Proofs of Prime Number Distribution Theorems and Goldbach Guess

素数分布定理与哥德巴赫猜想的证明

By Xie Bicheng

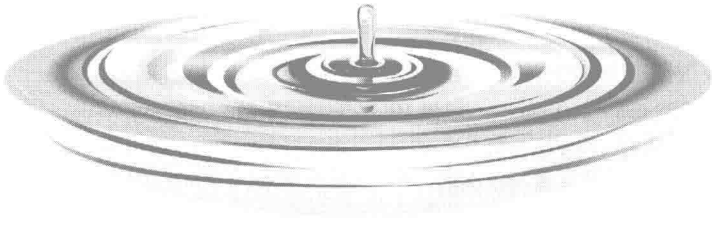
谢必成　著

电子科技大学出版社

Proofs of Prime Number Distribution Theorems and Goldbach Guess

素数分布定理与哥德巴赫猜想的证明

By Xie Bicheng
谢必成　著



电子科技大学出版社

图书在版编目（CIP）数据

素数分布定理与哥德巴赫猜想的证明：汉、英 / 谢必成著. ——成都：电子科技大学出版社，2015. 11

ISBN 978 - 7 - 5647 - 3338 - 4

Ⅰ. ①素… Ⅱ. ①谢… Ⅲ. ①素数 - 研究 - 汉、英 ②哥德巴赫猜想 - 研究 - 汉、英 Ⅳ. ① O156. 2

中国版本图书馆 CIP 数据核字（2015）第 269218 号

Proofs of Prime Number Distribution Theorems and Goldbach Guess

素数分布定理与哥德巴赫猜想的证明

By Xie Bicheng

谢必成　著

出　　版　电子科技大学出版社（成都市一环路东一段 159 号电子信息产业大厦　邮编：610051）

策划编辑　徐守铭

责任编辑　徐守仁　徐守铭

责任校对　王　坤

主　　页　www. uestcp. com. cn

电子邮箱　uestcp@ uestcp. com. cn

发　　行　新华书店经销

印　　刷　成都市新都华兴印务有限公司

成品尺寸　185 mm×260 mm　**印张** 10. 75　**字数** 250 千字

版　　次　2015 年 11 月第一版

印　　次　2015 年 11 月第一次印刷

书　　号　ISBN 978 - 7 - 5647 - 3338 - 4

定　　价　35. 00 元

New Preface (1)

In summer 2000, I obtained a series of important discoveries in the field of number theory. At 2 – 3 o'clock of Dec. 22 in 2005, I perfectly completed the exact description and proofs. What I want to say now is that, discovering the laws and exactly describing them is very important, but the perfect proving of them and from the proof, revealing the prime numbers distribution, from beginning to end, being (multiple and thoroughly) disseminated according to the fashion of congruence (members) in equal number, and the exact reason, are really extraordinarily difficult. The great mathematician Gauss once said, "Mathematics is the queen of science, but arithmetic is the queen of mathematics." I basically used arithmetic's method, passing the extremely arduous and tortuous path, with very hardship trek, step by step with exact inference and counting to obtain them. Plus my living a hardship life, this is really a cruel war about life or death...

It's a long story. About in Oct. 2000, I completed the first "proofs", and very quickly published about 90% contents of that paper on the Internet. And in Nov. 2002, I published the whole contents of that the paper. Then, I considered using the method of the famous Prime Theorem ($N/\ln N$) to add the interval, with the method I solved a little problem (of death causing) that had been asked by a famous mathematician. After the publication, I sought to read and appraise for examination several times (in fact, if it is really perfect proofs, one should be capable to exactly complete the examination by him/herself). In 2003, I had been run up against a stone wall several times, then I sobered down, and opened the book of *An Introduction about Theory Number* by Hua Luogeng, burying myself into the book. Though till now I still can not perfectly understand all of its secrets, I got a lot of information and increased the mathematical capacity. There is a theorem of Dirichlet at page 88 in *An Introduction about Theory Number* (1979 edition), i. e. "If a and b are mutually prime, then there are infinitely many prime numbers that as the form is $an + b$." Obviously, if my discovery that a series of laws in prime number distribution only in infinitely great scope is established, then the series new laws had been discovered must with the Dirichlet theorem have an overlapping suspicion. But using that the series laws, only in the infinitely great scope is established, to prove the "1 + 1" is really very high – sounding and impracticable... I turned back to go over some attached tables about the prime numbers and remainders, behind the "whole paper", it's easy to discover that I earliest found and summed up the laws that prime number distribution is (multiple and thoroughly) disseminated according to the fashion of congruence (members) in equal number not only in the infinitely great scope is established, but fully showed from the beginning, and is about to approach to perfection... In 2000, the theorems were proved to be established that only in infinitely great scope is right, but these proofs were very shallow, naive, and hadn't (far) go to the position. That famous mathematician said yet, "If you can solve the problem, I'll soon recommend." In that moment, I just began to truly discover the small question of his, there are only a few words, but each one has a thousand pieces of gold values. I deeply recognized that solving the problem is not absolutely easy, and it must be a fight which is the most brilliant in the mathematical history of human.

