INTRODUCTION TO THE THEORY OF QUANTIZED FIELDS

THIRD EDITION

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PREFACE TO THE THIRD EDITION

In this edition we have rewritten the chapters that discuss the methods of continuous integration and the renormalization group, which are two topics in theory that have become extremely important in recent years. We have also reworked and supplemented the sections on the complete Green functions.

This work was done in an atmosphere of friendly advice and fruitful discussions with our colleagues from the Steklov Mathematical Institute of the USSR Academy of Sciences and of the Laboratory of Theoretical Physics of the Joint Institute of Nuclear Studies, to whom we are extremely grateful.

N. N. BOGOLIUBOV
D. V. SHIRKOV

Dubna,
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PREFACE TO THE SECOND EDITION

In preparing the second edition of this book we concentrated on improving the exposition of the most important parts. Thus, for example, we have rewritten sections on the quantization of fields and on general rules for removal of divergencies. Fairly substantial changes were also made in the two last chapters (renormalization group and dispersion relations) where we have attempted to make the exposition easier to understand by freeing it of many complicating problems (for example, proof of the dispersion relationships at $t \neq 0$), since these complicating side issues are fairly well described in the specialized literature.

We are indebted to many colleagues of the Steklov Mathematical Institute of the USSR Academy of Sciences, the Laboratory of Theoretical Physics of the Joint Institute of Nuclear Studies, and of the Institute of Mathematics of the Siberian Department, USSR Academy of Sciences, for useful comments and advice in preparation of this second edition.

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PREFACE TO THE FIRST RUSSIAN EDITION

This monograph is an attempt to give a systematic presentation of the modern theory of quantized fields from its foundations right up to its most recent achievements.

In writing this book the authors were guided by their intention to present field theory from a unified point of view combining internal logical consistency and closure with completeness of the material covered. We also aimed, wherever possible, to introduce the maximum degree of clarity into the basic assumptions of the theory employed at the present stage of its development. At the same time, naturally, particular attention was devoted to the mathematical correctness of the arguments, as a result of which the extent of coverage of the applications of the theory to calculations of specific physical phenomena can be claimed to be only methodologically complete.

The authors also wanted to treat sufficiently fully the most promising approaches developed quite recently. We hope that because of this, the book will prove to be useful not only to persons first undertaking the study of quantum field theory, but also to theoreticians working in this domain of physics.

The chapter "Dispersion Relations," which presents the most recent results, was included in the book as a supplement in view of the current interest in this topic.

The authors wish to thank the staff and the post-graduate students of the Division of Theoretical Physics of the V. A. Steklov Mathematical Institute of the Academy of Sciences, U.S.S.R., and of the Chair of Statistical Physics and Mechanics of the Physics Faculty of the M. V. Lomonosov Moscow State University, for their remarks and suggestions made during the preparation of the manuscript. We are particularly grateful to B. V. Medvedev, who contributed a number of valuable comments on different parts of the book.

N. N. BOGOLIUBOV
D. V. SHIRKOV

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INTRODUCTION

General Plan. This book contains a systematic exposition of the fundamentals of the modern theory of quantized fields, making full use of all the necessary mathematical concepts. Most of the book (the first seven chapters) consists of a logical development of the formalism based on the Lagrangian formulation of the theory of free fields and the axiomatic introduction of the scattering matrix for interacting fields.

Chapter 1 consists of a presentation of the apparatus of nonquantized relativistic free fields, based on the Lagrangian formalism and the Noether theorem.

The free fields are then quantized in Chapter 2 on the basis of the correspondence principle and without recourse to the canonical formalism. Chapter 3 discusses the properties of commutation and causal functions, and also certain mathematical problems that arise during multiplication of singular functions.

Chapter 4 gives the general theory of the scattering matrix and a development of the perturbation-theory apparatus. In this we follow the ideas of Heisenberg, Feynman, and, especially, Stueckelberg, and define the scattering matrix without recourse to the Hamiltonian formalism, but taking as our basis the physical requirements of covariance, unitarity, and causality. The condition of causality, which is formulated explicitly with the aid of the variational derivative, plays an important role in this development. In other words, we give a constructive development of the perturbation-theory expansion of the S-matrix within the framework of the axiomatic approach. Careful analysis of the arbitrariness that arises when singular Green's functions are multiplied together is used to obtain the most general expression for the scattering matrix, and this subsequently forms the basis for the procedure used to remove divergences.

