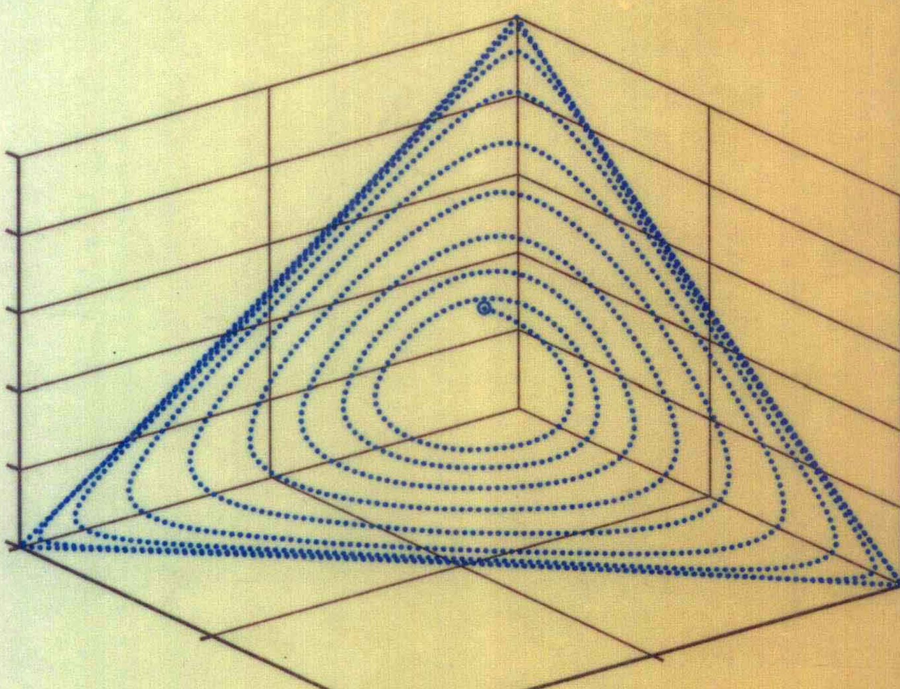
Undergraduate Texts in Mathematics

Saber Elaydi

AN INTRODUCTION TO DIFFERENCE EQUATIONS

Third Edition

差分方程导论 第3版



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Saber Elaydi
Department of Mathematics
Trinity University
San Antonio, Texas 78212
USA

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Preface to the Third Edition

In contemplating the third edition, I have had multiple objectives to achieve. The first and foremost important objective is to maintain the accessibility and readability of the book to a broad readership with varying mathematical backgrounds and sophistication. More proofs, more graphs, more explanations, and more applications are provided in this edition.

The second objective is to update the contents of the book so that the reader stays abreast of new developments in this vital area of mathematics. Recent results on local and global stability of one-dimensional maps are included in Chapters 1, 4, and Appendices A and C. An extension of the Hartman–Grobman Theorem to noninvertible maps is stated in Appendix D. A whole new section on various notions of the asymptoticity of solutions and a recent extension of Perron's Second Theorem are added to Chapter 8. In Appendix E a detailed proof of the Levin–May Theorem is presented. In Chapters 4 and 5, the reader will find the latest results on the larval–pupal–adult flour beetle model.

The third and final objective is to better serve the broad readership of this book by including most, but certainly not all, of the research areas in difference equations. As more work is being published in the *Journal of Difference Equations and Applications* and elsewhere, it became apparent that a whole chapter needed to be dedicated to this enterprise. With the prodding and encouragement of Gerry Ladas, the new Chapter 5 was born. Major revisions of this chapter were made by Fozi Dannan, who diligently and painstakingly rewrote part of the material and caught several errors and typos. His impact on this edition, particularly in Chapters 1, 4, and Chapter 8 is immeasurable and I am greatly indebted to him. My thanks go to Shandelle Henson, who wrote a thorough review of the book and suggested the inclusion of an extension of the Hartman–Groman Theorem, and to Julio Lopez and his student Alex Sepulveda for their comments and discussions about the second edition.