For a series of exact laws, there certainly has a series of exact reasons. But without principal and subsidiary causes, how dare I have courage to wage a battle? But it is strangely seems that God of Fate unceasingly shines on me who is a man of bad luck. I didn't stop, but continued to ponder upon the question all along. About at Oct. 2004, I surprisingly obtained a very important inspiration. Then a month later, I overcame some fear and then went in to a cruel battle... which lasted long time practice (I spent one year), and which was very tortuous and painful... In that year, there were a few days, I felt my body was floating, my soul was flying to the vault of heaven...

I am very happy to obtain an inspiration, which is decisive to the most difficult problem in mathematics. But as that is a very high mountain of iron hoop, how could it be shaken by a few inspirations? There wasn't any absurd matter in the paper, only an ordinary pen, and a start from the very ordinary angle. I tried to carry on the ordinary analysis and calculation, step by step on the solid ground. Obviously, to solve these problems, naturally, there is no lack of extremely going deep careful analysis, and soul - stirring battles, to raise some new concepts, some new expressions and calculating methods on base that have got new discoveries of a series of close relation. By utilizing the methods that are extremely clever, perfect, high - quality, exact to reach the position of satisfactory logic - inference, uncanny workmanship, I got through every blood vessel, and obtained striking key points under the help of God. It's worthy of people's expectations.

Compared with the "whole paper" of 2002, this paper: (1) revised the description of every prime number distribution theorem; (2) added theorem α at the front of theorem 8; (3) added correction original proof to where there are mistakes and deficiency in each prime number distribution theorem, from theorem 8 on, in the first part, borrowed the original proof, and continued to point out their mistakes and to be a steady and sure proof again (It may say that, my original proofs of prime number distribution theorems were at the first step only. In fact, the part of the newly adding proof is a basic and perfect proof) ... for theorem 8 shows the representative character more, all the odd prime numbers in proper order that divide by 3 is changed into divide respectively by 7; (4) the newly adding proof of theorem 9 and theorem 10 need the same method of theorem 8, because of all the proofs of prime number distribution theorems having the resembled character, the proofs used as supplements in subsequent theorems are omitted; (5) theorem 15 has simple, clear and perfect proof; (6) revised the proof of theorem 16, and reduced the analysis in former proving; (7) the proof of theorem 18 is slightly revised; for making people easier to understand and have a more perceptual cognition, two examples also was added, which show that when the even is more big, there are more sums of the not – like two prime numbers respectively equal is the even, this shows no difference with which pointed out in my proofs. Further more, compared with the original manuscript in 2000, this book has twice as the length.

It is easy to see that, either the discoveries of the series laws, or (needs) exact descriptions... all of them can't leave the great "Goldbach Guess". For hundreds of years, people have gained many great discoveries and achievements in the exceedingly difficult and very unusual way of proving the Goldbach Guess. It lets us know that raising a science problem to have original and innovative effects is even more than providing it.

But with what are discovered and proved to tally perfectly, it is perfectly satisfied.

