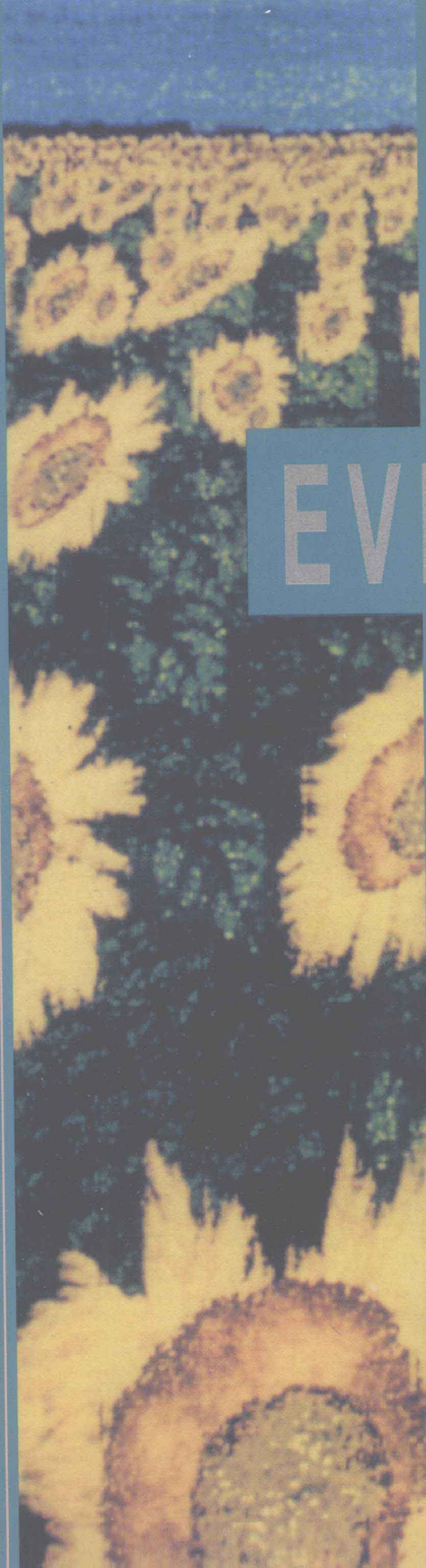
MICHAEL F. BARNSLEY

FRACTALS EVERYWHERE

SECOND EDITION

无处不在的分形

第2版



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Iterated Systems, Inc. Atlanta, Georgia

Revised with the assistance of Hawley Rising III
Answer key by Hawley Rising III

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I dedicate the second edition of this book to my daughter
Diana Gabriel Barnsley

Foreword to the Second Edition

Much has changed in the world of fractals, computer graphics and modern mathematics since the first edition of *Fractals Everywhere* appeared. The company Iterated Systems, Inc., founded by Michael Barnsley and Alan Sloan, is now competing in the image compression field with both hardware and software products that use fractal geometry to compress images. Indeed, there is now a plethora of texts on subjects like fractals and chaos, and these terms are rapidly becoming "household words."

The fundamental approach to fractal geometry through iterated function systems remains sound as an introduction to the subject. This edition of *Fractals Everywhere* leaves this approach largely as it stands. One still needs a grounding in concepts in metric space theory and eventually (see Chapter IX) measure theory to get a working understanding of the subject. However, there have been several additions to help ease and broaden the reader's development.

Primary to these is the addition of answers to the mathematical problems. These were done largely by starting at one end of the book writing the answers until the other cover was reached. Most of the answers found in the key have been worked over at least twice, in hopes improving the accuracy of the key. Every effort has been make to rely solely on the material presented ahead of each problem, although in a few of the harder problems some concepts have been introduced in the answers themselves. These are not considered necessary to the development of the main thread of the text; however, if the reader finds some areas of mathematics touched on in looking at the presented solutions which extend the feeling for the subject, the key has served its purpose.

In addition the the answer key, there have been some other changes as well. In Chapter III, section 11, the main theorem has been qualified. The reader with more mathematical background will recognize that the additional Lipshitz condition satisfies the need for equicontinuity in Theorem 11.1. This is not the only way to satisfy it, just the clearest in terms of the presumed mathematical background.

There have been problems added to several chapters to develop the idea of Cartesian products of code spaces. This was done because it helps bridge the gap between IFS theory and the reversible systems found in physical chaos, and because it presents an interesting way of looking the the Random Iteration Algorithm in Chapter IX. The thread of these problems begins in Chapter II, leads up to the baker's transformation in Chapter IV, and is completed as an example in Chapter IX. Additional problems were added in Chapter III to develop some basic properties of eigenvalues and eigenvectors, which can be useful in examining dynamics both from the point of view described in the text, and elsewhere. It is hoped that with these additional tools those readers whose goals are application-oriented will come away with more at their disposal, while the text itself will retain its readable style.