I am grateful to all the participants of the AbiTuMath Program and to its coordinator Andreas Ruffing for using the second edition as the main reference in their activities and for their valuable comments and discussions. Special thanks go to Sebastian Pancratz of AbiTuMath whose suggestions improved parts of Chapters 1 and 2. I benefited from comments and discussions with Raghib Abu-Saris, Bernd Aulbach, Martin Bohner, Luis Carvahlo, Jim Cushing, Malgorzata Guzowska, Sophia Jang, Klara Janglajew, Nader Kouhestani, Ulrich Krause, Ronald Mickens, Robert Sacker, Hassan Sedaghat, and Abdul-Aziz Yakubu. It is a pleasure to thank Ina Lindemann, the editor at Springer-Verlag for her advice and support during the writing of this edition. Finally, I would like to express my deep appreciation to Denise Wilson who spent many weekends typing various drafts of the manuscript. Not only did she correct many glitches, typos, and awkward sentences, but she even caught some mathematical errors.

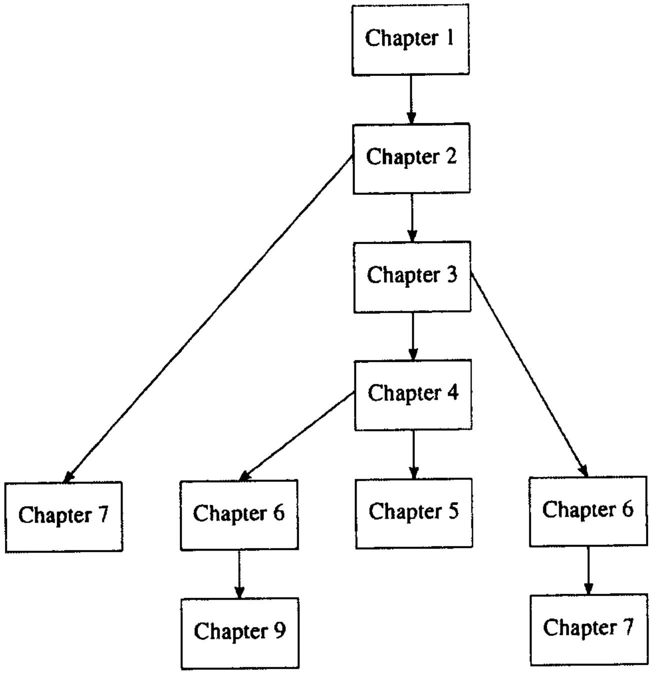
I hope you enjoy this edition and if you have any comments or questions, please do not hesitate to contact me at selaydi@trinity.edu.

San Antonio, Texas Saber N. Elaydi
April 2004

Suggestions for instructors using this book.

The book may be used for two one-semester courses. A first course may include one of the following options but should include the bulk of the first four chapters:

1. If one is mainly interested in stability theory, then the choice would be Chapters 1–5.
2. One may choose Chapters 1–4, and Chapter 8 if the interest is to get to asymptotic theory.
3. Those interested in oscillation theory may choose Chapters 1, 2, 3, 5, and 7.
4. A course emphasizing control theory may include Chapters 1–3, 6, and 10.



The diagram above depicts the dependency among the chapters.

Preface to the Second Edition

The second edition has greatly benefited from a sizable number of comments and suggestions I received from users of the first edition. I hope that I have corrected all the errors and misprints in the book. Important revisions were made in Chapters 1 and 4. In Chapter 1, I added two appendices (Global Stability and Periodic Solutions). In Chapter 4, I added a section on applications to mathematical biology. Influenced by a friendly and some not so friendly comments about Chapter 8 (previously Chapter 7: Asymptotic Behavior of Difference Equations), I rewrote the chapter with additional material on Birkhoff's theory. Also, due to popular demand, a new chapter (Chapter 9) under the title "Applications to Continued Fractions and Orthogonal Polynomials" has been added. This chapter gives a rather thorough presentation of continued fractions and orthogonal polynomials and their intimate connection to second-order difference equations. Chapter 8 (Oscillation Theory) has now become Chapter 7. Accordingly, the new revised suggestions for using the text are as follows.