Xie Bicheng

Feb. 28, 2006

新的前言（一）

2000 年夏天我在数论领域内取得了一系列重大的发现，直到 2005 年 12 月 22 日凌晨 2～3点才真正完成了对它们的准确描述及严密完善的证明. 我现在要说的是，发现规律并准确地描述它们是非常重要的；但真正圆满成功地证明它们，并从证明中揭示出素数分布从开始至无穷一直都具有严格而准确的按同余均匀分布的规律性及其确切的原因，其实是异乎寻常艰难的. 大数学家高斯有句名言：“数学是（科学的）女王，而算术是数学的女王.” 我是基本上运用算术的方法，通过极其曲折、极其艰难困苦的跋涉，一步步准确地推理与计算得出来的，加上我生活上的困难，这简直就是一场生与死的残酷战争……

说来话长，2000 年 10 月左右，我完成了第一次“证明”，并很快在网上向外界公开了这一系列发现及证明的 90% 以上的内容. 2002 年又公开了“全文”，自认为运用有名的素数定理（$N/\ln N$）加区间的方法解决了一位著名数学家对我的论文提出的一个“致命的小问题”. 公开后我又几次寻求评审与认定（其实真正成功的证明，自己应当完全能够评定它）. 至 2003 年，数次碰壁后我冷静了下来，重新打开了华罗庚先生的巨著《数论导引》一书，埋头学习了起来. 虽然至今我仍不能完全领会其中所有的奥妙，但从中也获得了不少的信息，增长了数学上的才干. 在《数论导引》1979 年版第 88 页上有 Dirichlet 氏一定理：若 a，b 互素，则形如 $an+b$ 之素数之个数无限. 显然，如果我发现的这一系列素数分布规律只有在无穷大时才成立，那么这一系列新发现必然与此定理有重叠之嫌. 而运用这一系列无限大时才成立的定理来证明“1 + 1”未免太迂阔了……我返转来审视我于 2000 年夏天计算出来的多份素数分布表，不难看出，我早先提出的素数按同余均匀分布的规律并不是在无穷大时才成立，而是从一开始就充分表现了出来，并且很快就趋向完善……我在 2000 年所做的证明充其量是在无限大的范围内证明了定理的成立，而且这个证明是非常粗浅、幼稚、不到位的，根本没有克服“致命的小问题”. 此数学家还向我讲了一句：“若你能解决此问题，我就推荐.” 我此时才真正理解到他这个小问题，短短几个字，真乃字字千金. 也深刻认识到，要解决此问题，绝非易事，它必将是人类数学史上一次极为辉煌的战斗.

一系列确切的规律性必有其一系列确切的原因. 但无因缘，我怎敢轻启战端？奇怪的是，好像命运之神照定了我这个命运多舛之人，我一直也摆脱不了对它的忧思. 2004 年 10 月左右，在周围一片“哦嚯”声中，我竟又一次获得重要灵感，一个月后我才克服了恐惧，投入了又一场残酷的战争……其时间之漫长（耗时一年）、经历道路的曲折与痛苦实难用言语表达……其中数日感到身体漂浮，魂飞天外……

获得灵感是件令人愉快的事，但科学上真正艰难问题的决战，巨型高大的铁围山岂是

区区几个灵感所能撼动．本书中并无怪异的东西，一支平常的笔，从极平常的角度入手，进行平常的分析与计算，仅仅做到步步脚踏实地，处处准确严密．显然，解决这样的问题，其中自然不缺极其深入、细致的分析，惊心动魄的战斗．在一系列紧密相关的新发现基础上提出的一些新的概念、新的表达与计算方法，极其巧妙、完善，又运用刚刚到位的非常圆满的逻辑推理方法，鬼斧神工，疏通根根经脉，还须神助点睛之笔，使龙成翔天，方不负众望．

本书比较 2002 年公开的“全文”修改补充了以下内容：①修改了对所有素数分布定理的描述；②在定理 8 前添加了定理 α；③由于是纠正，补充原素数分布定理证明中的缺点与不足，从定理 8 的证明起，前半部借用了原证明，接着指出缺点与不足，再接着踏踏实实证明下去……为了使定理 8 更具代表性，由所有素数依次去除以 3，改为依次去除以 7（可以说我原来关于素数分布定理的证明只是完成了第一步；从另一方面讲，我后面的补充证明其实就是完善的证明）；④定理 9、定理 10 的补充证明也是运用的同一个方法，由于我的所有的素数分布定理的证明均有相似性，再后面的定理证明中当补充的部分一笔带过；⑤对定理 15 做了简单、清晰且严密的证明；⑥对定理 16 的证明进行了修正，并大大精减了证明前的分析；⑦对定理 18（即“1 + 1”）的证明，稍微做了修改，为了让人们更便于理解和有进一步的感性认识，加入了两个例子，从中不难看出，偶数越大将会成为越多的不同素数对之和，与我证明中提示的一致．本书比较 2000 年的原文篇幅也扩大了一倍以上．

