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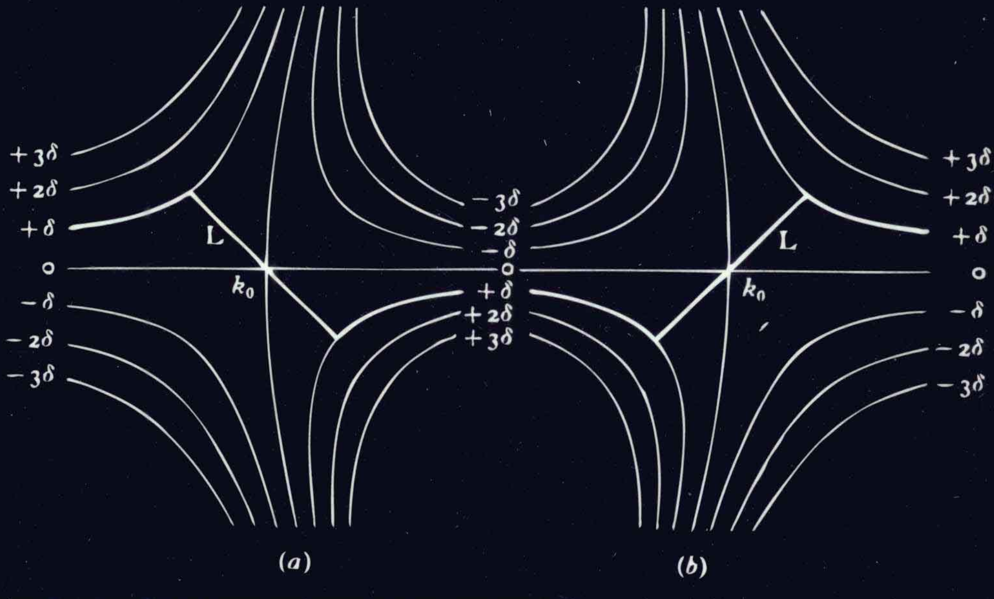
Waves in Fluids



流体中的波

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WAVES IN FLUIDS

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WAVES IN FLUIDS

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Waves in Fluids, 1st ed. (978-0-521-01045-0) by James Lighthill first published by Cambridge University Press 1978

PREFACE

The aims of this book are set out in the prologue. The main subject matter is developed in chapters 1–4. Several further topics are sketched briefly in the epilogue.

Although no references are included in the text, an annotated bibliography is designed to take the reader through the book's subject matter, indicating where he or she can read more about each topic mentioned. This is followed by a notation list showing the meanings of the principal symbols used.

Pages 470 to 486, which constitute the bibliography, have subsidiary page designations *A* to *Q*, which are used for bibliographical references throughout the Author Index and Subject Index.

Within each chapter, mathematical equations are numbered consecutively: (1), (2), (3), etc. The numbering then ***begins again from*** (1) in the next chapter, or in the epilogue. When, in any chapter, we refer to a numbered equation, we mean the equation of that number ***in the same chapter***.

By contrast, figures are numbered continuously (from 1 to 117) throughout the book. Exercises for the reader are given at the end of each chapter.

Cambridge JAMES LIGHTHILL
1978

PROLOGUE

This book is designed as a comprehensive introduction to the science of wave motions in fluids (that is, in liquids and gases); an area of knowledge which forms an essential part of the dynamics of fluids, as well as a significant part of general wave science; and, also, has important applications to the sciences of the environment and of engineering. The subject's extent and variety are enormous: the different types of waves in fluids, the different fundamental ideas that have been developed to interpret their properties, and the different applications of those properties are so extensive that a comprehensive introduction in one volume demands very careful selection.

The design adopted for the book, in four chapters and an epilogue, has two principal aims. First, as the chapter titles suggest, it allows an analysis in depth of four important and representative types of waves in fluids (sound waves; one-dimensional waves in fluids; water waves; internal waves) to precede brief descriptions of some other important types in the epilogue. At the same time, the subject matter of the four chapters is chosen so that, as far as possible, all the most generally useful fundamental ideas of the science of waves in fluids can be developed at length, one after another. The main exceptions are certain very difficult, advanced ideas which could not, even in a comprehensive introduction, be treated so fully; they are merely sketched, with references to more extensive treatments, in the epilogue.