Next, in Chapter 5, we take the lowest-order diagrams of spinor electrodynamics to illustrate the practical evaluation of Feynman integrals and the removal of the simplest ultraviolet divergencies in perturbation theory is then discussed in its fullest form, and a rigorous theory is presented of the renormalization of the S-matrix. A classification of renormalizability of theories is given within the framework of perturbation theory.

Chapter 6 contains an application of general renormalization theory in any order of perturbation theory to spinor electrodynamics, to the theories of scalar and pseudoscalar meson fields with interaction, and to the theory of pseudoscalar interaction of nucleons and pseudoscalar mesons. Particular attention is devoted to the renormalization structure of complete Green's functions. Schwinger's equations are also obtained for these functions.

In Chapter 7, the S-matrix is used to investigate the evolution of a system of quantized fields in time, and the Tomonaga-Schwinger equation is derived. The dynamic variables are introduced, and generalizations of free-field operators to the case of interaction are constructed. The Dirac equation is obtained with the radiative corrections included, and an account is given of the theory of the Lamb shift.

The three concluding chapters discuss some of the general methods in the theory of quantized fields that are not organically connected with the perturbation-theory expansion.

Chapter 8 contains an account of the method of functional integration. This method is based on a particular representation of complete Green's functions in terms of functional integrals, and is very general. Its possibilities are demonstrated by considering spinor electrodynamics. A generalization of Ward's identity is obtained, the gauge transformation of the electron Green's function is derived, and the structure of its infrared singularities is determined. Studies of non-Abelian gauge fields have made this method increasingly popular. It has become crucially important as an instrument of theoretical analysis of quantized-field models, not based on perturbation theory.

In Chapter 9, we present the renormalization group method which is based on the group character of multiplicative renormalizations. The formalism of functional and differential equations is developed, and considerable attention is devoted to the analysis of asymptotic behavior in the ultraviolet region, which is of considerable practical importance.

Finally, Chapter 10 is devoted to the method of dispersion relations. Our presentation, based on the axiomatic approach, contains a derivation of the spectral representations of single-particle Green's functions and of the dispersion relations for pion-nucleon scattering. Extensive use is made of variational derivatives of the S-matrix with respect to the quantized fields (i.e., currents) and the matrix elements of their products. Enough material is presented to enable the reader to achieve an adequate understanding of the theory of functions of many complex variables, as used for the analytic continuation of generalized functions.

During the twenty-year history of the theory of dispersion relations, these methods at first seemed exotic, but eventually found extensive application in quantum field theory and now form the basis for most of its results (connection between spin and statistics, the CPT theorem, and so on).

In its broad sense, the method of dispersion relations, i.e., the method of investigating the basic variables of the theory in terms of their analytic properties, has itself undergone considerable development during this period, and has found varied and physically meaningful applications. Some of them, for example, the high-energy properties of the scattering amplitude (the Pomeranchuk and Logunov theorems, the Froissart limit) are based on rigorously proved analytic properties. Others use additionally postulated propositions (for example, the double spectral representations of Mandelstam). All these problems are, however, outside the framework of our presentation.

Some Notation. We now introduce some of the notation used in this book. All the components of four-vectors are chosen to be real. The metric is defined by the Minkowski tensor, taken with reversed sign:

$$g^{mn}=0 \quad \text{for} \quad m\neq n; \quad g^{00}=-g^{11}=-g^{22}=-g^{33}=1,$$

i.e., the product of two contravariant four-vectors a and b with components $\{a^0, a^1, a^2, a^3\}=\{a^0, \boldsymbol{a}\}$ and $\{b^0, b^1, b^2, b^3\}=\{b^0, \boldsymbol{b}\}$ is defined as follows:

$$ab \equiv \sum_{0\leq m,n\leq 3} g^{mn}a^m b^n = a^0b^0 - a^1b^1 - a^2b^2 - a^3b^3 = a^0b^0 - \boldsymbol{ab}.$$

