

TP 13
M 178

8862153

Computer-Aided Design of Feedback Control Systems for Time Response

Clarence J Maday



E8862153

First published 1987 by The
Instrument Society of America,
67 Alexander Drive, P.O. Box 12277,
Research Triangle Park, NC 27709, USA.

Published in Great Britain by Kogan Page,
120 Pentonville Road, London N1 9JN

© The Instrument Society of America 1987
All rights reserved.

British Library Cataloguing in Publication Data

Maday, Clarence J.

Computer-aided design of feedback control
systems for time response.

1. Automatic control — Data processing

I. Title

629.8'312

TJ213

ISBN 1 85091 421 4

Printed in Great Britain by
Billing & Sons Ltd., Worcester

Computer-Aided Design of
Feedback Control
Systems for
Time Response

Contents



List of Figures	vii
List of Tables	xiii
Preface	xv
Chapter 1 Introduction	1
Previous Work	5
Chapter 2 The Integral Error with State Feedback Method	9
Reference	14
Chapter 3 Saturation Operation with Linear Feedback	15
Objectives	16
Automatic Control	17
Optimal Linear Feedback Controller	18
Problem Statement	19
Solution Method	19
Discussion and Conclusion	41
Reference	41

(v)

Chapter 4 Interaction and Noninteraction in Multi-Input, Multi-Output Systems	43
Introduction	43
Decoupling Filters with Proportional-Integral-Derivative Control	45
Integral Error Plus State Feedback Control	48
Comparison Procedure	49
Results	51
Proportional-Integral-Derivative Control with Ziegler-Nichols Gains	52
Proportional-Integral-Derivative Control for Minimum Integrated Absolute Value of the Error	55
Integral Error Plus State Feedback Control	55
Discussion	58
References	58
Chapter 5 Pole Assignment in Hybrid Analog/Discrete Systems	61
Introduction	61
Objective	62
State of the Art	62
Problem Statement	63
Solution Technique	66
Simulation Results	72
Conclusion	96
Chapter 6 The Reduced-Order Error Observer	97
Objective	98
Error Observer	99
Extension to Higher-Order Systems	102
References	107
Appendix	109
CONTROL	109
CAPSTAN	114
DIG	116

List of Figures

2-1	Single-input/single-output first-order analog plant with state feedback only.	11
2-2	Single-input/single-output first-order analog plant with state feedback and one stage of integral compensation.	11
2-3	Single-input/single-output second-order analog plant with state feedback only.	11
2-4	Single-input/single-output second-order analog plant with state feedback and one stage of integral compensation.	11
2-5	Single-input/single-output second-order analog plant with state feedback and two stages of integral compensation.	13
2-6	Plant with ordinary PID controller.	14
3-1	Simple dc servo motor with feedback control.	20
3-2	Block diagram of dc servo motor.	21
3-3	Block diagram of dc servo motor with error compensation, cascaded integrators, and state feedback.	23
3-4	Block diagram of dc servo motor with one stage of integral compensation and state feedback.	25
3-5	Response of dc servo motor to unit step input — no constraints.	27
3-6	Schematic of logic sequence for limited integrator.	28
3-7	Response of constrained servo motor to unit step input; specified repeated poles at $s = -200$.	31
3-8	Response of constrained servo motor to unit step input; specified repeated poles at $s = -300$.	31
3-9	Response of constrained servo motor to unit step input; specified repeated poles at $s = -350$.	32

3-10	Response of constrained servo motor to unit step input of six radians; specified repeated poles at $s = -250$.	33
3-11	Tape drive motor system.	35
3-12	Block diagram of tape drive motor system with cascaded error compensation and state feedback.	37
3-13	Response of tape drive system to step input of 100 rpm; initial speed is 100 rpm.	38
3-14	Response of tape drive system for $J_L = 4.18 \text{ oz-in.-sec}^2$; feedback gains same as for Figure 3-12.	39
3-15	Response of tape drive system for $J_L = 12.54 \text{ oz-in.-sec}^2$; feedback gains same as for Figure 3-12.	40
3-16	Response of tape drive system to step disturbance, $T_D = 811 \text{ oz-in.}$	40
4-1	Two-input/two-output plant and controller with coupling	45
4-2	Single-input/single-output plant with PID. (Proportional-integral-derivative) control.	47
4-3	Single input/single output plant with IESF (Integral error plus state feedback) control.	48
4-4	Three-pole plant with IESF (Integral error plus state feedback) control	50
4-5	Response of interacting system with PID (Proportional-integral-derivative) control Ziegler-Nichols gains. Unit step input at r_2 .	53
4-6	Response of decoupled system with PID (Proportional-integral-derivative) control Ziegler-Nichols gains. Unit step input at r_2 .	53
4-7	Response of interacting system with PID (Proportional-integral derivative) control Ziegler-Nichols gains. Plant time constants changed 25%. Unit step input at r_2 .	54
4-8	Response of decoupled system with PID (Proportional-integral derivative) control Ziegler-Nichols gains. Plant time constants changed 25%. Unit step input at r_2 .	54
4-9	Response of interacting system with PID control, minimum IAE gains. Unit step input at r_2 .	56
4-10	Response of decoupled system with PID control, minimum IAE gains. Unit step input at r_2 .	56
4-11	Response of interacting system with PID control, minimum IAE gain. Plant time constants changed 25%. Unit step input at r_2 .	57
4-12	Response of decoupled system with PID control, minimum IAE gains. Plant time constants changed 25%. Unit step input at r_2 .	57