Thus, each chapter is designed *both* to analyse the main type of wave system named in its title *and* to develop an important body of fundamental ideas of general application to waves in fluids. The ideas developed in each chapter are especially important for the wave system of that chapter, but are applicable to wave science generally, and to other systems of waves in fluids in particular. Therefore, later chapters include some applications of the ideas they have developed to the wave systems of earlier chapters; as when methods developed in chapter 2 are used to analyse the generation and propagation of supersonic booms, or when methods developed in chapter 4 are used to analyse the wavemaking resistance of ships.

Practical applications are continually indicated: to noise-abatement research in chapters 1 and 2, to areas as diverse as hydraulics and circulation physiology in chapter 2, to oceanography and ocean exploitation in chapters 3 and 4, and to numerous parts of atmospheric science in chapter 4. Some even more diverse applications are indicated in the epilogue.

Within wave science as a whole, the nature of waves in fluids is characterised especially by their ability to interact with complex fluid flow fields. Such interactions are therefore described at length in this book (see, in particular, sections 1.10, 2.14, 3.9, 3.10, 4.6, 4.7 and 4.12).

Some ideas of fundamental importance treated early in chapter 1 are the property of linearity (direct linear superposability of different wave motions); the concept of energy transport by waves; and the differing character of propagation in one, two and three dimensions. Next, two quite distinct sets of ideas (complementary of their use) are developed: (i) for sources small in comparison with the length of the waves generated ('compact sources'), and (ii) for fluid systems on a scale large compared with the wavelength; both are applied to noise-source problems. In later chapters, both sets of ideas are taken still further; see especially section 4.9 for compact sources, and section 4.5 for the general ray-tracing technique applicable to systems with properties varying gradually on a scale of wavelengths.

In the meantime, sections 1.11 and 1.12 describe an intermediate régime. Finally, chapter 1, like every chapter in the book, includes an account of processes associated with wave *attenuation*: processes involving dissipation in the body of the fluid (section 1.13); or dissipation near either a solid boundary (section 2.7), or a free boundary (section 3.5); or the generation of steady streaming motions by wave attenuation (section 4.7).

Chapter 2 resumes in detail the theme of one-dimensional propagation, and shows how a common treatment is possible for a wide range of seemingly quite different systems; including propagation of sound in ducts, of the blood pulse in arteries, and of 'long' water waves in open channels. Fundamental ideas treated in the first half of chapter 2 include (i) the different effects on a wave of *discontinuous* or *gradual* changes in the properties of the containing tube or channel, (ii) the application of that knowledge to propagation in branching systems, and (iii) the study of a variety of types of resonance which can occur.

The second half of chapter 2 gives an extended treatment of nonlinear effects; and, in particular, of those effects which generate a local steepening of waveforms. Shock waves and other essentially discontinuous waves involving a balance between steepening and dissipative effects are described at length, and methods are outlined for tracing the development of complex signals including shock waves. Finally, one-dimensional nonlinear theory is applied to the propagation of signals along those abstract 'ray tubes' whose properties were first encountered in chapter 1.

The subject of water waves, broached in chapter 2 only as far as 'long' waves (waves of length far exceeding the water depth) and their associated discontinuities (the 'hydraulic jumps') are concerned, is pursued much further in chapter 3. Beyond the basic dynamics of surface waves with gravity or surface tension as the restoring force, chapter 3 is concerned to introduce also the special properties of 'dispersive' waves. The fundamental distinction between phase velocity and group velocity is developed for general dispersive systems that are isotropic (with propagation properties independent of direction, although varying with wavelength). The subject is approached from three complementary standpoints (sections 3.6–3.8) and then applied to the analysis of surface waves generated by storms, or by obstacles in a stream, or by the motion of a ship through water.

