普通高等学校“十一五”省级规划教材

理工科核心课程双语规划教材

有限元方法模拟与MSC.AutoForge软件

FEM Simulation and Software MSC.AutoForge

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内 容 简 介

本书适用对象为材料成型及控制工程专业本科生以及无有限单元法学习经历的其他专业学生。内容包含两部分:第一部分介绍有限单元法(FEM)。这部分内容通过对一些最简单的力学问题的求解引入有限元法的基本概念,继而顺序介绍常用的1-D单元、2-D单元和3-D单元等,并讨论它们在结构分析中的应用。第二部分介绍非线性分析软件MSC. AutoFore的功能和用法以及如何用AutoForge分析典型金属塑性成型过程。本教材有针对性地设计了一个综合实验(钢板轧制过程模拟),目的是让学生在AutoForge环境下经历完整的操作过程,熟悉前处理(建模)、求解和后处理步骤及参数设置,为日后运用其他类似软件分析工程实际问题打下基础。

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Preface

In the past decades, the Finite Element Method (FEM) has been developed into a key indispensable technology in the modeling and simulation of various engineering systems. In the development of an advanced engineering system, engineers have to go through a very rigorous process of modeling, simulation, visualization, analysis, designing, prototyping, testing, and finally fabrication/construction. As such, techniques related to modeling and simulation in a rapid and effective way play an increasingly important role in building advanced engineering systems, and therefore, the practical application of FEM has multiplied manifolds in the past years.

This is a core undergraduate course offered in the Department of Materials Forming and Control Engineering, Anhui University of Technology. It was first taught on the Fall Semester 2001 and has been repeated every year since.

"In science there is only physics; all the rest is stamp collecting." (Lord Kelvin) The quote reflects the values of the mid-19th century. Even now, at the dawn of the 21st century, progress and prestige in the natural sciences favors fundamental knowledge. By contrast, engineering knowledge consists of three components: First, conceptual knowledge—understanding the framework of the physical world. Second, operational knowledge—methods and strategies for formulating, analyzing and solving problems, or "which buttons to push". And the last, integral knowledge—the synthesis of conceptual and operational knowledge for technology development.

The language that connects conceptual and operational knowledge is mathematics, and in particular the use of mathematical models. Most engineering programs in the USA correctly emphasize both conceptual and operational components. They differ, however, in how well the two are integrated.

The most successful curricula are those that address the tendency to "disconnection" that bedevils engineering students suddenly exposed to a vast array of subjects.

Integral knowledge is unique to the engineering profession. Synthesis ability is a personal attribute that cannot be coerced, only encouraged and cultivated, the same as the best music programs do not automatically produce Mozarts. Studies indicate no correlation between good engineers and good students. The best that can be done is to provide an adequate (and integrated) base of conceptual and operational knowledge to potentially good engineers. So the objectives of this FEM course are mainly as follows:

○ Understand the fundamental ideas of the FEM;

○ Know the behavior and usage of each type of elements covered in this course;

○ Be able to prepare a suitable FE model for given problems;

○ Can interpret and evaluate the quality of the results (know the physics of the problems);

○ Be aware of the limitations of the FEM (don't misuse the FEM — a numerical tool);

○ Be familiar with the procedure of analysis in MSC. AutoForge system, including pre-processing, solving and postprocessing, especially the preparation of FE model for typical analysis of plate rolling process so as to lay a basis for applying other FE codes to solve practical engineering problems.

This course embodies two parts. Part 1 is intended to introduce the Finite Element Method (FEM) for undergraduate students majoring in materials forming and control engineering or others who have no previous experience with this computational method. The course covers the basic concepts in the FEM using the simplest mechanics problems as examples, and lead to the discussions and applications of the 1-D bar and beam, 2-D plane and 3-D solid elements etc, in the analyses of structures. Part 2 is designed to introduce and show how to use MSC. AutoForge, one of powerful nonlinear analysis software, to simulate a typical process of metal forming both mechanically and thermo-mechanically. The proper usage of the FEM, as a popular numerical tool in engineering, is emphasized throughout the course.

This material is mainly quoted from the online document based on the lecture notes developed by the author, Dr Liu Yijun, since 1997 for the undergraduate course on the FEM in the mechanical engineering department at the University of Cincinnati. The ultimate goal of this e-book on the FEM is to make it readily available for students, researchers and engineers, worldwide, to help them learn subjects in the FEM and eventually solve their own design and analysis problems using the FEM.