I would like to thank Lyman Hurd for many useful discussions about the topological nature of nonempty compact sets, and John Elton for his patience while I ran many of my new examples and problems past him to check them and to check the "excitement level" of the additional material.

Hawley Rising

It seems now that deterministic fractal geometry is racing ahead into the serious engineering phase. Commercial applications have emerged in the areas of image compression, video compression, computer graphics, and education. This is good because it authenticates once again the importance of the work of mathematicians. However, sometimes mathematicians lose interest in wonderful areas once scientists and engineers seem to have the subject under control. But there is so much more mathematics to be done. What is a useful metric for studying the contractivity of the vector recurrent IFS of affine maps in $\mathbb{R}^2$? What is the information content of a picture? Measures, pictures, dreams, chaos, flowers and information theory—the hours of the days keep rushing by: do not let the beauty of all these things pass us by too.

Michael Fielding Barnsley

Acknowledgments

I acknowledge and thank many people for their help with this book. In particular I thank Alan Sloan, who has unceasingly encouraged me, who wrote the first Collage software, and who so clearly envisioned the application of iterated function systems to image compression and communications that he founded a company named *Iterated Systems Incorporated.* Edward Vrscay, who taught the first course in deterministic fractal geometry at Georgia Tech, shared his ideas about how the course could be taught, and suggested some subjects for inclusion in this text. Steven Demko, who collaborated with me on the discovery of iterated function systems, made early detailed proposals on how the subject could be presented to students and scientists, and provided comments on several chapters. Andrew Harrington and Jeffrey Geronimo, who discovered with me orthogonal polynomials on Julia sets. My collaborations with them over five years formed for me the foundation on which iterated function systems are built. Watch for more papers from us!

Les Karlovitz, who encouraged and supported my research over the last nine years, obtained the time for me to write this book and provided specific help, advice, and direction. His words can be found in some of the sentences in the text. Gunter Meyer, who has encouraged and supported my research over the last nine years. He has often given me good advice. Robert Kasriel, who taught me some topology over the last two years, corrected and rewrote my proof of Theorem 7.1 in Chapter II and contributed other help and warm encouragement. Nathanial Chafee, who read and corrected Chapter II and early drafts of Chapters III and IV. His apt constructive comments have increased substantially the precision of the writing. John Elton, who taught me some ergodic theory, continues to collaborate on exciting research into iterated function systems, and helped me with many parts of the book. Daniel Bessis and Pierre Moussa, who are filled with the wonder and mystery of science, and taught me to look for mathematical events that are so astonishing that they may be called miracles. Research work with Bessis and Moussa at Saclay during 1978, on the Diophantine Moment Problem and Ising Models, was the seed that grew into this book. Warren Stahle, who provided some of his experimental research results for inclusion in Chapter VI.

Graduate students John Herndon, Doug Hardin, Peter Massopust, Laurie Reuter, Arnaud Jacquin, and François Malassenet, who have contributed in many ways to this book. They helped me to discover and develop some of the ideas. Els Withers and Paul Blanchard, who supported the writing of this book from the start and suggested some good ideas that are used. The research papers by Withers on iterated functions are deep. Edwina Barnsley, my mother, whose house was always full of flowers. Her encouragement and love helped me to write this book. Thomas Stelson, Helena Wisniewski, Craig Fields, and James Yorke who, early on, supported the development of applications of iterated function systems. Many of the pictures in this text were produced in part using software and hardware in the DARPA/GTRC funded Computergraphical Mathematics Laboratory within the School of Mathematics at Georgia Institute of Technology.

George Cain, James Herod, William Green, Vince Ervin, Jamie Good, Jim Osborne, Roger Johnson, Li Shi Luo, Evans Harrell, Ron Shonkwiler, and James Walker who contributed by reading and correcting parts of the text, and discussing research. Thomas Morley, who contributed many hours of discussion of research and never asks for any return. William Ames who encouraged me to write this book and introduced me to Academic Press. Annette Rohrs, who typed the first drafts of Chapters II, III, and IV. William Kammerer, who introduced me to EXP, the technical word processor on which the manuscript was written, and who has warmly supported this project.

This book owes its deepest debt to Alan Barnsley, my father, who wrote novels and poems under the *nom-de-plume* Gabriel Fielding. I learnt from him care for precision, love of detail, enthusiasm for life, and an endless amazement at all that God has made.