Similarly, chapter 4 is primarily concerned to explain wave dispersion in systems that are *not* isotropic, including those internal gravity waves in stratified fluids which give the chapter its title. These are systems for which the group velocity and the phase velocity may be quite different in direction as well as in magnitude, with many important consequences. Several other fundamental ideas are also treated at length: 'trapped waves', caustics, wave-flow interaction, travelling wave sources in general and, finally, waveguides.

On the other hand, two groups of fundamental ideas of a higher order of difficulty are postponed to the epilogue, where indeed they are only sketched (with references). These include theories of the interaction between dispersive effects and nonlinear effects, and theories of the development of statistical assemblages of waves through nonlinear interactions.

Readers approaching this book are likely to possess basic knowledge of *dynamics*, including the elementary dynamics of vibrations, and the elementary dynamics of fluids. However, for these and for all other matters on which prior knowledge may be desirable, a selection of suitable texts is suggested in the bibliography.

The science of waves in fluids is here approached quantitatively, and with the aim of outlining, wherever possible, techniques of quantitative analysis. Subject to this aim, however, the extent of mathematical development has been kept to a minimum. No mathematics has been included for its own sake; furthermore, all the mathematical analyses which have been included have to the maximum possible extent been given clear physical interpretations. Nevertheless, the subject of waves seems to demand the use of complex variables; accordingly, the elementary theory of functions of a complex variable, and (for similar reasons) the elementary properties of Fourier integrals, are among the mathematical knowledge which our readers are either assumed to possess or else (perhaps) assisted to acquire through direction to suitable texts.

The most fundamental waves in fluids are sound waves (chapter 1), because they can exist in a fluid without any external force field needing to be present. Readers familiar with the elementary dynamics of vibrations know that a wave or any other vibrating system involves a balance between a restoring force and the inertia of the system. Most of the waves treated in this book involve external restoring forces; especially gravity (chapters 2, 3 and 4) but also surface tension (section 3.4) or tube elasticity (section 2.2). Other external forces, important for wave systems treated in the epilogue, include magnetic force fields and the Coriolis force felt by rotating fluids.

Sound waves propagate, however, independently of external forces. The restoring force balancing the fluid's inertia is provided entirely by the fluid's own compressibility. Because the compressibility properties of the fluid are the same in all directions, sound propagation is isotropic.

By contrast, most wave motions due to an *external* restoring force are anisotropic, and this is why the general theory of anisotropic propagation given in chapter 4 is so important. Waves on a horizontal water surface are an exception, because they are subject only to two-dimensional propagation in horizontal directions; and, evidently, a vertical external force such as gravity can make no distinction between different horizontal directions. On the other hand, when the source of those waves is a moving ship, they are made effectively anisotropic by the Doppler effect (section 4.12).

Those extensive parts of the dynamics of fluids that are strongly influenced by the properties of waves in fluids include many large and important fields of current research. These are found, for example, in modern aeronautical engineering, and other branches of engineering where flow noise is important, and in those parts of naval architecture and offshore-structure technology that interact with the wave properties of the sea surface.

Such research fields include, furthermore, the analysis of tides and surges in oceans and seas and estuaries; and the study of numerous ocean-current patterns of a wavelike nature. They include the analysis of several atmospheric propagation phenomena of great importance, from small-scale 'clear air turbulence' to large-scale wavelike wind patterns; and many properties of the air–sea interaction. Other active areas of geophysical study include wave propagation in the ionosphere, and in the liquid core of the earth, while astrophysical observations constantly reveal wavelike gaseous motions suitable for analysis by similar methods. This book, designed as a comprehensive introduction to waves in fluids, is intended to prepare readers to be able to enter any of these active research fields, by giving them that wide background of fundamental ideas in terms of which the specialised literature of such a field can be readily understood.

CONTENTS

Epilogue

Bibliography

(indexed as pages *A* to *Q*)

1. SOUND WAVES

1.1 The wave equation

As remarked in the prologue, it is a balance between the compressibility and the inertia of a fluid that governs the propagation of sound waves through it. The linear theory of this propagation is described in chapter 1.