Who Must Attend

Students majoring in materials forming and control engineering, and others who are in the areas relating to mechanical engineering, structure and component design, analysis, modeling and simulation.

Prerequisites

The course will be taught at an undergraduate student level. Some background in partial differential equations, matrix algebra, mechanics of engineering, mechanics of plastic working, fundamentals of metal rolling and computer programming will be assumed.

LI Shengzhi
October 20th, 2011

Contents

Chapter 1 Overview

This book is an introduction to the analysis of linear elastic structures by the Finite Element Method (FEM) and Code MSC. Autoforge. This Chapter presents an overview of where the book fits, and what finite elements are.

1.1 Where This Material Fits

The field of Mechanics can be subdivided into three major areas:

$$\text{Mechanics}\begin{cases}\text{Theoretical}\\ \text{Applied}\\ \text{Computational}\end{cases} \tag{1-1}$$

Theoretical mechanics deals with fundamental laws and principles of mechanics studied for their intrinsic scientific value. Applied mechanics transfers this theoretical knowledge to scientific and engineering applications, especially as regards the construction of mathematical models of physical phenomena. Computational mechanics solves specific problems by simulation through numerical methods implemented on digital computers.

REMARK

Paraphrasing an old joke about mathematicians. One may define a computational mechanician as a person who searches for solutions to given problems, an applied mechanician as a person who searches for problems that fit given solutions, and a theoretical mechanician as a person who can prove the existence of problems and solutions.

1.1.1 Computational Mechanics

Several branches of computational mechanics can be distinguished according to the physical scale of the focus of attention:

Computational Mechanics

$$\begin{cases}\text{Nanomechanics and micromechanics}\\ \text{Continuum mechanics}\begin{cases}\text{Solids and Structures}\\ \text{Fluids}\\ \text{Multi physics}\end{cases}\\ \text{Systems}\end{cases} \tag{1-2}$$

Nanomechanics deals with phenomena at the molecular and atomic levels of matter. As such it is closely interrelated with particle physics and chemistry. Micromechanics looks primarily at the crystallographic and granular levels of matter. Its main technological application is the design and fabrication of materials and microdevices.

Continuum mechanics studies bodies at the macroscopic level, using continuum models in which the microstructure is homogenized by phenomenological averages. The two traditional areas of application are solid and fluid mechanics. The former includes structures which, for obvious reasons, are fabricated with solids. Computational solid mechanics takes an applied-sciences approach, whereas computational structural mechanics emphasizes technological applications to the analysis and design of structures.

Computational fluid mechanics deals with problems that involve the equilibrium and motion of liquid and gases. Well developed related areas are hydrodynamics, aerodynamics, atmospheric physics, and combustion. Multiphysics is a more recent newcomer. This area is meant to include mechanical systems that transcend the classical boundaries of solid and fluid mechanics, as in interacting fluids and structures. Phase change problems such as ice melting and metal solidification fit into this category, as do the interaction of control, mechanical and electromagnetic systems.

Finally, system identifies mechanical objects, whether natural or artificial, that performs a distinguishable function. Examples of man-made systems are airplanes, buildings, bridges, engines, cars, microchips, radio telescopes, robots, roller skates and garden sprinklers. Biological systems, such as a whale, ameba or pine tree are included if studied from the viewpoint of biomechanics. Ecological, astronomical and cosmological entities also form systems. ①

In this progression of (1-2) the system is the most general concept. A system is studied by decomposition: its behavior is that of its components plus the

① Except that their function may not be clear to us. "The usual approach of science of constructing a mathematical model cannot answer the questions of why there should be a universe for the model to describe. Why does the universe go to all the bother of existing?" (Stephen Hawking).

interaction between the components. Components are broken down into subcomponents and so on. As this hierarchical process continues the individual components become simple enough to be treated by individual disciplines, but their interactions may get more complex. Consequently, there is a trade off art in deciding where to stop. ①

1.1.2 Statics vs. Dynamics

Continuum mechanics problems may be subdivided according to whether inertial effects are taken into account or not:

$$\text{Continuum mechanics} \begin{cases} \text{Statics} \\ \text{Dynamics} \end{cases} \tag{1-3}$$

In dynamics the time dependence is explicitly considered because the calculation of inertial (and/or damping) forces requires derivatives respect to actual time to be taken.

Problems in statics may also be time dependent but the inertial forces are ignored or neglected.

Static problems may be classified into strictly static and quasi-static. For the former time need not be considered explicitly; any historical time-like response-ordering parameter (if one is needed) will do. In quasi-static problems such as foundation settlement, creep deformation, rate-dependent plasticity or fatigue cycling, a more realistic estimation of time is required but inertial forces are still neglected.

