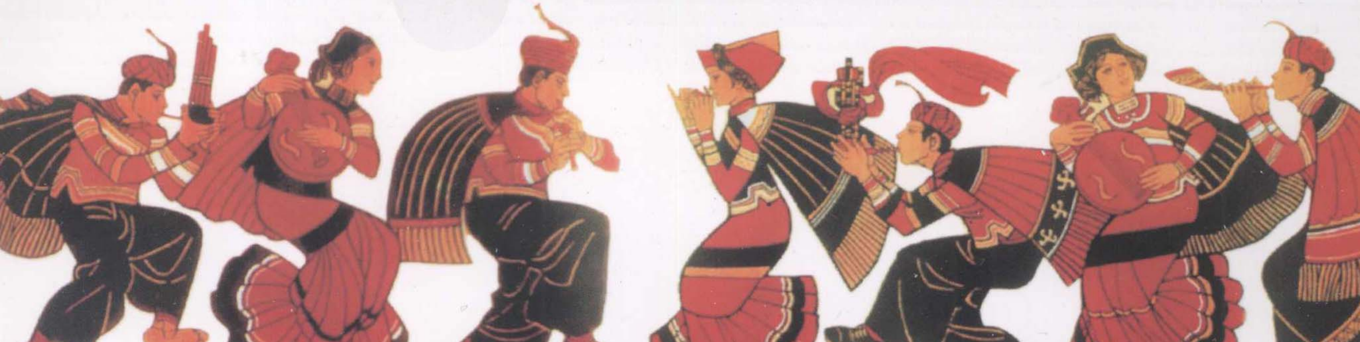
科技与民族发展

Technology and National Development 第一辑

王凤友/主编



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前　言

21 世纪是科学技术迅猛发展的世纪。民族地区经济社会翻天覆地的变化，与人类科学技术的进步有着紧密的联系。这种联系有两方面的含意:第一是科学技术在民族地区正得到广泛的运用;第二是民族地区的科学技术资源正越来越多地受到重视,得到开发。科学技术作为第一生产力,正在深刻地改变着民族地区的面貌。民族地区的各个领域和各个行业,进入了全面发展的历史时期。

斗转星移,白驹过隙,民族地区的沧桑巨变,正在推动贵州民族学院从一所以文科为主、面向贵州的高等院校,向立足西南、面向全国、文理兼容的综合性高校转型。数学、物理学、化学与环境科学、民族医学与药学、计算机与信息工程、建筑工程学等自然科学类和工程类专业,开始成为贵州民族学院重点建设的学科,这必将在民族地区高等学校发展的历史上写下熠熠生辉的一页。

为展现贵州民族学院在自然科学各学科领域的研究成果,我们选编了《科技与民族发展》论文第一辑。科技进步与民族发展,这是历史赋予我们的职责。这一领域的教师和科研人员,都将为此躬耕无悔,奉献一生。

由于时间紧,书中疏漏在所难免,敬请广大读者批评指正。

《科技与民族发展》编辑委员会

2011 年 11 月

《科技与民族发展》编辑委员会

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MORREY TYPE SPACES AND MAXIMAL OPERATOR

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Abstract A Morrey type space is defined and the corresponding theorem of maximal operator is established.

Key words Morrey type space; Hardy-Littlewood maximal function

1. Introduction

Let $\mathbb{R}^n$ be the n-dimensional Euclidean space. For given $x \in \mathbb{R}^n$ and $r > 0$, denote by $B(x, r)$ the ball of center x and radius r in $\mathbb{R}^n$.

For $0 < p \leq \infty$ and $\lambda \in \mathbb{R}$, the Morrey space $M(p,\lambda)$ is the space of all functions f in $L_p^{loc}(\mathbb{R}^n)$ satisfying

$$\|f\|_{M(p,\lambda)} = \sup_{x \in \mathbb{R}^n,\, r>0} r^{-\frac{\lambda}{p}} \|f\|_{Lp(B(x,r))} < \infty .$$

The Weak Morrey space $M^*(p,\lambda)$ is the space of all $f \in L_p^{loc}(\mathbb{R}^n)$ such that

$$\|f\|_{M^*(p,\lambda)} = \sup_{x \in \mathbb{R}^n,\, r>0} r^{-\frac{\lambda}{p}} \|f\|_{WLp(B(x,r))} < \infty .$$

Here $\|f\|_{WLp(\Omega)}$ is defined as

$$\|f\|_{WLp(\Omega)} = \sup_{r>0} r \,|\{x \in \Omega : |f(x)| > r\}|^{1/p} < \infty .$$

Clearly, $M(p,0) = L_p(\mathbb{R}^n)$, $M(p,n) = L_\infty(\mathbb{R}^n)$.

