CAMBRIDGE SERIES IN CHEMICAL ENGINEERING

Advanced Transport Phenomena

Fluid Mechanics and Convective Transport Processes

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ADVANCED TRANSPORT PHENOMENA

Advanced Transport Phenomena is ideal as a graduate textbook. It contains a detailed discussion of modern analytic methods for the solution of fluid mechanics and heat and mass transfer problems, focusing on approximations based on scaling and asymptotic methods, beginning with the derivation of basic equations and boundary conditions and concluding with linear stability theory. Also covered are unidirectional flows, lubrication and thin-film theory, creeping flows, boundary-layer theory, and convective heat and mass transport at high and low Reynolds numbers. The emphasis is on basic physics, scaling and nondimensionalization, and approximations that can be used to obtain solutions that are due either to geometric simplifications, or large or small values of dimensionless parameters. The author emphasizes setting up problems and extracting as much information as possible short of obtaining detailed solutions of differential equations. The book also focuses on the solutions of representative problems. This reflects the author's bias toward learning to think about the solution of transport problems.

L. Gary Leal is professor of chemical engineering at the University of California in Santa Barbara. He also holds positions in the Materials Department and in the Department of Mechanical Engineering. He has taught at UCSB since 1989. Before that, from 1970 to 1989 he taught in the chemical engineering department at Caltech. His current research interests are focused on fluid mechanics problems for complex fluids, as well as the dynamics of bubbles and drops in flow, coalescence, thin-film stability, and related problems in rheology. In 1987, he was elected to the National Academy of Engineering. His research and teaching have been recognized by a number of awards, including the Dreyfus Foundation Teacher-Scholar Award, a Guggenheim Fellowship, the Allan Colburn and Warren Walker Awards of the AIChE, the Bingham Medal of the Society of Rheology, and the Fluid Dynamics Prize of the American Physical Society. Since 1995, Professor Leal has been one of the two editors of the AIP journal *Physics of Fluids* and he has also served on the editorial boards of numerous journals and the Cambridge Series in Chemical Engineering.

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Preface

This book represents a major revision of my book *Laminar Flow and Convective Transport Processes* that was published in 1992 by Butterworth-Heinemann. As was the case with the previous book, it is about fluid mechanics and the convective transport of heat (or any passive scalar quantity) for simple Newtonian, incompressible fluids, treated from the point of view of classical continuum mechanics. It is intended for a graduate-level course that introduces students to fundamental aspects of fluid mechanics and convective transport processes (mainly heat transfer and some single solute mass transfer) in a context that is relevant to applications that are likely to arise in research or industrial applications. In view of the current emphasis on small-scale systems, biological problems, and materials, rather than large-scale classical industrial problems, the book is focused more on viscous phenomena, thin films, interfacial phenomena, and related topics than was true 14 years ago, though there is still significant coverage of high-Reynolds-number and high-Peclet-number boundary layers in the second half of the book. It also incorporates an entirely new chapter on linear stability theory for many of the problems of greatest interest to chemical engineers.

The material in this book is the basis of an introductory (two-term) graduate course on transport phenomena. It starts with a derivation of all of the necessary governing equations and boundary conditions in a context that is intended to focus on the underlying fundamental principles and the connections between this topic and other topics in continuum physics and thermodynamics. Some emphasis is also given to the limitations of both equations and boundary conditions (for example "non-Newtonian" behavior, the "no-slip" condition, surfactant and thermocapillary effects at interfaces, etc.). It should be noted, however, that though this course starts at the very beginning by deriving the basic equations from first principles, and thus can be taken successfully even without an undergraduate transport background, there are important topics from the undergraduate curriculum that are *not* included, especially macroscopic balances, friction factors, correlations for turbulent flow conditions, etc. The remainder of the book is concerned with how to solve transport and fluids problems analytically; but with a lot of emphasis on basic physics, scaling, nondimensionalization, and approximations that can be used to obtain solutions that are due either to geometric simplifications or large or small values of dimensionless parameters.