The book may be used with considerable flexibility. For a one-semester course, one may choose one of the following options:

(i) If you want a course that emphasizes stability and control, then you may select Chapters 1, 2, and 3, and parts of Chapters 4, 5, and 6. This is perhaps appropriate for a class populated by mathematics, physics, and engineering majors.

(ii) If the focus is on the applications of difference equations to orthogonal polynomials and continued fractions, then you may select Chapters 1, 2, 3, 8, and 9.

I am indebted to K. Janglajew, who used the book several times and caught numerous glitches and typos. I am very grateful to Julio Lopez and his students, who helped me correct some mistakes and improve the exposition in Chapters 7 and 8. I am thankful to Raghib Abu-Saris, who caught some errors and typos in Chapter 4. My thanks go to Gerry Ladas, who assisted in refining Chapter 8, and to Allan Peterson, who graciously used my book and caught some mistakes in Chapter 4. I thank my brother Hatem Elaydi who read thoroughly Chapter 6 and made valuable revisions in the exercises. Many thanks to Fozi Dannan, whose comments improved Chapters 1, 4, and 9. Ronald Mickens was always there for me when I needed support, encouragement, and advice. His impact on this edition is immeasurable. My special thanks to Jenny Wolkowicki of Springer-Verlag.

I apologize in advance to all those whom I did not mention here but who have helped in one way or another to enhance the quality of this edition.

It is my pleasure to thank my former secretary, Constance Garcia, who typed the new and revised material.

San Antonio, Texas
April 1999

Saber N. Elaydi

Preface to the First Edition

This book grew out of lecture notes I used in a course on difference equations that I have taught at Trinity University for the past five years. The classes were largely populated by juniors and seniors majoring in mathematics, engineering, chemistry, computer science, and physics.

This book is intended to be used as a textbook for a course on difference equations at both the advanced undergraduate and beginning graduate levels. It may also be used as a supplement for engineering courses on discrete systems and control theory.

The main prerequisites for most of the material in this book are calculus and linear algebra. However, some topics in later chapters may require some rudiments of advanced calculus and complex analysis. Since many of the chapters in the book are independent, the instructor has great flexibility in choosing topics for a one-semester course.

This book presents the current state of affairs in many areas such as stability, Z-transform, asymptoticity, oscillations, and control theory. However, this book is by no means encyclopedic and does not contain many important topics, such as numerical analysis, combinatorics, special functions and orthogonal polynomials, boundary value problems, partial difference equations, chaos theory, and fractals. The nonselection of these topics is dictated not only by the limitations imposed by the elementary nature of this book, but also by the research interest (or lack thereof) of the author.

Great efforts were made to present even the most difficult material in an elementary format and to write in a style that makes the book accessible to students with varying backgrounds and interests. One of the main features of the book is the inclusion of a great number of applications in economics, social sciences, biology, physics, engineering, neural networks, etc. Moreover, this book contains a very extensive and carefully selected set of exercises at the end of each section. The exercises form an integral part of the text. They range from routine problems designed to build basic skills to more challenging problems that produce deeper understanding and build technique. The asterisked problems are the most challenging, and the instructor may assign them as long-term projects. Another important feature of the book is that it encourages students to make mathematical discoveries through calculator/computer experimentation.

Chapter 1 deals with first-order difference equations, or one-dimensional maps on the real line. It includes a thorough and complete analysis of stability for many popular maps (equations) such as the logistic map, the tent map, and the Baker map. The rudiments of bifurcation and chaos theory are also included in Section 1.6. This section raises more questions and gives few answers. It is intended to arouse the reader's interest in this exciting field.