Michael Barnsley

Contents

Chapter I

Introduction

Fractal geometry will make you see everything differently. There is danger in reading further. You risk the loss of your childhood vision of clouds, forests, galaxies, leaves, feathers, flowers, rocks, mountains, torrents of water, carpets, bricks, and much else besides. Never again will your interpretation of these things be quite the same.

The observation by Mandelbrot [Mandelbrot 1982] of the existence of a "Geometry of Nature" has led us to think in a new scientific way about the edges of clouds, the profiles of the tops of forests on the horizon, and the intricate moving arrangement of the feathers on the wings of a bird as it flies. Geometry is concerned with making our spatial intuitions objective. Classical geometry provides a first approximation to the structure of physical objects; it is the language that we use to communicate the designs of technological products and, very approximately, the forms of natural creations. Fractal geometry is an extension of classical geometry. It can be used to make precise models of physical structures from ferns to galaxies. Fractal geometry is a new language. Once you can speak it, you can describe the shape of a cloud as precisely as an architect can describe a house.

This book is based on a course called "Fractal Geometry," which has been taught in the School of Mathematics at the Georgia Institute of Technology for two years. The course is open to all students who have completed two years of calculus. It attracts both undergraduate and graduate students from many disciplines, including mathematics, biology, chemistry, physics, psychology, mechanical engineering, electrical engineering, aerospace engineering, computer science, and geophysical science. The delight of the students with the course is reflected in the fact there is now a second course, entitled "Fractal Measure Theory." The courses provide a compelling vehicle for teaching beautiful mathematics to a wide range of students.

Here is how the course in Fractal Geometry is taught. The core is Chapter II, Chapter III, sections 1–5 of Chapter IV, and sections 1–3 of Chapter V. This is followed by a collection of delightful special topics, chosen from Chapters VI, VII, and VIII. The course is taught in 30 one-hour lectures.

Chapter II introduces the basic topological ideas that are needed to describe subsets to spaces such as $\mathbb{R}^2$. The framework is that of metric spaces; this is adopted because metric spaces are both rigorously and intuitively accessible, yet full of suprises. They provide a suitable setting for fractal geometry. The concepts introduced include openness, closedness, compactness, convergence, completeness, connectedness, and equivalence of metric spaces. An important theme concerns properties that are preserved under equivalent metrics. Chapter II concludes by presenting the most exciting idea: a metric space, denoted $\mathcal{H}$, whose elements are the nonempty compact subsets of a metric space. Under the right conditions this space is complete, sequences converge, and fractals can be found!

Chapter III deals with transformations on metric spaces. First, the goal is to develop intuition and practical experience with the actions of elementary transformations on subsets of spaces. Particular attention is devoted to affine transformations and Möbius transformations in $\mathbb{R}^2$. Then the contraction mapping principle is revealed, followed by the construction of contraction mappings on $\mathcal{H}$. Fractals are discovered as the fixed points of certain set maps. We learn how fractals are generated by the application of "simple" transformations on "simple" spaces, and yet they are geometrically complicated. We explain what an iterated function system (IFS) is, and how it can define a fractal. Iterated function systems provide a convenient framework for the description, classification, and communication of fractals. Two algorithms, the "Chaos Game" and the Deterministic Algorithm, for computing pictures of fractals are presented. Attention is then turned to the inverse problem: given a compact subset of $\mathbb{R}^2$, fractal, how do you go about finding a fractal approximation to it? Part of the answer is provided by the Collage Theorem. Finally, the thought of the wind blowing through a fractal tree leads to discovery of conditions under which fractals depend continuously on the parameters that define them.

Chapter IV is devoted to dynamics on fractals. The idea of addresses of points on certain fractals is developed. In particular, the reader learns about the metric space to which addresses belong. Nearby addresses correspond to nearby points on the fractal. This observation is made precise by the construction of a continuous transformation from the space of addresses to the fractal. Then dynamical systems on metric spaces are introduced. The ideas of orbits, repulsive cycles, and equivalent dynamical systems are described. The concept of the shift dynamical system associated with an IFS is introduced and explored. This is a visual and simple idea in which the author and the reader are led to wonder about the complexity and beauty of the available orbits. The equivalence of this dynamical system with a corresponding system on the space of addresses is established. This equivalence takes no account of the geometrical complexity of the dance of the orbit on the fractal. The chapter then moves towards its conclusion, the definition of a chaotic dynamical system and the realization that "most" orbits of the shift dynamical system on a fractal are chaotic. To this end, two simple and delightful ideas are shown to the reader. The Shadow Theorem illustrates how apparently random orbits may actually be the "shadows" of deterministic motions in higher-dimensional spaces. The Shadow*ing* Theorem demonstrates how a rottenly inaccurate orbit may be trailed by a precise orbit, which clings like a secret agent. These ideas are used to make an explanation of why the "Chaos Game" computes fractals.

