} = L^m \times L^pT^{-p} \times M^qL^{-3q} \times T^{-r} \times M^sL^{-s}T^{-s}$$

Equating powers of M $\quad 1 = q + s$

$L \quad 1 = m + p - 3q - s$

$T \quad -2 = -p - r - s$

The equation given in the problem indicates that it is desirable to solve for m, p and q in terms of r and s.

$$q = 1 - s, \quad p = 2 - r - s$$
$$m = 1 - p + 3q + s = 2 + r - s$$

Substituting in equation (1)

$$F = \Sigma A d^{2+r-s} v^{2-r-s} \rho^{1-s} n^r \mu^s$$

Regrouping by powers

$$F = \Sigma A \rho d^2 v^2 \left(\frac{\mu}{\rho d v}\right)^s \left(\frac{dn}{v}\right)^r$$

which can be written

$$F = \rho d^2 v^2 \phi \left\{\frac{\mu}{\rho d v}, \frac{dn}{v}\right\}$$

where ϕ means "a function of".

1.7 Buckingham's Pi theorem

State Buckingham's Π theorem and apply it to the problem of example **1.6**.

Solution. Buckingham's Π theorem states that if there are n variables in a problem and these variables contain m primary dimensions (for example M, L and T) the equation relating the variables will contain $n - m$ dimensionless groups. Buckingham referred to these dimensionless groups as Π_1, Π_2, etc., and the final equation obtained is

$$\Pi_1 = \phi(\Pi_2, \Pi_3, \ldots \Pi_{n-m})$$

Thus in example **1.5** there are seven variables with three primary dimensions so that the final equation

$$\frac{R}{\rho V^2 l^2} = \phi \left\{\frac{\rho V l}{\mu}, \frac{V}{\sqrt{(K/\rho)}}, \frac{V}{\sqrt{(lg)}}\right\}$$

is formed of four dimensionless groups.

In the problem of example **1.6** there are six variables, F, ρ, d, v, η and n and three primary dimensions. The equation relating the variables will therefore be formed of $6 - 3 = 3$ dimensionless groups and will be

$$\Pi_1 = \phi(\Pi_2, \Pi_3)$$

The dimensionless groups can be formed as follows:

(1) Choose a number of variables equal to the number of primary dimensions and including all these dimensions, in this case F, ρ and v.

(2) Form dimensionless groups by combining the variables selected in (1) with each of the others in turn.

Combining F, ρ and v with d to form a dimensionless group:

$$\Pi_1 = \frac{F}{\rho v^2 d^2}$$

Combining F, ρ and v with n, $\quad \Pi_2 = \dfrac{Fn^2}{\rho v^4}$

Combining F, ρ and v with μ, $\quad \Pi_3 = \dfrac{F\rho}{\mu^2}$

Since $\quad \Pi_1 = \phi(\Pi_2, \Pi_3)$,

$$\frac{F}{\rho v^2 d^2} = \phi\left(\frac{Fn^2}{\rho v^4}, \frac{F\rho}{\mu^2}\right) \tag{1}$$

The groups can be rearranged to obtain the desired form by cross-multiplying. Rewrite equation (1) as

$$\frac{F}{\rho v^2 d^2} = \left(\frac{Fn^2}{\rho v^4}\right)^a \left(\frac{F\rho}{\mu^2}\right)^b \times \text{constant}$$

Multiplying both sides by $\left(\dfrac{F}{\rho v^2 d^2}\right)^{-a-b}$ gives

$$\left(\frac{F}{\rho v^2 d^2}\right)^{1-a-b} = \left(\frac{Fn^2}{\rho v^4} \cdot \frac{\rho v^2 d^2}{F}\right)^a \left(\frac{F\rho}{\mu^2} \cdot \frac{\rho v^2 d^2}{F}\right)^b \times \text{constant}$$

$$\left(\frac{F}{\rho v^2 d^2}\right)^{1-a-b} = \left(\frac{dn}{v}\right)^{2a} \left(\frac{\mu}{\rho v d}\right)^{-2b} \times \text{constant}$$

$$\frac{F}{\rho v^2 d^2} = \phi\left\{\frac{dn}{v}, \frac{\mu}{\rho v d}\right\}$$

Problems

1 Show that the frictional torque L required to rotate a disc of diameter d at an angular velocity ω in a fluid of viscosity μ and density ρ is given by

$$\frac{L}{d^5 \omega^2 \rho} = \phi\left\{\frac{\rho d^2 \omega}{\mu}\right\}$$

2 The rate of flow Q of a gas through a sharp-edged orifice depends on the diameter d of the orifice, the difference in pressure P between the two sides of the orifice, the density ρ and the kinematic viscosity v of the gas. Show by the method of dimensions that

$$Q = d^2 \left(\sqrt{\frac{P}{\rho}}\right) \phi \left\{\frac{v}{d}\sqrt{\frac{\rho}{P}}\right\}$$

3 Show that the power P developed by a hydraulic turbine is given by

$$P = \rho N^3 D^5 \phi \left\{\frac{N^2 D^2}{gH}\right\}$$

where ρ is the mass density of the fluid, N the speed of rotation, D the diameter of the rotor and H the available head.