1.1.3 Linear vs. Nonlinear

A classification of static problems that is particularly relevant to this book is

$$\text{Statics} \begin{cases} \text{Linear} \\ \text{Nonlinear} \end{cases}$$

Linear static analysis deals with static problems in which the response is linear in the cause-and-effect sense. For example: if the applied forces are doubled, the displacements and internal stresses also double. Problems outside this domain are classified as nonlinear.

① Thus in breaking down a car engine, say, the decomposition does not usually proceed beyond the components you can buy at a parts shop.

1.1.4 Discretization Methods

A final classification of CSM static analysis is based on the discretization method by which the continuum mathematical model is discretized in space, i.e., converted to a discrete model of finite number of degrees of freedom:

$$\text{Spatial discretization method}\begin{cases}\text{Finite Element Method (FEM)}\\ \text{Boundary Element Method (BEM)}\\ \text{Finite Difference Method (FDM)}\\ \text{Finite Volume Method (FVM)}\\ \text{Spectral Method}\\ \text{Mesh-Free Method}\end{cases}\qquad (1\text{-}4)$$

For linear problems finite element methods currently dominate the scene, with boundary element methods posting a strong second choice in specific application areas. For nonlinear problems the dominance of finite element methods is overwhelming.

Classical finite difference methods in solid and structural mechanics have virtually disappeared from practical use. This statement is not true, however, for fluid mechanics, where finite difference discretization methods are still important. Finite-volume methods, which address finite volume method conservation laws, are important in highly nonlinear problems of fluid mechanics. Spectral methods are based on transforms that map space and/or time dimensions to spaces where the problem is easier to solve.

A recent newcomer to the scene is the mesh-free method. It is finite different method on arbitrary grids constructed through a subset of finite element techniques and tools.

1.1.5 FEM Variants

The term Finite Element Method actually identifies a broad spectrum of techniques that share common features outlined in (1-3) and (1-4). Two subclassifications that fit well applications to structural mechanics are

$$\text{FEM Formulation}\begin{cases}\text{Displacement}\\ \text{Equilibrium}\\ \text{Mixed}\\ \text{Hybrid}\end{cases}\qquad \text{FEM Solution}\begin{cases}\text{Stiffness}\\ \text{Flexibility}\\ \text{Mixed (a.k.a. Combined)}\end{cases}\qquad (1\text{-}5)$$

The distinction between these subclasses requires advanced technical concepts, and will not be covered here.

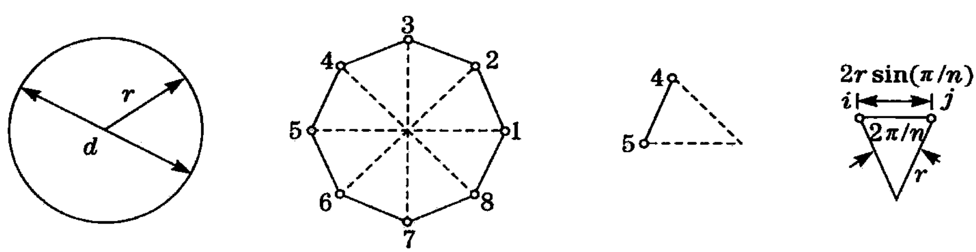
Using the foregoing classification, we can state the topic of this book more precisely: the computational analysis of linear static structural problems by the Finite Element Method. Of the variants listed in (1-5), emphasis is placed on the displacement formulation and stiffness solution. This combination is called the Direct Stiffness Method or DSM.

1.2 What Does a Finite Element Look Like

The finite element method (FEM), or finite element analysis (FEA), is based on the idea of building a complicated object with simple blocks, or, dividing a complicated object into small and manageable pieces. Application of this simple idea can be found everywhere in everyday life, as well as in engineering.

The subject of this book is FEM. But what is a finite element? The concept will be partly illustrated through a truly ancient problem: find the perimeter L of a circle of diameter d. Since $L = \pi d$, this is equivalent to obtaining a numerical value for π.

Draw a circle of radius r and diameter $d = 2r$ as in Fig. 1-1(a). Inscribe a regular polygon of n sides, where $n = 8$ in Fig. 1-1(b). Rename polygon sides as elements and vertices as nodal points or nodes. Label nodes with integers 1, ..., 8. Extract a typical element, say that joining nodes 4~5 as shown in Fig. 1-1(c). This is an instance of the generic element i - j shown in Fig. 1-1(d).