Morrey space $M(p,\lambda)$ is first introduced by Morrey in 1938 in [14]. It plays an important role in the study of partial differential equations, especially the local behavior of the solutions of elliptic partial differential equations. We refer the reader to the papers [1, 3, 15, 16] and the references therein.

For $f \in L_1^{loc}(\mathbb{R}^n)$, its Hardy-Littlewood maximal function $M(f)$ is defined as

$$M(f)(x) = \sup_{t>0} |B(x,t)|^{-1} \int_{B(x,t)} |f(y)|\,dy,$$

where $|B(x,t)|$ is the volume of the ball $B(x,t)$.

The following theorem is from [3].

Theorem 1.1. *Suppose* $0 < \lambda < n$.

(1) *If* $1 < p < \infty$, *then* M *is bounded on* $M(p,\lambda)$.

(2) M *is bounded from* $M(1,\lambda)$ *to* $M^*(1,\lambda)$.

In this paper, we consider a general Morrey type space $M_F(p,\lambda)$ and the related weak Morrey type space $M_F^*(1,\lambda)$. Our main goal is to establish a theorem for the boundedness of the maximal operator on these spaces. It is worth to note that the Morrey type space introduced in this paper includes the space $M_{p\theta,\lambda}$ introduced by Adams in [2], which has been studied and applied by many researches (see for example, [13, 10, 11, 12, 4, 5, 8]). For example, the maximal operator M is proved to be bounded on this space.

2. Morrey type space and some basic properties

Suppose $(F_i, \|\cdot\|_{F_i}), i = 1,2$, and $(F, \|\cdot\|_F)$ are Banach spaces of certain complex-valued measurable functions on $(0,\infty)$. We are particularly interested in the following two properties:

(I) $F_1F_2 \subseteq F$, i. e., if $f\in F_1$, $g\in F_2$, then $fg\in F$.

(II) If $F_1F_2\subseteq F$, then the Hölder inequality holds, i. e., the inequality

$$\|fg\|_F \le \|f\|_{F_1}\|g\|_{F_2}$$

holds for all $f\in F_1$ and $g\in F_2$.

Definition 2.1. *For $0 < p\le\infty$ and $\lambda\in\mathbb{R}$, the Morrey type space $M_F(p,\lambda)$ is the space which consists of all $f\in L_p^{loc}(\mathbb{R}^n)$ satisfying*

$$\|f\|_{M_F(p,\lambda)} = \sup_{x\in\mathbb{R}^n} \| r^{-\frac{\lambda}{p}} \|f\|_{Lp(B(x,r))} \|_F.$$

The weak Morrey type space $M_F^*(p,\lambda)$ is the space which consists of all $f\in L_p^{loc}(\mathbb{R}^n)$ satisfying

$$\|f\|_{M_F^*(p,\lambda)} = \sup_{x\in\mathbb{R}^n} \| r^{-\frac{\lambda}{p}-\frac{\lambda}{\theta}} \|f\|_{WL_p(B(x,r))} \|_F.$$

It is clear that if $F = L\infty(0,\infty)$, then $\|\cdot\|_{M_F(p,\lambda)} = \|\cdot\|_{M(p,\lambda)}$.

Lemma 2.1. *Suppose $1\le p\le q\le\infty$ and F is a Banach space of certain complex-valued measurable functions on $(0,\infty)$. If $f\in M_F(q,\lambda)$, then $f\in M_F(p,\lambda+n(\frac{q}{p}-1))$ and*

$$\|f\|_{M_F(p,\lambda+n(\frac{q}{p}-1))} \le v_n^{\frac{1}{p}-\frac{1}{q}} \|f\|_{M_F(q,\lambda)}. \tag{1}$$

Here v_n is the volume of the unit ball in $\mathbb{R}^n$.

Proof. By Hölder inequality, we have

$$\begin{aligned}\|f\|_{L_p(B(x,r))} &\le \|f\|_{L_q(B(x,r))} |B(x,r)|^{\frac{1}{p}-\frac{1}{q}} \\ &= v_n^{\frac{1}{p}-\frac{1}{q}} \|f\|_{L_q(B(x,r))} r^{\frac{n}{p}-\frac{n}{q}}.\end{aligned}$$

Using above inequality, together with the definition of the Morrey type space, we can drive the desired result.