4-13	Response of coupled system with IESF control. Unit step input at r_2 .	59
4-14	Response of coupled system with IESF control. Plant time constants changed 25%. Unit step input at r_2 .	59
5-1	Vector-matrix block diagram of continuous plant controlled by a discrete controller.	64
5-2	Block diagram of complete system—spring/mass plant controlled by a digital controller. Integral compensation is by Euler integration.	69
5-3	Response and actuator force for step input. $\omega_n = 1.0$, $\zeta = 0$, $T = 0.25$, repeated poles at $z = 0.5$. Gains calculated on the basis of continuous approximation.	74
5-4	Response and actuator force for step input. $\omega_n = 1.0$, $\zeta = 0$, $T = 0.25$, repeated poles at $z = 0.5$. Compare with continuous approximation in Figure 5-1.	74
5-5	Response and actuator force for step input. $\omega_n = 1.0$, $\zeta = 0$, $T = 0.1$, repeated poles at $z = 0$.	75
5-6	Response and actuator force for step input. $\omega_n = 1.0$, $\zeta = 0$, $T = 0.2$, repeated poles at $z = 0$.	75
5-7	Response and actuator force for step input. $\omega_n = 1.0$, $\zeta = 0$, $T = 0.3$, repeated poles at $z = 0$.	76
5-8	Response and actuator force for step input. $\omega_n = 1.0$, $\zeta = 0$, $T = 0.49$, repeated poles at $z = 0$.	76
5-9	Response and actuator force for step input. $\omega_n = 1.0$, $\zeta = 0$, $T = 0.5$ (corresponds to Nyquist frequency), repeated poles at $z = 0$.	77
5-10	Response and actuator force for step input. $\omega_n = 1.0$, $\zeta = 0.1$, $T = 0.5$ (corresponds to Nyquist frequency), repeated poles at $z = 0$.	77
5-11	Response and actuator force for step input. $\omega_n = 1.0$, $\zeta = 0$, $T = 0.51$, repeated poles at $z = 0$.	78
5-12	Response and actuator force for step input. $\omega_n = 1.0$, $\zeta = 0$, $T = 0.75$, repeated poles at $z = 0$.	78
5-13	Response and actuator force for step input. $\omega_n = 1.0$, $\zeta = 0.1$, $T = 1.0$, repeated poles at $z = 0$.	79
5-14	Response and actuator force for step input. $\omega_n = 1.0$, $\zeta = 0.707$, $T = 1.0$, repeated poles at $z = 0$.	79
5-15	Response and actuator force for step input. $\omega_n = 1.0$, $\zeta = 0$, $T = 0.1$, repeated poles at $z = 0.5$.	80
5-16	Response and actuator force for step input. $\omega_n = 1.0$, $\zeta = 0$, $T = 0.49$, repeated poles at $z = 0.5$.	80
5-17	Response and actuator force for unit ramp input. $\omega_n = 1.0$, $\zeta = 0$, $T = 0.01$, repeated poles at $z = 0$.	81
5-18	Response and actuator force for unit ramp input. $\omega_n = 1.0$, $\zeta = 0$, $T = 0.25$, repeated poles at $z = 0.5$.	81