(a) continuum object (b) a discrete approximation (inscribed regular polygon) (c) disconnected element (d) generic element

Fig. 1-1 The "find π" problem treated with FEM concepts

The element length is $L_{ij} = 2r \sin(\pi/n)$. Since all elements have the same length, the polygon perimeter is $Ln = nL_{ij}$, whence the approximation to π is $\pi_n = Ln/d = n\sin(\pi/n)$.

Values of π_n obtained for $n = 1, 2, 4, \cdots, 256$ are listed in the second column of Table 1-1. As can be seen the convergence to π is fairly slow. However, the sequence can be transformed by Wynn's ε algorithm① into that shown in the third column. The last value displays 15-place accuracy.

Table 1-1 Rectification of Circle by Inscribed Polygons (Archimedes FEM)

n	$\pi_n = n \sin(\pi/n)$	Extrapolated by Wynn-ε	Exact π to 16 places
1	0.000 000 000 000 000		
2	2.000 000 000 000 000		
4	2.828 427 124 746 190	3.414 213 562 373 096	
8	3.061 467 458 920 718		
16	3.121 445 152 258 052	3.141 418 327 933 211	
32	3.136 548 490 545 939		
64	3.140 331 156 954 753	3.141 592 658 918 053	
128	3.141 277 250 932 773		
256	3.141 513 801 144 301	3.141 592 653 589 786	3.141 592 653 589 793

Some of the key ideas behind the FEM can be identified in this simple example. The circle, viewed as a source mathematical object, is replaced by polygons. These are discrete approximations to the circle. The sides, renamed as elements, are specified by their end nodes. Elements can be separated by disconnecting the nodes, a process called disassembly in the FEM. Upon disassembly a generic element can be defined, independently of the original circle, by the segment that connects two nodes i and j. The relevant element property: length L_{ij}, can be computed in the generic element independently of the others, a property called local support in the FEM. Finally, the desired property: the polygon perimeter, is obtained by reconnecting n elements and adding up their length; the corresponding steps in the FEM being assembly and solution, respec-

① A widely used extrapolation algorithm that speeds up the convergence of many sequences. See: Wimp J. Sequence Transformations and Their Applications [M]. New York: Academic Press, 1981.

tively. There is of course nothing magic about the circle; the same technique can be used to rectify any smooth plane curve. ①

This example has been offered in the FEM literature to adduce that finite element ideas can be traced to Egyptian mathematicians from circa 1 800 BC, as well as Archimedes' famous studies on circle rectification by 250 BC. But comparison with the modern FEM, as covered in Chapters 2~3, shows this to be a stretch. The example does not illustrate the concept of degrees of freedom, conjugate quantities and local-global coordinates. It is guilty of circular reasoning: the compact formula $\pi = \lim_{n\to\infty} n \sin(\pi/n)$ uses the unknown π in the right hand side. ②Reasonable people would argue that a circle is a simpler object than, say, a 128-sided polygon. Despite these flaws the example is useful in one respect: showing a fielder's choice in the replacement of one mathematical object by another. This is at the root of the simulation process described in the next section.

1.3 The FEM Analysis Process

A model-based simulation process using FEM involves doing a sequence of steps. This sequence takes two canonical configurations depending on the environment in which FEM is used. These are reviewed next to introduce terminology.

In Fig. 1-2, the mathematical model (top) is the source of the simulation process. Discrete model and solution follow from it. The ideal physical system (should one go to the trouble of exhibiting it) is inessential.

1.3.1 The Mathematical FEM

The process steps are illustrated in Fig. 1-2. The process centerpiece, from which everything emanates, is the mathematical model. This is often an ordinary or partial differential equation in space and time. A discrete finite element model is generated from a variational or weak form of the mathematical model. ③

① A similar limit process, however, may fail in three or more dimensions.

② This objection is bypassed if n is advanced as a power of two, as in Table 1-1, by using the half-angle recursion $\sqrt{2}\sin\alpha = \sqrt{1-\sqrt{1-\sin^2 2\alpha}}$, started from $2\alpha = \pi$ for which $\sin\pi = -1$.

③ The distinction between strong, weak and variational forms is discussed in advanced FEM courses. In the present course such forms will be stated as recipes.

This is the discretization step. The FEM equations are processed by an equation solver, which delivers a discrete solution(or solutions).