Chapter V introduces the concept of fractal dimension. The fractal dimension of a set is a number that tells how densely the set occupies the metric space in which it lies. It is invariant under various stretchings and squeezings of the underlying space. This makes the fractal dimension meaningful as an experimental observable; it possesses a certain robustness and is independent of the measurement units. Various theoretical properties of the fractal dimension, including some explicit formulas, are developed. Then the reader is shown how to calculate the fractal dimension of real-world data, and an application to a turbulent jet exhaust is described. Lastly the Hausdorff-Besicovitch dimension is introduced. This is another number that can be associated with a set. It is more robust and less practical than the fractal dimension. Some mathematicians love it; most experimentalists hate it; and we are intrigued.

Chapter VI is devoted to fractal interpolation. The aim of the chapter is to teach the student practical skill in using a new technology for making complicated curves and fitting experimental data. It is shown how geometrically complex graphs of continuous functions can be constructed to pass through specified data points. The functions are represented by succinct formulas. The main existence theorems and computational algorithms are provided. The functions are known as fractal interpolation functions. It is explained how they can be readily computed, stored, manipulated and communicated. "Hidden variable" fractal interpolation functions are introduced and illustrated; they are defined by the shadows of the graphs of three-dimensional fractal paths. These geometrical ideas are extended to introduce space-filling curves.

Chapter VII gives an introduction to Julia sets, which are deterministic fractals that arise from the iteration of analytic functions. The objective is to show the reader how to understand these fractals, using the ideas of Chapters III and IV. In so doing we have the pleasure of explaining and illustrating the Escape Time Algorithm. This algorithm is a means for computergraphical experimentation on dynamical systems that act on two-dimensional spaces. It provides illumination and coloration, a seachlight to probe dynamical systems for fractal structures and regions of chaos. The algorithm relies on the existence of "repelling sets" for continuous transformations which map open sets to open sets. The applications of Julia sets to biological modelling and to understanding Newton's method are considered.

Chapter VIII is concerned with how to make maps of certain spaces, known as parameter spaces, where every point in the space corresponds to a fractal. The fractals depend "smoothly" on the location in the parameter space. How can one make a picture that provides useful information about what kinds of fractals are located where? If both the space in which the fractals lie and the parameter space are two-dimensional, the parameter space can sometimes be "painted" to reveal an associated Mandelbrot set. Mandelbrot sets are defined, and three different examples are explored, including the one discovered by Mandelbrot. A computergraphical technique for producing images of these sets is described. Some basic theorems are proved.

Chapter IX is an introduction to measures on fractals and to measures in general. The chapter is an outline that can be used by a professor as the basis of a course in fractal measure theory. It can also be used in a standard measure theory course as a source of applications and examples. One goal is to demonstrate that measure theory is a workaday tool in science and engineering. Models for real-world images can be made using measures. The variations in color and brightness, and the complex textures in a color picture, can be successfully modelled by measures that can be written down explicitly in terms of succinct "formulas." These measures are desirable for image engineering applications, and have a number of advantages over non-negative "density" functions. Section 1 provides an intuitive description of measures and motivates the rest of the chapter. The context is that of Borel measures on compact metric spaces. Fields, sigma-fields, and measures are defined. Carathéodory's extension theorem is introduced and used to explain what a Borel measure is. Then the integral of a continuous real-valued function, with respect to a measure, is defined. The reader learns to evaluate some integrals. Next the space $\mathcal{P}$ of normalized Borel measures on a compact metric space is defined. With an appropriate metric, $\mathcal{P}$ becomes a compact metric space. Succintly defined contraction mappings on this space lead to measures that live on fractals. Integrals with respect to these measures can be evaluated with the aid of Elton's ergodic theorem. The book ends with a description of the application of these measures to computer graphics.

This book teaches the tools, methods, and theory of *deterministic* geometry. It is useful for describing *specific* objects and structures. Models are represented by succinct "formulas." Once the formula is known the model can be reproduced. We do not consider statistical geometry. The latter aims at discovering general statistical laws that govern families of similar-looking structures, such as *all* cumulus clouds, *all* maple leaves, or *all* mountains.

In deterministic geometry, structures are defined, communicated, and analyzed, with the aid of elementary transformations such as affine transformations, scalings, rotations, and congruences. A fractal set generally contains infinitely many points whose organization is so complicated that it is not possible to describe the set by specifying directly where each point in it lies. Instead, the set may be defined by "the relations between the pieces." It is rather like describing the solar system by quoting the law of gravitation and stating the initial conditions. Everything follows from that. It appears always to be better to describe in terms of relationships.