5-19	Response and actuator force for unit ramp input. $\omega_n = 1.0, \zeta = 0, T = 0.4$, repeated poles at $z = 0$.	82
5-20	Response and actuator force for harmonic input, $\sin 0.2\pi t$. $\omega_n = 1.0, \zeta = 0, T = 0.1$, repeated poles at $z = 0$.	82
5-21	Response and actuator force for harmonic input, $\sin 0.2\pi t$. $\omega_n = 1.0, \zeta = 0, T = 0.25$, repeated poles at $z = 0$.	83
5-22	Response and actuator force for harmonic input, $\sin 20\pi t$. $\omega_n = 1.0, \zeta = 0, T = .005$, repeated poles at $z = 0$.	83
5-23	Response and actuator force for harmonic input, $\sin 20\pi t$. $\omega_n = 1.0, \zeta = 0, T = 0.01$, repeated poles at $z = 0$.	84
5-24	Response and actuator force for harmonic input, $\sin 20\pi t$. $\omega_n = 1.0, \zeta = 0, T = 0.02$, repeated poles at $z = 0$.	84
5-25	Response and actuator force for step disturbance of 39.48. $\omega_n = 1.0, \zeta = 0, T = 0.2$, repeated poles at $z = 0.5$.	85
5-26	Response and actuator force for step disturbance of 39.48. $\omega_n = 1.0, \zeta = 0, T = 0.25$, repeated poles at $z = 0.5$.	85
5-27	Response and actuator force for step disturbance of 39.48. $\omega_n = 1.0, \zeta = 0, T = 0.3$, repeated poles at $z = 0.5$.	86
5-28	Response and actuator force for step disturbance of 39.48. $\omega_n = 1.0, \zeta = 0, T = 0.2$, repeated poles at $z = 0$.	86
5-29	Response and actuator force for step disturbance of 39.48. $\omega_n = 1.0, \zeta = 0, T = 0.25$, repeated poles at $z = 0$.	87
5-30	Response and actuator force for step disturbance of 39.48. $\omega_n = 1.0, \zeta = 0, T = 0.3$, repeated poles at $z = 0$.	87
5-31	Response and actuator force for step disturbance of 39.48. $\omega_n = 1.0, \zeta = 0, T = 0.4$, repeated poles at $z = 0$.	88
5-32	Response and actuator force for harmonic disturbance force, $39.48 \sin \pi t$. $\omega_n = 1.0, \zeta = 0, T = 0.1$, repeated poles at $z = 0$. Input is zero.	88
5-33	Response and actuator force for harmonic disturbance force, $39.48 \sin 4\pi t$. $\omega_n = 1.0, \zeta = 0, T = 0.1$, repeated poles at $z = 0$. Input is zero.	89
5-34	Response and actuator force for harmonic disturbance force, $39.48 \sin 4\pi t$. $\omega_n = 1.0, \zeta = 0, T = 0.05$, repeated poles at $z = 0$. Input is zero.	89

5-35	Response and actuator force for harmonic disturbance force, $39.48 \sin 2\pi t$, $\omega_n = 1.0$, $\zeta = 0$, $T = 0.1$, repeated poles at $z = 0$. Input is zero.	90
5-36	Response and actuator force for step input. Model $\omega_n = 1.0$, actual $\omega_n = 1.15$, $\zeta = 0$, $T = 0.1$, repeated poles at $z = 0.5$.	90
5-37	Response and actuator force for step input. $\omega_n = 1.0$, model $\zeta = 0$, actual $\zeta = 0.5$, $T = 0.1$, repeated poles at $z = 0.5$.	91
5-38	Response and actuator force for step input. $\omega_n = 1.0$, $\zeta = 0$, model $T = 0.25$, actual $T = 0.3$, repeated poles at $z = 0$.	91
5-39	Response and actuator force for step input. $\omega_n = 1.0$, $\zeta = 0$, model $T = 0.1$, actual $T = 0.13$, repeated poles at $z = 0$.	92
5-40	Response and actuator force for step input. Model $\omega_n = 1.0$, actual $\omega_n = 1.15$, $\zeta = 0$, $T = 0.1$, repeated poles at $z = 0$.	92
5-41	Response and actuator force for step input. $\omega_n = 1.0$, model $\zeta = 0$, actual $\zeta = 0.5$, $T = 0.1$, repeated poles at $z = 0$.	93
5-42	Response and actuator force for step input. $\omega_n = 1.0$, $\zeta = 0$, model $T = 0.25$, actual $T = 0.3$, repeated poles at $z = 0.1$.	93
5-43	Response and actuator force for step input. $\omega_n = 1.0$, $\zeta = 0$, model $T = 0.25$, actual $T = 0.2$, repeated poles at $z = 0.1$.	94
5-44	Response and actuator force for step input. $\omega_n = 1.0$, $\zeta = 0$, model $T = 0.1$, actual $T = 0.13$, repeated poles at $z = 0.5$.	94
5-45	Response and actuator force for step input. $\omega_n = 1.0$, $\zeta = 0$, model $T = 0.25$, actual $T = 0.3$, repeated poles at $z = 0.5$.	95
5-46	Response and actuator force for step input. $\omega_n = 1.0$, $\zeta = 0$, model $T = 0.25$, actual $T = 0.2$, repeated poles at $z = 0.5$.	95
6-1	"Error observer" concept.	99
6-2	Linear quadratic servomechanism.	100
6-3	Digital equivalent of Figure 6-1.	101
6-4	Plant with error observer and one stage of integral compensation.	101
6-5	Sampled-data equivalent of Figure 6-4.	101
6-6	Vector/matrix block diagram for multivariable system.	103
6-7	First order pole with state feedback.	103
6-8	Second order plant with an error observer.	104