THE SCOPE OF THIS BOOK

No single book can encompass all topics, and the present book is no exception. We consider only laminar flows and transport processes involving laminar flows, for incompressible, Newtonian fluids. Specifically, we do not consider turbulent flows. We do not consider compressibility effects, nor do we consider numerical methods, except by means of a brief introduction to boundary integral techniques for creeping flows. Further, we do not consider non-Newtonian flows, except for a few limited homework examples, nor even the basic constitutive equations for non-Newtonian fluids except briefly in the introductory chapter, Chapter 2, primarily in the context of thinking about why fluids may exhibit non-Newtonian behavior and hence what the limitations of the Newtonian fluid approximation may be. We do consider both flow and convective transport processes, but with the latter generally posed as a heat transfer problem. We shall see, however, that much of the same analysis and principles apply to mass transfer when there is a single solute. Finally, multicomponent mass transfer is not considered, and in the graduate transport sequence of classes would often be taught as a separate class.

The goal of this book is to provide a fundamental understanding of the governing principles for flow and convective transport processes in Newtonian fluids, and some of the modern tools and methods for "analysis" of this class of problems. By "analysis," I mean both what one can achieve from a qualitative point of view without actually solving differential equations and boundary conditions, as well as detailed analytic solutions obtained generally from an asymptotic point of view. There is a strong emphasis on the derivation of basic equations and boundary conditions, including those relevant to a fluid interface. I also focus on complete descriptions of the solutions of representative problems rather than an exhaustive summary of all possible problems. This is because of the importance that I place on learning how to think about transport problems, and how to actually solve them, rather than just being told that some problem exists with a certain solution, but without adequate details to really understand how to achieve that solution or to generalize from the current problem to a related but presently unanticipated extension.

An important tool that we develop in this book is the use of characteristic scales, nondimensionalization, and asymptotic techniques, in the analysis and understanding of transport processes. At the most straightforward level, asymptotic methods provide a systematic framework to generate approximate solutions of the nonlinear differential equations of fluid mechanics, as well as the corresponding thermal energy (or species transport) equations. Perhaps more important than the detailed solutions enabled by these methods, however, is that they demand an extremely close interplay between the mathematics and the physics, and in this way contribute a very powerful understanding of the physical phenomena that characterize a particular problem or process. The presence of large or small dimensionless parameters in appropriately nondimensionalized equations or boundary conditions is indicative of the relative magnitudes of the various physical mechanisms in each case, and is thus a basis for approximation via retention of the dominant terms.

There is, in fact, an element of truth in the suggestion that asymptotic approximation methods are nothing more than a sophisticated version of dimensional analysis. Certainly it is true, as we shall see, that successful application of scaling/nondimensionalization can provide much of the information and insight about the nature of a given fluid mechanics or transport process without the need either to solve the governing differential equations or even be concerned with a detailed geometric description of the problem. The latter determines the magnitude of numerical coefficients in the correlations between dependent and independent dimensionless groups, but usually does not determine the form of the correlations. In this sense, asymptotic theory can reduce a whole class of problems, which differ only in the geometry of the boundaries and in the nature of the undisturbed flow, to the evaluation of a single coefficient. When the body or boundary geometry is simple, this can be done by means of detailed solutions of the governing equations and boundary conditions. Even when the geometry is too complex to obtain analytic solutions, however, the general asymptotic framework is unchanged, and the correlation between dimensionless groups is still reduced to determination of a single constant, which can now be done (in principle) by means of a single experimental measurement.

It is important, however, not to overstate what can be accomplished by asymptotic (and related analytic) techniques applied either to fluid mechanics or heat (and mass) transfer processes. At most, these methods can treat limited regimes of the overall parameter domain for any particular problem. Furthermore, the approximate solutions obtained can be no more general than the framework allowed in the problem statement; that is, if we begin by seeking a steady axisymmetric solution, an asymptotic analysis will produce only an approximation for this class of solutions and, by itself, can guarantee neither that the solution is unique within this class nor that the limitation to steady and axisymmetric solutions is representative of the actual physical situation. For example, even if the geometry of the problem is completely axisymmetric, there is no guarantee that an axisymmetric solution exists for the velocity or temperature field, or if it does, that it corresponds to the motion or temperature field that would be realized in the laboratory. The latter may be either time dependent or fully three dimensional or both. In this case, the most that we may hope is that these more complex motions may exist as a consequence of instabilities in the basic, steady, axisymmetric solution, and thus that the conditions for departure from this basic state can be predicted within the framework of classical stability theories. The important message is that analytic techniques, including asymptotic methods, are not sufficient by themselves to understand fluid mechanics or heat transfer processes. Such techniques would almost always need to be supplemented by some combination of stability analysis or, more generally, by experimental or computational studies of the full problem.