Lemma 2.2. (*Hölder inequality*) *Suppose F, F_1, F_2 are Banach spaces of certain complex-valued measurable functions on $(0,\infty)$, and satisfy properties (I) and (II). Suppose $0 < p, p_1, p_2 \le\infty$,*

$\lambda,\lambda_1,\lambda_2\in\mathbb{R}^n$, *and* $\frac{1}{p}=\frac{1}{p_1}+\frac{1}{p_2}, \frac{\lambda}{p}=\frac{\lambda_1}{p_1}+\frac{\lambda_2}{p_2}$. *If* $f\in M_{F_1}(p_1,\lambda_1)$ *and* $g\in M_{F_2}(p_2,\lambda_2)$, *then* $fg\in M_F(p,\lambda)$ *and*

$$\|fg\|_{M_F(p,\lambda)}\le \|f\|_{M_{F_1}(p_1,\lambda_1)}\|g\|_{M_{F_2}(p_2,\lambda_2)}.$$

Proof. By Hölder inequality, we have

$$\|fg\|_{L_p(B(x,r))}\le \|f\|_{L_{p_1}(B(x,r))}\|g\|_{L_{p_2}(B(x,r))}.$$

Therefore

$$\begin{aligned}\|fg\|_{M_F(p,\lambda)} &= \sup_{x\in\mathbb{R}^n}\left\| r^{-\frac{\lambda}{p}}\|fg\|_{L_p(B(x,r))}\right\|_F\\ &\le \sup_{x\in\mathbb{R}^n}\left\| r^{-\frac{\lambda_1}{p_1}}r^{-\frac{\lambda_2}{p_2}}\|f\|_{L_{p_1}(B(x,r))}\|g\|_{L_{p_2}(B(x,r))}\right\|_F\\ &\le \sup_{x\in\mathbb{R}^n}\left\| r^{-\frac{\lambda_1}{p_1}}\|f\|_{L_{p_1}(B(x,r))}\right\|_{F_1}\left\| r^{-\frac{\lambda_2}{p_2}}\|g\|_{L_{p_2}(B(x,r))}\right\|_{F_2}\\ &\le \|f\|_{M_{F_1}(p_1,\lambda_1)}\|g\|_{M_{F_2}(p_2,\lambda_2)}.\end{aligned}$$

3. Maximal operator on $M_F(p,\lambda)$

The following theorem is inspired by the work of V. I. Burenkov and H. V. Guliyev [4], where the boundedness of the maximal operator on the local Morrey type space are investigated.

Theorem 3.1. *Suppose* $1\le p<\infty$, $\lambda<n$, *and* F *is a Banach space of certain complex-valued measurable functions on* $(0,\infty)$.

(1) *If* $1<p<\infty$, *then the maximal operator* M *is bounded on the Morrey type space* $M_F(p,\lambda)$, *i. e., there exists a constant* $C>0$ *such that*

$$\|M(f)\|_{M_F(p,\lambda)}\le C\|f\|_{M_F(p,\lambda)}$$

holds for all $f\in M_F(p,\lambda)$.

(2) *The maximal operator* M *is bounded from the Morrey type space* $M_F(1,\lambda)$ *to the weak Morrey type space* $M_F^*(1,\lambda)$, *i. e., there exists a constant* $C>0$ *such that*

$$\|M(f)\|_{M_F^*(1,\lambda)}\le C\|f\|_{M_F(1,\lambda)}$$

holds for all $f\in M_F(1,\lambda)$.

Proof. For fixed $u\in\mathbb{R}^n$ and $r>0$, denote

$R_0(u,r)=B(u,2r)$ and $R_j(u,r)=B(u,2^{j+1}r)\setminus B(u,2^jr)$, $j=1,2,\cdots$.