Use of a linear theory, for waves of any kind, implies that we consider disturbances so weak that in equations of motion we can view them as small quantities whose products are neglected. Such products of small quantities occur, for example, in the well-known expression for the acceleration of a fluid element:

$$\partial \mathbf{u}/\partial t + \mathbf{u}\cdot\nabla\mathbf{u}, \tag{1}$$

where $\mathbf{u}$ is the vector velocity field. In this expression (significant whenever inertia is important, as it is for practically all waves in fluids) the linear term $\partial \mathbf{u}/\partial t$ represents the local rate of change of $\mathbf{u}$ at a fixed point, while the nonlinear term $\mathbf{u}\cdot\nabla\mathbf{u}$ describes how the element's velocity changes owing to its changing position in space. This 'convective rate of change' of $\mathbf{u}$ involves products of its spatial gradients with components of $\mathbf{u}$ itself, and so is neglected in a linear theory.

In this chapter, then, disturbances are supposed weak enough for such nonlinear contributions to inertial effects, together with nonlinear terms in the restoring forces (here, those associated with compressibility), to be neglected. Investigations of just *how* weak disturbances need to be for the theory to be reasonably good, and of what detailed effect on stronger disturbances the nonlinear terms may have, are postponed to chapter 2.

In this section, taking into account compressibility and inertia but no other properties of the fluid, we obtain the linearised equations of the theory of sound in their simplest form, a very useful one. We postpone consideration of how sound waves are influenced by effects neglected here (especially viscosity, heat conduction, external forces including gravity, and inhomogeneities such as stratification) to section 1.2 and later parts of the book.

The inertial nature of a fluid of density ρ is expressed when we apply to a small fluid element Newton's second law of motion. This demands that the product of the mass per unit volume ρ and of the acceleration (1) is the force on the element per unit volume, which in the absence of external forces is due solely to those internal stresses through which neighbouring fluid acts on it. When viscous stresses are neglected, this force per unit volume is simply minus the gradient ∇p of the fluid pressure p; thus

$$\rho(\partial \mathbf{u}/\partial t + \mathbf{u}\cdot\nabla\mathbf{u}) = -\nabla p. \tag{2}$$

Compressibility implies that the density of a fluid element may change, in accordance with the well-known equation of continuity:

$$\partial\rho/\partial t + \mathbf{u}\cdot\nabla\rho + \rho\nabla\cdot\mathbf{u} = 0. \tag{3}$$

The first two terms in (3) make up the total rate of change of ρ for the element. Thus, the *divergence* $\nabla\cdot\mathbf{u}$ of the velocity field is identified by (3) as the rate of increase of volume of an element moving in that velocity field, divided by the volume; in other words (since the element's mass is conserved) *minus* the rate of increase of density divided by the density. At the same time, an alternative interpretation of equation (3), based on grouping the second and third terms together as $\nabla\cdot(\rho\mathbf{u})$, is also possible and is used later (section 1.10).

We linearise these equations by regarding as small quantities all departures from a state in which the fluid has uniform density ρ_0 and is at rest. In the absence of external forces this implies that the pressure also takes a uniform value, say p_0.

Equations (2) and (3), with products of small quantities neglected, become the linearised equations of momentum

$$\rho_0\,\partial\mathbf{u}/\partial t = -\nabla p \tag{4}$$

and of continuity

$$\partial\rho/\partial t = -\rho_0\nabla\cdot\mathbf{u}. \tag{5}$$

These forms result from the neglect in (2) of $\mathbf{u}\cdot\nabla\mathbf{u}$ as already discussed, and the similar neglect in (3) of $\mathbf{u}\cdot\nabla\rho$, both involving products of small velocities with small gradients. At the same time the factor ρ in one term of each equation is replaced by ρ_0, the error being the product of a small quantity $(\rho-\rho_0)$ with another small quantity ($\partial\mathbf{u}/\partial t$ or $\nabla\cdot\mathbf{u}$). From this there result local rates of change of velocity $\mathbf{u}$ and density ρ directly proportional to pressure gradient and to velocity divergence, respectively.

One quantity that on the linear theory of sound behaves extremely simply is the vorticity; that is,

$$\boldsymbol{\Omega} = \nabla\times\mathbf{u}, \tag{6}$$