I want to thank my many colleagues and students who have contributed to this work for many years. I would also like to thank the users of the first edition who made substantial suggestions for improvement. I look forward to the reader's reaction to this new version.

L. Gary Leal
Santa Barbara

Acknowledgments

I want to thank a number of people who contributed to this book. Most important among these were Professor G. M. Homsy, and several years of graduate students from my own classes at the University of California at Santa Barbara, who used this book in preprint form and provided much useful input on topics that required better explanation, typos, etc. In addition, these students had the first "opportunity" to work many of the problems at the end of each chapter, and this led to a number of important changes in the problem statements. I specifically appreciate their patience in this latter endeavor. I also owe a major debt of gratitude to number of faculty around the country, who had taught graduate transport classes from my previous book and provided detailed comments on the proposed contents and format of this new book. In addition, several of these individuals also contributed problems from their own classes, which they kindly allowed me to use in this new book. For this major contribution, I thank David Leighton from Notre Dame, John Brady from the California Institute of Technology, Roger Bonnecaze from the University of Texas at Austin, and James Oberhauser from the University of Virginia. In addition, Professor Howard Stone from Harvard University provided very useful notes on the dynamics of thin films from his own class, and also kindly read several of the new sections. Finally, I thank Cambridge University Press, and particularly Peter Gordon, for their patience in waiting for me to finish this book. The last 10 percent took at least 50 percent of the time! I take full responsibility for the contents of this book.

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A Preview

A. A BRIEF HISTORICAL PERSPECTIVE OF TRANSPORT PHENOMENA IN CHEMICAL ENGINEERING

"Transport phenomena" is the name used by chemical engineers to describe the subjects of fluid mechanics and heat and mass transfer. The earliest step toward the inclusion of specialized courses in fluid mechanics and heat or mass transfer processes within the chemical engineering curriculum probably occurred with the publication in 1923 of the pioneering text *Principles of Chemical Engineering* by Walker, Lewis, and McAdams.[1] This was the first major departure from curricula that regarded the techniques involved in the production of specific products as largely unique, to a formal recognition of the fact that certain physical or chemical processes, and corresponding fundamental principles, are common to many widely differing industrial technologies.

A natural outgrowth of this radical new view was the gradual appearance of fluid mechanics and transport in both teaching and research as the underlying basis for many of the unit operations. Of course, many of the most important unit operations take place in equipment of complicated geometry, with strongly coupled combinations of heat and mass transfer, fluid mechanics, and chemical reaction, so that the exact equations could not be solved in a context of any direct relevance to the process of interest. Hence, insofar as the large-scale industrial processes of chemical technology were concerned, even at the unit operations level, the impact of fundamental studies of fluid mechanics or transport phenomena was certainly less important than a well-developed empirical approach (and this remains true today in many cases). Indeed, the great advances and discoveries of fluid mechanics during the first half of the twentieth century took place almost entirely without the participation (or even knowledge) of chemical engineers.

Gradually, however, chemical engineers began to accept the premise that the generally "blind" empiricism of the "lumped-parameter" approach to transport processes at the unit operations scale should at least be supplemented by an attempt to understand the basic physical principles. This finally led, in 1960, to the appearance of the landmark textbook of Bird, Stewart, and Lightfoot,[2] which not only introduced the idea of detailed analysis of transport processes at the continuum level, but also emphasized the mathematical similarity of the governing field equations, along with the simplest constitutive approximations for fluid mechanics and heat and mass transfer. The presentation of Bird *et al.* was primarily focused on results and solutions rather than on the methods of solution or analysis. However, the combination of the more fundamental approach that it pioneered within the chemical engineering community and the appearance of chemical engineers with very strong mathematics backgrounds produced the most recent transitions in our ways of thinking about and understanding transport processes.