4 Define viscosity and state the units in which it is measured. Show by applying the principle of dimensional homogeneity that the resistance to motion of a sphere through a viscous fluid is given by $R = K\mu dv$ where μ is the viscosity; d the diameter of the sphere; v the velocity; and K a numerical coefficient.

5 Assuming that for turbulent flow in a rough pipe the resistance per unit area of solid boundary τ is dependent upon the viscosity μ, the density ρ, the velocity V of the fluid, the diameter D of the pipe and the size of roughness k, show that

$$\frac{\tau}{\rho V^2} = \phi \left\{\frac{VD\rho}{\mu}, \frac{k}{D}\right\}$$

6 In the rotation of similar discs in a fluid in which the motion of the fluid is turbulent, show by the method of dimensions that a rational formula for the frictional torque M of a disc of diameter D rotating at speed N in a fluid of viscosity μ and density ρ is

$$M = D^5 N^2 \rho \phi \left(\frac{\mu}{D^2 N \rho}\right)$$

7 Derive a general expression for the resistance to motion of a partially submerged body through a liquid in terms of the Froude and Reynolds numbers. How is this expression used to compare the resistance of a ship model with the full-size ship? Explain the assumptions usually made and quote experiments which justify them.

Describe in detail either (a) the production of a ship model; or (b) a method of measuring the resistance force when the model is towed through still water.

8 Describe, with the help of diagrams, the operation of a film lubricated journal bearing.

Specify the conditions necessary for strict geometrical similarity and show, by using the method of dimensions, that if temperature effects are neglected, the moment of frictional resistance for geometrically similar journal bearings can be expressed by

$$M = \mu ND^3\phi\left(\frac{\mu N}{P}\right)$$

where μ is the viscosity of the lubricant, N is the speed of journal rotation, P is the load per unit projected area and D is the diameter of the journal.

Hence show that the moment of frictional resistance for all geometrically similar bearings running at "corresponding speeds" is proportional to PD^3.

9 Prove that the viscous resistance F of a sphere of diameter d, moving at constant speed v through a fluid of density ρ and viscosity μ, may be expressed as

$$F = \frac{\mu^2}{\rho}\phi\left(\frac{\rho vd}{\mu}\right)$$

Show that Stokes' result for low velocities, $F = 3\pi\mu vd$, is in agreement with this general formula.

A sample of emery powder was shaken in water contained in a glass beaker and then allowed to settle. It was found that the water cleared in 1 min 40 s when the depth was 18 cm. Calculate the minimum diameter of the particles, assuming them all spherical and taking the specific gravity of emery as 4·0 and the coefficient of viscosity of water as 0·012 poise.
Answer 0·00364 cm

10 Prove that the total resistance R to flow in a length l of a pipe of diameter d is given by

$$R = ldv^2\rho\phi\left(\frac{\mu}{\rho vd}\right)$$

where ρ is the density of the fluid and v its mean velocity. From the above equation show that the loss of head h in a length l of a pipe can be expressed as klv^n/d^{3-n}. Show that this can be applied to viscous flow if $n = 1$ and $k = 32\,\mu/pg$.

11 The discharge through a small orifice is dependent upon the head over the orifice H, the gravitational acceleration g, the diameter of the orifice D, the viscosity μ, the density ρ, the surface tension σ of the fluid and the roughness k. Find the dimensionless groups upon which the coefficient of discharge depends.

Answer $\dfrac{\rho D\sqrt{(gH)}}{\mu}, \dfrac{D}{H}, \dfrac{\sigma}{\rho g H^2}, \dfrac{k}{H}$

12 Show, by applying the method of dimensions, that a rational formula for the resistance of geometrically similar bodies moving, partially submerged, with uniform velocity V in a fluid having density ρ and viscosity μ is $R = \rho L^2V^2.\ \phi(N,F)$ where N denotes the Reynolds number and F denotes the Froude number.

State the particular forms of resistance associated with each of