Bold type is used to represent ordinary three-vectors. The transition from contravariant a^i to covariant a_i components (lowering of index) is achieved with the aid of the metric tensor

$$a_m = \sum g^{mn}a^n = g_{mn}a^n; \qquad g_{mn}=g^{mn},$$

i.e.,

$$a_0 = a^0, \quad a_\alpha = -a^\alpha, \quad \alpha = 1, 2, 3, \quad a_m = \{a^0, -\boldsymbol{a}\}.$$

Repeated indices imply summation (the summation symbol is not explicitly indicated). As a rule, indices representing summation over all four components 0, 1, 2, 3 are indicated by Latin letters, and those over the three space components by Greek letters. For example,

$$\begin{aligned} ab &= g^{mn}a_m b_n = a^n b_n = a_m b^m, \\ \boldsymbol{ab} &= a_\alpha b_\alpha = a^\alpha b^\alpha. \end{aligned}$$

By raising (or lowering) one of the indices of the metric tensor we obtain the Kronecker symbol:

$$g_{nm}g^{mk} = \delta_n^k.$$

Indices referring to groups of internal symmetries (for example, isospin indices) are usually represented by Latin letters at the beginning of the alphabet ($a, b, \ldots$).

Generalizing notation is occasionally used for (operator) field functions in addition to the individualized notation introduced at the relevant points in our presentation. Fields with integer spin (Bose fields) are denoted by φ, and fields with half-integer spin (Fermi fields) are denoted by the symbol ψ. To emphasize the generality of the discussion, continuous space-time coordinates are occasionally combined with discrete coordinates into a single argument, indicated by a Greek letter, for example,

$$\varphi_a(x) \to \varphi(\xi), \qquad \psi_a(x) \to \psi(\xi), \qquad \xi = (x, a).$$

Integration with respect to ξ is defined as follows:

$$\int d\xi = \sum \int d^4x.$$

The special notation $u(\xi)$ is used for the most general analysis, valid for both Bose and Fermi fields.

The symbol $\hat{a}$ represents the contraction of the components of the four-vector a_m and the Dirac matrices γ^m:

$$\hat{a} = a^m \gamma_m.$$

The following abbreviated notation is occasionally used for derivatives:

$$\frac{\partial \varphi_a}{\partial x^n} = \partial_n \varphi_a = \varphi_{a;\, n}; \qquad \frac{\partial u}{\partial x_n} = \partial^n u = u^{;n},$$

where it is, of course, understood that

$$\varphi_a^{;n} = g^{nm} \varphi_{a;\, m}.$$

The d'Alembert operator

$$\Box = \Delta - \partial_0^2$$

is represented by

$$\Box = -\partial^n \partial_n.$$

Throughout this book we use the system of units in which the velocity of light and Planck's constant divided by 2π, are both equal to unity, i.e.,

$$c = \hbar = 1.$$

In this system of units, energy, momentum, and mass have the dimensions of the reciprocal of length, and the time $x_0 = t$ has the dimensions of length.

The four-dimensional Fourier transformation is usually written in the form

$$f(x) \sim \int e^{-ipx} \tilde{f}(p)\, dp, \qquad \tilde{f}(p) \sim \int e^{ipx} f(x)\, dx.$$

The sign of the argument of the exponential is chosen so that it is consistent with the quantum-mechanical formula

$$f(x^0, \boldsymbol{x}) = f(t, \boldsymbol{x}) \sim \int e^{-iEt} \tilde{f}(E, \boldsymbol{x})\, dE.$$

Accordingly, the three-dimensional Fourier transformation has the form

$$\varphi(\boldsymbol{x}) \sim \int e^{ipx} \tilde{\varphi}(\boldsymbol{p})\, d\boldsymbol{p}, \qquad \tilde{\varphi}(\boldsymbol{p}) \sim \int e^{-ipx} \varphi(\boldsymbol{x})\, d\boldsymbol{x}.$$

The only exception occurs in the case of formulas for the positive-frequency parts of field functions and the positive-frequency parts of Green's functions. Different normalizing factors (powers of 2π) are used for the Fourier transformations at different places in the book.

Bibliographic references are given in the form of the surname of the author followed by the year of publication in parenthesis. Full bibliographic references can be found at the end of the book.