For $k\ge 0$, let

$$T_{u,r}^k(f)(x)=\begin{cases}f(x), & \text{if } x\in R_k(u,r),\\ 0, & \text{if } x\notin R_k(u,r).\end{cases}$$

Clearly $f(x)=\sum_{k=0}^{\infty}T_{u,r}^k(f)(x)$, $\forall x\in\mathbb{R}^n$, and

$$\|M(f)\|_{L_p(B(u,r))}\le\sum_{k=0}^{\infty}\|M(T_{u,r}^k(f))\|_{L_p(B(u,r))}.$$

We estimate $\| M(T^0_{u,r}(f)) \|_{L_p(B(u,r))}$ first. We need the well-known Fefferman-Stein maximal inequality (see for example [9]), which states that there exists a constant $C>0$, such that

$$\| M(f) \|_{L_p(\mathbb{R}^n)} \le C \|f\|_{L_p(\mathbb{R}^n)},$$

holes for all $f \in L_p(\mathbb{R}^n)$.

Using the above inequality, we have

$$\begin{aligned} \| M(T^0_{u,r}(f)) \|_{L_p(B(u,r))} &\le \| M(T^0_{u,r}(f)) \|_{L_p(\mathbb{R}^n)} \\ &\le C \| T^0_{u,r}(f) \|_{L_p(\mathbb{R}^n)} \\ &= C \|f\|_{L_p(B(u,2r))}. \end{aligned}$$

Here, $C>0$ is independent of $u \in \mathbb{R}^n$, $r>0$ and $f \in L_p(\mathbb{R}^n)$.

We now estimate $\| M(T^k_{u,r}(f)) \|_{L_p(B(u,r))}$ for all $k \ge 1$. For given $x \in B(u,r)$, let

$$t_x = \inf\{ t: B(x,t) \cap B(u,2^{k+1}r) \ne \emptyset \}.$$

It is clear that $t_x \ge 2^k r$. Hence we have

$$\begin{aligned} M(T^k_{u,r}(f))(x) &= \sup_{t>0} |B(x,t)|^{-1} \int_{B(x,t)} |T^k_{u,r}(f)(y)|dy \\ &\le |B(x,t_x)|^{-1} \int_{\mathbb{R}^n} |T^k_{u,r}(f)(y)|dy \\ &\le C(2^k r)^{-n} \int_{\mathbb{R}^n} |T^k_{u,r}(f)(y)|dy \\ (\text{Höölder}) &\le C(2^k r)^{-n/p} \|f\|_{L_p(B(u,2^{k+1}r))}. \end{aligned}$$

Combining estimates above, we obtain

$$\begin{aligned} & r^{-\frac{\lambda}{p}} \| M(f) \|_{L_p(B(u,r))} \\ &\le C \sum_{k=0}^{\infty} r^{-\frac{\lambda}{p}} \| M(T^k_{u,r}(f)) \|_{L_p(B(u,r))} \\ &\le Cr^{-\frac{\lambda}{p}} \|f\|_{L_p(B(u,2r))} + Cr^{-\frac{\lambda}{p}} \sum_{k=0}^{\infty} (2^k)^{-n/p} \|f\|_{L_p(B(u,2^{k+1}r))}, \end{aligned}$$

which is

$$r^{-\frac{\lambda}{p}} \| M(f) \|_{L_p(B(u,r))} \le C \sum_{k=0}^{\infty} (2^{\frac{n-\lambda}{p}})^{-k} (2^{k+1}r)^{-\frac{\lambda}{p}} \|f\|_{L_p(B(u,2^{k+1}r))}.$$

Applying the norm $\|\cdot\|_F$ to both side of above inequality, we have

$$\begin{aligned} \left\| r^{-\frac{\lambda}{p}} \| M(f) \|_{L_p(B(u,r))} \right\|_F &\le C \sum_{k=0}^{\infty} (2^{\frac{n-\lambda}{p}})^{-k} \left\| (2^{k+1}r)^{-\frac{\lambda}{p}} \|f\|_{L_p(B(u,2^{k+1}r))} \right\|_F \\ &\le C \sum_{k=0}^{\infty} (2^{\frac{n-\lambda}{p}})^{-k} \|f\|_{M_F(p,\lambda)} \\ &\le C \|f\|_{M_F(p,\lambda)}. \end{aligned}$$

The series above converges because of $\lambda < n$.

Similarly, we can prove the (2).

In Theorem 3.1, if we choose $F = L_\infty(0,\infty)$, we obtain Theorem 1.1.

MORREY 型函数空间与极大函数算子

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[摘　要]本文引入一类 Morrey 型函数空间，并证明其上的极大函数算子是有界的。

[关键词]Morrey 型函数空间　极大函数算子

References

[1]D R Adams. A note on Riesz potentials[J]. Duke Math.,1975,42: 765 -778.