In Chapter 2 we give solution methods for linear difference equations of any order. Then we apply the obtained results to investigate the stability and the oscillatory behavior of second-order difference equations. At the end of the chapter we give four applications: the propagation of annual plants, the gambler's ruin, the national income, and the transmission of information.

Chapter 3 extends the study in Chapter 2 to systems of difference equations. We introduce two methods to evaluate A^n for any matrix A. In Section 3.1 we introduce the Putzer algorithm, and in Section 3.3 the method of the Jordan form is given. Many applications are then given in Section 3.5, which include Markov chains, trade models, and the heat equation.

Chapter 4 investigates the question of stability for both scalar equations and systems. Stability of nonlinear equations is studied via linearization (Section 4.5) and by the famous method of Liapunov (Section 4.6). Our exposition here is restricted to autonomous (time-invariant) systems. I believe that the extension of the theory to nonautonomous (time-variant) systems, though technically involved, will not add much more understanding to the subject matter.

Chapter 5 delves deeply into Z-transform theory and techniques (Sections 5.1, 5.2). Then the results are applied to study the stability of Volterra difference scalar equations (Sections 5.3, 5.4) and systems (Sections 5.5, 5.6). For readers familiar with differential equations, Section 5.7 provides a comparison between the Z-transform and the Laplace transform. Most of the results on Volterra difference equations appear here for the first time in a book.

Chapter 6 takes us to the realm of control theory. Here, we cover most of the basic concepts including controllability, observability, observers, and stabilizability by feedback. Again, we restrict the presentation to autonomous (time-invariant) systems, since this is just an introduction to this vast and growing discipline. Moreover, most practitioners deal mainly with time-invariant systems.

In Chapter 7 we give a comprehensive and accessible study of asymptotic methods for difference equations. Starting from the Poincaré Theorem, the chapter covers most of the recent development in the subject. Section 7.4 (asymptotically diagonal systems) presents an extension of Levinson's Theorem to difference equations, while in Section 7.5 we carry our study to nonlinear difference equations. Several open problems are given that would serve as topics for research projects.

Finally, Chapter 8 presents a brief introduction to oscillation theory. In Section 8.1, the basic results on oscillation for three-term linear difference equations are introduced. Extension of these results to nonlinear difference equations is presented in Section 8.2. Another approach to oscillation theory, for self-adjoint equations, is presented in Section 8.3. Here we also introduce a discrete version of Sturm's Separation Theorem.

I am indebted to Gerry Ladas, who read many parts of the book and suggested many useful improvements, especially within the section on stability of scalar difference equations (Section 4.3). His influence through papers and lectures on Chapter 8 (oscillation theory) is immeasurable. My thanks go to Vlajko Kocic, who thoroughly read and made many helpful comments about Chapter 4 on Stability. Jim McDonald revised the chapters on the Z-transform and control theory (Chapters 5 and 6) and made significant improvements. I am very grateful to him for his contributions to this book. My sincere thanks go to Paul Eloe, who read the entire manuscript and offered valuable suggestions that led to many improvements in the final draft of the book. I am also grateful to Istvan Gyori for his comments on Chapter 8 and to Ronald Mickens for his review of the whole manuscript and for his advice and support. I would like to thank the following mathematicians who encouraged and helped me in numerous ways during the preparation of the book: Allan Peterson, Donald Bailey, Roberto Hasfura, Haydar Akca, and Shunian Zhang. I am grateful to my students Jeff Bator, Michelle MacArthur, and Nhung Tran, who caught misprints and mistakes in the earlier drafts of this book. My special thanks are due to my student Julie Lundquist, who proofread most of the book and made improvements in the presentation of many topics. My thanks go to Constance Garcia, who skillfully typed the entire manuscript with its many, many revised versions. And finally, it is a pleasure to thank Ina Lindemann and Robert Wexler from Springer-Verlag for their enthusiastic support of this project.

San Antonio, Texas
December 1995

Saber N. Elaydi

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