6-9	Three-pole plant and controller with final output feedback only.	105
6-10	Three-pole plant with controller—feedback from two states.	105
6-11	Three-pole plant with controller—feedback unavailable from second pole.	106
6-12	Cascaded control system.	106

List of Tables

4-1	Performance Criteria for PID and IESF Controllers	52
-----	---	----

Preface



The material in this “how-to” book builds upon a first course in classical control. Appropriate concepts from modern control are used. Explanations are based upon the author’s insight and intuition gained through his experience with dynamical systems. In particular, the author tries to show how error compensation networks are developed. In most classical control courses, this topic is usually scheduled near the end, which often means it is not covered. This problem is addressed directly here because the subject of error compensation unifies the presentations in the book.

Frequency domain analysis herein is limited to pole assignment. Time domain analysis through numerical simulation dominates the presentation. In linear systems, the solution to the eigenvalue problem gives only a glimpse of the system response. In large systems, we can determine stability and a bit about oscillatory behavior. The usual worth of a picture — 1000 words — certainly holds true here. In nonlinear systems, numerical simulation may be the only available tool. The first system in Chapter 3 is nonlinear and emphasizes this. This is also true for linear digital systems where the sampling frequency is only two to five times greater than the highest system frequency. Such cases are described in Chapter 5. Frequency domain analysis for

digital systems generally provides only little insight into the transient behavior of the system. The situation becomes virtually intractable when one tries this approach to evaluate the robustness of the system.

The material in Chapter 6 on the error observer has not been published or described before. Beyond its novelty, the error observer establishes a neat connection between classical control and modern control.

Chapter 3 is similar to "Synthesis and Design of Feedback Control Systems for Time Response I" presented at the American Control Conference in 1984 and printed in the Proceedings, pp 1332-1337. These proceedings are copyrighted by American Automatic Control Council and are distributed through IEEE Service Center, 445 Hoes Lane, Piscataway, NJ 08854.

Clarence J. Maday
Raleigh, NC

Introduction

Historical accounts trace the use of self-regulating or feedback control systems at least as early as the third century B.C. Given the ingenuity of man, we can conjecture that earlier uses may have gone unnoticed because the inventor neglected to document such efforts. The recorded history of automatic control, however, is fascinating, and the interested reader can refer to one of the available monographs for a detailed account. We note only that today we tend to think primarily in terms of classical control and modern control. The topic usually called classical control was developed in a period that extended from the late 1930's to the early 1950's. Pontryagin's *Maximum Principle* renewed interest in state-space methods, and the general availability of high speed digital computers contributed to the development of what is called modern control theory. It has been noted in the literature that the term *modern* may be somewhat inappropriate because state-space or state variable techniques date back to the work of Poincare, Gibbs and Lyapunov around the beginning of this century. Nonetheless, the application of these methods to the analysis of control systems has proven to be valuable, and there is every indication that new developments will continue. It is generally agreed that *modern control* includes the state variable methods just indicated, as well as optimal control and digital control.

Optimal control has given us the linear quadratic regulator (LQR), and its counterpart, the linear quadratic servomechanism, both of which use complete state feedback. Difficulties are encountered when one or more of the states are inaccessible, i.e., they cannot be measured in the physical setting. The concept of the observer, which synthesized the missing states, was put forward to overcome this problem.

Digital control or sampled-data systems experienced a surge of interest during the 1950's when much of the theory was developed. Interest in these systems is renewed at the time of this writing because of the availability of very high speed microprocessors. Moreover, digital systems offer the potential of handling numbers with precision and with accuracy.

The need for feedback in electromechanical systems, chemical process control, aerospace guidance systems, internal combustion engines, and precision machining is accepted today. Feedback control systems are widely used and may be taken for granted. In view of this, it seems appropriate to reexamine the purpose of a feedback controller and to question our expectations of such a system. In short, what do we expect of an automatic feedback control system? For single-input, single-output (SISO) systems we expect the feedback system to be better than the sum of its constituent components. That is, the closed-loop performance of a controller should be superior to its open-loop performance. We expect also that the system will track faithfully any reasonable input command. Moreover, it should continue to perform well with approximate models of physical systems, and it should be insensitive to external disturbances; that is, it should be robust.

Let us consider a seemingly simple example that highlights these requirements and also serves to introduce more sophisticated concepts. The temperature control of a house is the example. The control element is an on-off heating/cooling device controlled by a thermostat, which is also an on-off device, often with a thermal anticipatory provision. The model of the house is generally taken to be a first-order one whose elements are known only approximately. Outdoor temperature variations and opened doors and windows are external and usually unpredictable disturbances. Yet the system works well even if it does not lend itself to linear analysis. We keep in mind the on-off character of the controller as a desirable feature for a controller because this