[2]D R Adams. Lectures on L_p-potential theory[C]. Umea U, Report (No. 2), 1981:1 -74.

[3]F Chiarenza and M Frasca. Morrey spaces and Hardy- Littlewood maximal function[J]. Rend. Math. 1987, 7: 273 -279.

[4]V. I. Burenkov, H V Guliyev. Necessary and sufficient conditions for boundedness of the maximal operator in the local Morrey-type spaces[J]. Studia Mathematica, 2004,163(2): 157 -176.

[5]V I Burenkov, H V Guliyev, V S Guliyev, Necessary and sufficient conditions for boundedness of the fractional maximal operators in the local Morrey-type spaces[J]. J. Comput. Appl. Math., 2007, 208(1): 280 -301.

[6]V I Burenkov, V S Guliyev. Necessary and sufficient conditions for the boundedness of the Riesz potential in local Morrey-type spaces[J]. Potential Anal.,2009, 30(3): 211 -249.

[7]V Burenkov, A Gogatishvili, V S Guliyev, R Mustafayev. Boundedness of the fractional maximal operator in local Morrey-type spaces[J]. Complex Var. Elliptic Equ. 2010, 55(8 -10): 739 -758.

[8] V I Burenkov, V S Guliyev, A Serbetci, T V Tararykova. Necessary and sufficient conditions for the boundedness of genuine singular integral operators in local Morrey-type spaces[J]. Eurasian Mathematical Journal, 2010, 1(1): 32 -53.

[9]Ch Feerman and E M Stein. Some maximal inequalities[J]. Amer. J. Math.,1971, 93: 105 -115.

[10]V S Guliyev. Integral operators on function spaces on the homogeneous groups and on domains in R^n. (Russian), Doctor's degree dissertation. Mat. Inst. Steklov, Moscow, 1994.

[11]V S Guliyev, Function spaces, Integral Operators and Two Weighted Inequalities on Homogeneous Groups [C]. Some Applications (Russian), Baku, 1999.

[12]V. S. Guliyev, R. Ch. Mustafayev, Fractional integrals in spaces of functions defined on spaces of homogeneous type(Russian)[J]. Anal. Math. 1998, 24(3): 181 -200.

[13]G Lu. Embedding theorems on Campanato-Morrey spaces for vector fields and applications[J]. C. R. Acad. Sci. Paris, 1995, 320: 429 -434.

[14]C B Morrey, On the solutions of quasi-linear elliptic partial differential equations[J]. Trans. Amer. Math. Soc. 1938, 43: 126 -166.

[15]J Peetre. On the theory of $L^{p\theta,\lambda}$ spaces[J]. Journal Funct. Analysis 1969, 4: 71 -87.

[16]S Spanne. Sur l'interpolation entre les espaces $L_k^{p,\Phi}$[C]. Ann. Schola Norm. Sup. Pisa. 1966, 20: 625 -648.

基于 EViews 拟合 ARMA 模型的注记*

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[摘　要]基于 EViews 计算的 ARMA 模型估计，提出4个常发生误会的注意点。

[关键词]EViews　ARMA 模型　残差 ARMA 模型　SARIMA 模型

1　引　言

在《时间序列分析》课程的教学与应用中，EViews 是首选的软件，ARMA 模型是重点。在理论探索方面，对 ARMA 模型各有各的表述习惯，这不影响理论之发展。在应用中，要结合计算软件的假定才不会出错。下面结合计量经济学软件 EViews，对 ARMA 模型进行四点说明，以免在今后的教学科研少出错。

2　关于 ARMA 模型的注记

设时间序列 $X_t \sim ARMA(p,q)$，王黎明[1]、易丹辉[3]和 Box&Jenkins[6]记为

$$X_t - c - \phi_1 X_{t-1} - \phi_2 X_{t-2} - \cdots - \phi_p X_{t-p} = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots - \theta_q \varepsilon_{t-q} \tag{2.1}$$

简记为 $\Phi(L)X_t = \Theta(L)\varepsilon_t$，$\Phi(z) = 1 - \phi_1 z - \phi_2 z^2 - \cdots - \phi_p z^p$ 自回归多项式，其中 L 为滞后算子 $L^k X_t = X_{t-k}$，$\Theta(z) = 1 - \phi_1 z - \phi_2 z^2 - \cdots - \phi_q z^q$ 为移动平均多项式，$\{\varepsilon_t\}$为白噪声。这样表述有一个好处，即自回归多项式和移动平均多项式是对称的、一致的，但表述为回归模型时，则符号不一致。而且，如果要结合 EViews 中的最小二乘估计法（OLS）应用，符号容易出错。因为 EViews 关于 ma 说明如下

Moving average error specification. The ma specification may be added in an ls or tsls specification to indicate a moving average error component. ma(1) indicates the first order component, ma(2) indicates the second order component, and so on.