不管是发现这些规律，还是准确地对它们进行描述……所有这些都离不开伟大的哥德巴赫猜想．数百年来，人们在证明哥德巴赫猜想这一非常艰难、极不平常的道路上取得了很多伟大的发现和成就．它让我们认识到，一个科学问题的提出比证明它们更加具有原始创新的意义．

令人感到非常满意的是，这样的发现用如此（新）的证明方法刚刚完全吻合．

谢必成

2006 年 2 月 28 日

New Preface (2)

After that, I spent nearly one year to translate it into English. At the beginning of 2007, I submitted the manuscripts to some mathematical Professors in a well-known university of England. After a few months, the Professors gave me two suggestions. One was telling me that the initial tentative of theorems should be discussed. Thus, I added the proofs of the initial tentative to the paper soon. Because of using mathematical abbreviations in the suggestion two, I didn't understand it until Jun. 2009, that the professor suggested me to point out the simple proof of theorem 15 should be expressed in a detailed and strict way. Actually theorem 15 occupies a very important place in my paper. Obviously, it is very difficult to demonstrate the proof process of theorem 15, and I was facing tackle key problems again. It is lucky that I kept a quite good feeling that I got some new finds, especially some important ones before long, so theorem 15 (i. e. theorem 6 in this paper) could be completed in Sep. 2009. But the proof wasn't perfect for its complicated and perplexing expressions. In spring 2010, after I pushed out a new assisting theorem β, theorem 6 (i. e. theorem 15 of the past) is re-proved in the updated manuscript with the method that are penetrating, clear, and easy to understand. So far, the whole proof of theorem 6 reached a level of satisfactory and perfection.

Compared with the preceding manuscript, I took off some unnecessary, repeated theorems, paragraphs, to make the paper more succinct.

Here I'd like to take this opportunity to thank for all the following gentlemen who gave important helps to me within my proving period: my dear uncle Du Chuanyong (graduated from Mathematics Department, Sichuan University), Professor He Dake (classmate of my uncle Du), and some mathematical Professors of England.

This paper dedicate to the honest, kind, and diligent people in the world.

Xie Bicheng

Apr. 8, 2010

新的前言（二）

接着我用了一年时间将其翻译成英文，于 2007 年初向英国一所著名大学投稿，很快得到一个投稿编号及两条意见. 一条是指出定理的起始条件应当讨论（我很快补充了对起始条件的论证）；第二条中用的数学简写符号，我几乎一直读不懂，直到 2009 年 6 月初才清楚，它是指我简略证明的定理 15（即现定理 6）必须详细、严密论证. 是啊此定理在整书中占有非常重要的位置，显然对它详细而严密的论证绝非易事，我面临又一次攻关. 好在我静下了心，很快有了一些发现，特别是获得了一个关键性的发现，于 2009 年 9 月完成了对它详细而严密的论证. 但它算不了完美的证明，语言繁复让人费解. 2010 年初春，我在推出了一个辅助定理 β 后，使此证明变得清晰、明了起来，整个证明也达到圆满、完美的程度.

在此期间我又发现原书中有些定理不必要，有的意义重叠……因此又对稿件进行了必要的精简.

从尘世上分析，凭自己的学历，以前我并未想到能走到这一步. 我之所以能走到这一步，与我的表叔——四川大学数学系毕业的杜传勇先生的鼓励、支持、指导，与西南交通大学教授何大可先生（我表叔的同窗）的点拨，与牛津大学数学教授们的宝贵意见是分不开的. 在此谨向他们表示我衷心的感谢.

谨以本书献给世界上所有诚实、善良、勤劳的人们.