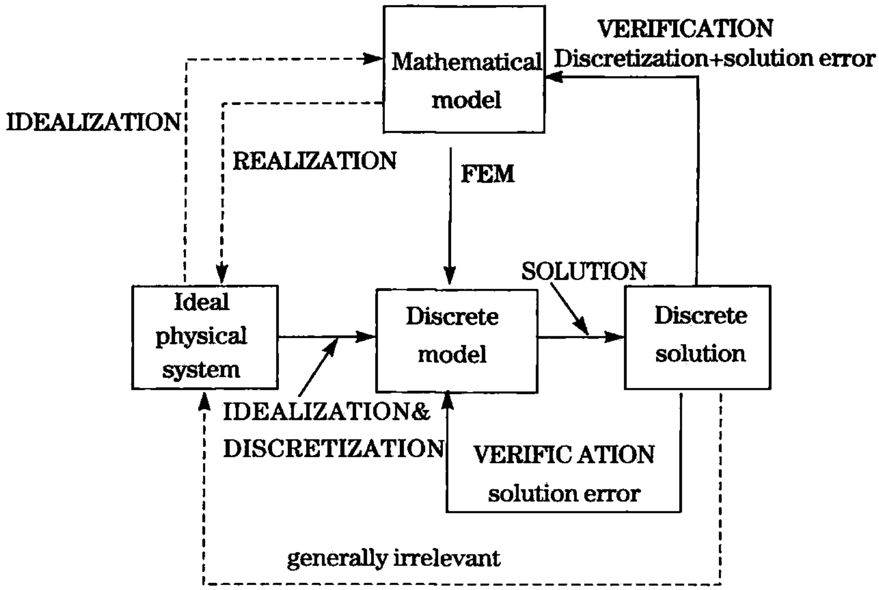


Fig. 1-2 The Mathematical FEM

On the left Fig. 1-2 shows an ideal physical system. This may be presented as a realization of the mathematical model; conversely, the mathematical model is said to be an idealization of this system. For example, if the mathematical model is the Poisson's equation, realizations may be a heat conduction or a electrostatic charge distribution problem. This step is inessential and may be left out. Indeed FEM discretizations may be constructed without any reference to physics.

The concept of error arises when the discrete solution is substituted in the "model" boxes. This replacement is generically called verification. The solution error is the amount by which the discrete solution fails to satisfy the discrete equations. This error is relatively unimportant when using computers, and in particular direct linear equation solvers, for the solution step. More relevant is the discretization error, which is the amount by which the discrete solution fails to satisfy the mathematical model.① Replacing into the ideal physical system would in principle quantify modeling errors. In the mathematical FEM this is largely irrelevant, however, because the ideal physical system is merely that: a figment of the imagination.

1.3.2 The Physical FEM

The second way of using FEM is the process illustrated in Fig. 1-3. The centerpiece is now the physical system to be modeled. Accordingly, this sequence is called the Physical FEM. The processes of idealization and discretization are carried out concurrently to produce the discrete model. The solution is computed as before.

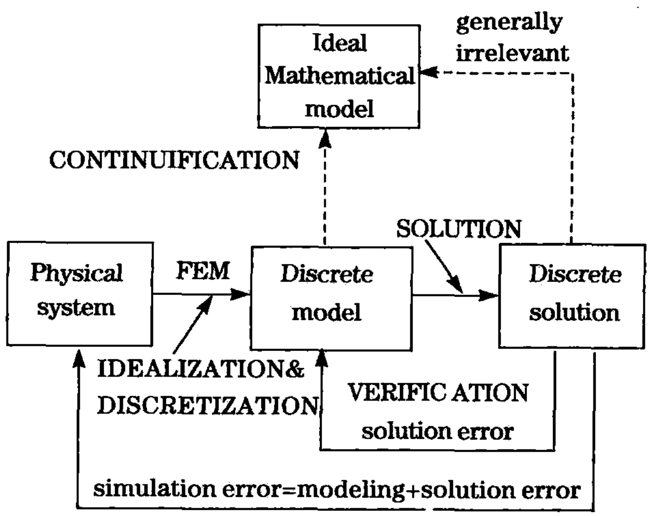


Fig. 1-3 The Physical FEM

In Fig. 1-3 is the Physical FEM. The physical system (left) is the source of the simulation process. Discrete model and solution follow from it. The ideal mathematical model (should one go to the trouble of exhibiting it) is inessential.

Just like Fig. 1-2 shows an ideal physical system, Fig. 1-3 depicts anideal mathematical model. This may be presented as a continuum limit or "continuifi-

① This error can be computed in several ways, the details of which are of no importance here.