Examples

ls m1 c tb3 tb3(-1) ma(1) ma(2)

regresses M1 on a constant, TB3, and TB3 lagged once with first order and second order moving average error components.

EViews 帮助文档的上述例子是说命令“ls m1 c tb3 tb3(-1) ma(1) ma(2)”的功用是估计如下回归式：

$$m1_t = c + c_1 tb3_t + c_2 tb3_{t-1} + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$$

所以，为避免符号出错，应该表述为[2,4,5]：$X_t \sim ARMA(p,q)$，即

* 贵州省科学技术基金资助，项目编号：黔科合J字[2011]2108号。

$$X_t = c + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \cdots + \phi_p X_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q} \quad (2.2)$$

$$X_t = \{c + \phi_1 X_{t-1} + \phi_2 X_{t-2} + \cdots + \phi_p X_{t-p} + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q}\} + \varepsilon_t$$

自回归多项式 $\Phi(z)$ 同上，但移动平均式项式和前面有所不同，应是 $\Theta(z) = 1 + \phi_1 z + \phi_2 z^2 + \cdots + \phi_q z^q$。这样的表述与一般回归模型相同，所以在 EViews 中用 OLS 估计(2.2)式的命令为

$$ls \quad X \quad c \quad ar(1) \quad ar(2) \quad \cdots \quad ar(p) \quad ma(1) \quad ma(2) \quad \cdots \quad ma(q) \quad (2.3)$$

这样(2.3)中 ma(k)即是(2.2)中 θ_k。如果使用模型(2.1)，则(2.3)中的 ma(k) $= -\theta_k$。

(2.3)的实质是对残差 $\hat{u}_t = X_t - c$ 拟合 $ARMA(p,q)$ 模型，这样理解对下文有帮助。

3 拟合 ARMA 时使用 ar(k)，ma(k)

一般地，在 EViews 中拟合线性回归模型 $y \sim x_1, x_2, \cdots, x_p$，其命令如下

$$ls \quad y \quad c \quad x_1 \quad x_2 \quad \cdots \quad x_p \quad (3.1)$$

$X_t \sim ARMA(p,q)$，当 $q=0$ 时称之 $AR(p)$ 模型，记为 $X_t \sim AR(p)$。拟合 $AR(p)$ 的 EViews 命令如下

$$ls \quad X \quad c \quad X(-1) \quad X(-2) \quad \cdots \quad X(-p) \quad (3.2)$$

EViews 中，$X(-k)$ 表示 X_{t-k}. (3.2)式表示拟合线性回归模型

$$X_t = c + \phi_{t-1} X_{t-1} + \phi_{t-2} X_{t-2} + \quad \cdots \quad + \phi_{t-p} X_{t-p}。$$

在 EViews 拟合 AR(P)还可以输入命令

$$ls \quad X \quad c \quad ar(1) \quad ar(2) \quad \cdots \quad ar(p) \quad (3.3)$$

(3.2)和(3.3)有相同的效果。但当 $q>0$ 时，模型未拟合之前 ε_{t-k} 未知，不能像(3.2)一样实名表出，只能用 ma(k)表示 ε_{t-k} 的系数。例如拟合一个 ARMA(3,2)模型时，只能如下设置

$$ls \quad X \quad c \quad ar(1) \quad ar(2) \quad ar(3) \quad ma(1) \quad ma(2) \quad (3.4)$$

总而言之，拟合自回归模型可以(3.2)与(3.3)两式通用，即回归因子可以实名表出，也可用 ar(k)表出，而拟合 ARMA 模型只能如(3.4)使用。(3.3)式的实质是用最小二乘法(OLS)对残差 $X-c$ 拟合一个 AR 模型；(3.4)式的实质是用最小二乘法(OLS)对残差 $X-c$ 拟合一个 ARMA(2,2)模型，不能如(1.2)引用实名变量。