谢必成

2010 年 4 月 8 日

New Preface (3)

My paper is truly perfection. Before several days, I found finally a small question (i. e. "Remainder Change Value") to be in the paper, it has been neglect. I deeply know that when proving of a topic of mathematics, any small neglect can lead to a whole failure in proving of the topic. It's an unusual thing, neglected it, but didn't influence prove wholly still along right direction to advance. Obviously the question isn't fatal, and it is easy to solve too. From finding it to basically solving it in my head, I spent about half an hour only. Obvious I worry to have a few minutes too. Very simple an example of the "Remainder Change Value" is: "when using all odd numbers that divided by 3 with remainder of 1 to be multiplied by 5 respectively to form a new number series, the new number series contains (and only contains) all the odd numbers that divided by 5 with remainder of 0 within all the odd numbers that divided by 3 with remainder of 2; while using all odd numbers that divided by 3 with remainder of 2 to be multiplied by 5 respectively to form a number series, the number series contains (and only contains) all the odd numbers that divided by 5 with remainder of 0 within all the odd numbers that divided by 3 with remainder of 1. (It's truly perfect exchange!)" About the concept of the "Remainder Change Value", read the explanation in step (4) of sum up proof of theorem 3, please. I have revised ten small parts in English of the book for solving the neglectful question, so (adding Chinese parts) made the book to increase more than 4 pages. It let us know that a perfect paper itself must have its strict, perfect, and steady system, when it face to blame or difficult position, we can easily find a method to return natural character of its perfection.

On the other hand to say, my original proof (in Apr. 2010) was s prefect basic proof in fact. Please think by thinking that, in fact, I always need to prove the situations (i. e. matters) of n kinds to be established at every time. The proof's method was to prove among any kind situation to be established first, then using the same method, other the situations of $n-1$ kinds have been proved. The first select the kind situation to produce the "Remainder Change Value", but it always changed into a remainder of other kind (within the n kinds). I have proved the other kind to be established before, i. e. have proved among a kind situation to be established in fact. And their in, the methods of changing remainders of m kinds are: any kind change (go) out to have (and only have) one chance, it change back to have (and only have) one chance, too; and all the changing remainders value are within the m kinds all.

Xie Bicheng

Sept. 2015

新的前言（三）

我的著作已近交付出版印刷了，但就在此时，我终于发现了其中存在一个“余数换值”的小问题被忽略了. 我深知证明数学题，任何一小处的忽略都可能导致整个证明的失败. 奇怪的是忽略了它，并不影响整个证明沿正确的方向前进. 显然此问题并不致命，也容易解决. 从发现它到我在头脑中基本解决它，大概就花费了半个多小时. 显然我也着急了两三分钟. “余数换值”最简单的例子是：“当用除以 3 余 1 的所有奇数分别去乘以 5，所得的奇数列包括（并且仅仅包括）了除以 3 余 2 的数列中所有除以 5 余 0 的奇数. 而用除以 3 余 2 的所有奇数分别去乘以 5，所得的奇数列包括（并且仅仅包括）了除以 3 余 1 的数列中所有除以 5 余 0 的奇数.（这刚好是一种完美的调换啊!）”关于“余数换值”的概念请见定理 3 的综合证明内第（4）步中的解释. 为修正这一忽略了的问题，在本书的中、英文中各有 10 处被修正，从而使本书增加了 4 页多的篇幅. 这让人想到一部完美的著作，它自有严格、完美、稳定的体系，当它面对责难或困境时，是很容易寻找到方法恢复它完善的本质.

另一方面讲，我原来的证明（在 2010 年 4 月）其实就是基本完善的证明. 请想想，我以前每次总是要证 n 种情况成立. 证明的方法是先证明其中一种成立，另 $n-1$ 种情况用同样的方法就证明了. 先选出的一种余数变换了，但它总是变换成了 n 种中的另一种，我显然是证明了另一种成立，即证明了其中一种成立. 而且在其中，各起 m 种余数的变换方法是：任一种换出去有（并且仅有）一次机会，它换回来也有（并且仅有）一次机会；而且都是在这 m 种内转换.