4 残差 ARMA 模型

所谓残差 ARMA 模型，是指拟合一个模型后，其残差是自相关的，是一个 ARMA(p,q)过程，张晓峒[1]称之为“回归与 ARMA 组合模型”，易丹辉[2]称之为“残差序列的 ARMA 模型”，高铁梅[3]称之为“回归扰动项序列相关模型“。其一般形式为

$$y_t = X_t\beta + \Phi^{-1}(L)\Theta(L)\varepsilon_t \quad (4.1)$$

其中$\{\varepsilon_t\}$是白噪声。究其实质，模型(4.1)的回归残差(扰动项)是自相关的，回归残差是一个 ARMA 模型：

$$\Phi(B)(y_t - X_t\beta) = \Theta(B)\varepsilon_t \quad (4.2)$$

分解写法如下：

$$\begin{cases} y_t = X_t\beta + u_t \\ u_t \sim ARMA(p,q) \end{cases} \quad (4.3)$$

依(2.3)含义说明,模型(4.2)的估计方法如下

$$ls\ y\ c\ x_1\ x_2\ \cdots\ x_k\ ar(1)\ \cdots\ ar(p)\ ma(1)\ ma(2)\ \cdots\ ma(q) \tag{4.4}$$

例如,张晓峒[1:310.],中国储蓄存款总额与 GDP 关系研究,做了两步估计:先估计 1),令 $\hat{u}=resid$,根据模型定阶法再对残差估计 2)式

$$1)\ \ln Y_t = -8.865 + 1.7647\ln GDP_t + \hat{u}_t$$

$$P\text{ 值}\quad (<0.000)\quad (<0.000)$$

$$R^2=0.9918, DW=0.23, T=42 (Pr>F)<0.000$$

$$2)\ \hat{u}_t = -0.0094 + 1.1792\hat{u}_{t-1} - 0.3574\hat{u}_{t-2}$$

$$P\text{ 值}\quad (0.5598)\quad (<0.000)\quad (0.0205)$$

$$R^2=0.8113, DW=1.65, (Pr>F)<0.000$$

根据 EViews 中 ar(k)和 ma(k)的定义改写(4.4)式,在 EViews 命令行中输入命令

$$ls\ \ln y\ c\ \ln GDP\ ar(1)\ ar(2)$$

即混合估计了协整方程和残差 ARMA 模型,得到:

$$\ln Y_t = -8.87367 + 1.7445\ln GDP_t + 1.1838\hat{u}_{t-1} - 0.3512\hat{u}_{t-2}$$

$$P\text{ 值}\quad (<0.000)\quad (<0.000)\quad (<0.000)\quad (<0.0273)$$

$$R^2=0.9985, DW=1.6413, T=42, (Pr>F)<0.000$$

当然,为了对回归残差定阶,第一步还是要做的。做第一步的目的是得到回归残差序列 $\hat{u}_t$,然后做回归残差 $\hat{u}_t$ 的 ACF 与 PACF,然后定阶,然后估计回归残差的 ARMA 模型。

5 SARIMA 模型中使用 sar(k)与 sam(k)

在经济领域中,季节性序列是常见的,处理季节性序列用季节 ARIMA 模型(seasonal ARIMA model, SARIMA)。一般描述见张晓峒[2]第 302 页。

$$\Phi_p(L)A_P(L^s)(\Delta^d\Delta_s^D y_t) = \Theta_q(L)B_Q(L^s)\varepsilon_t$$

称之为 $(p,d,q)\times(P,D,Q)$ 乘积季节模型,SARIMA 模型的具体表现为“稀疏系数”的 ARIMA模型,EViews 定义季节系数为 sar(k),sma(k)。例如

$$ls\quad d\log(y,1,12)\quad ar(1)\quad sar(12)\quad ma(1)\quad sma(12)$$

表示要估计乘积季节模型 $(1-\phi_1 L)(1-\alpha_1 L^{12})\Delta\Delta_{12}(\log(y)) = (1+\theta_1 L)(1+\beta_1 L^{12})\varepsilon_t$。而非估计

$$x_t = d\log(y,1,12)$$

$$x_t = \phi_1 x_{t-1} + \phi_{12}x_{t-12} + \theta_1\varepsilon_{t-1} + \theta_{12}\varepsilon_{t-12}$$

6 总　结

Eviews 的最小二乘法编程,是按标准回归式进行的,所以基于 EViews 的最小二乘估计计算要符合标准回归方程书写,才不出错。