谢必成

2015 年 9 月

Contents
目　　录

Proof of Goldbach Guess "1 +1"

哥德巴赫猜想“1 +1”的证明

Miscellaneous

其　他

The Symbol Manual

1. In the book, each letter may represent different meaning in the different theorems. Please note its meaning is appointed in each theorem.

Such as:

n can be used to ①represent an odd number; ②orderly represent all the non-negative integers; ③represent an odd prime number;...

m can be used to ①represent a natural number; ②represent the nth prime number;...

...

2. The circle^2 shows using n circle in the same girth, making one circle on top of another one to form a circle^2 in appointing way;

The circle^3 shows using n circle^2 in the same girth, making one circle^2 on top of another one to form a circle^3 in appointing way;

The circle^4 shows using n circle^3 in the same girth, making one circle^3 on top of another one to form a circle^4 in appointing way;

...

The circle^n shows using n circle^{n-1} in the same girth, making one circle^{n-1} on top of another one to form a circle^n in appointing way.

It has a point exception that circle^3 shows using n circle in the same girth, making one circle on top of another one to form a circle^3 in appointing way in theorem 6.

The circle^x can represent over any one (as): circle^2, circle^3,..., circle^{n-1}.

3. About symbol "...;...;": when there are a lot of compound sentences and these compound sentences use ";" mark apart, because within each compound sentence omitted a part have been used "···," mark. For express clear and readers easy reading, so when among many compound sentences have been omitted, I use "...;...;" marks to express.

4. All circle and circle (circumference) mean circumference of circle.

符号说明

1. 每个字母在本书内所代表的意义在不同定理中有所不同,请以各个定理中的指定为准. 如:

n 在不同定理中可以代表①一个奇数;②依次代表所有非负整数;③一个奇素数;……

m 在不同定理中可以代表①一个自然数;②第 n 个素数;……

……

2. 圆^2 表示 n 个同周长的圆按一定方法重叠在一起形成一个圆^2(n 代表某一自然数,且 $n>1$);

圆^3 表示 n 个同周长的圆^2 按一定方法重叠在一起形成一个圆^3;

圆^4 表示 n 个同周长的圆^3 按一定方法重叠在一起形成一个圆^4;

……

圆^n 表示 n 个同周长的圆$^{n-1}$按一定方法重叠在一起形成一个圆^n.

只有在定理 6 中有一种例外,即圆^3 表示 n 个同周长的圆重叠形成.

圆^x 可以代表上面任何一个重叠形成的圆.

3. 关于符号"…;…;"的说明:当定理的证明中有很多并列句,这些并列句都是用";"分开的,由于每个并列句内都有省略部分,已经用了"…,",因此为了表达清晰和便于读者区别,当这些并列句的上层句子也是并列关系时,用"…;…;"表示上层句子中的省略部分.

4. 本书中所有的圆、圆(周),含义都是"圆周".

Abstract

In summer 2000, the author discovered a series of laws on the distribution of prime number. From beginning to end, they are (multiple and thoroughly) disseminated according to the fashion of congruence (members) in equal number. The author can prove them, because of having obtained a series of important discoveries again (in winter 2005): on the odd number axis, along with the composite odd numbers of various kinds continuously being taken out step by step, all surplus odd numbers were divided into various kinds of "corresponding congruent odd numbers"; from beginning to end, the "corresponding congruent odd numbers" of every kind distribution's distance are equal on the layer upon layer "circle^x" (circumference). Further, the author calculated out: the distribution laws that the congruent prime numbers of various kinds appear in order, to keep all along the pattern as are well-distributed. We can judge that prime numbers' appearance isn't random, and it strictly has the fashion (pattern). On the layer upon layer "circle^x", every prime number is according to exact seat (place) to appear out. Also, the author proved the Goldbach Guess i.e. "1 + 1".

Keywords

prime number distribution
congruent prime numbers
1 + 1
Goldbach Guess
circle^4

Acknowledgements

Here the author is glad to take this opportunity to thank the following gentlemen that gave important helps to me in my proving time: Du Chuanyong (graduated from Mathematics Department, Sichuan University), Professor He Dake (classmate of my uncle Du), and some mathematical Professors of England. And sincere thanks for some wise American mathematicians who gave me many valuable suggestions.