计算机程序设计艺术

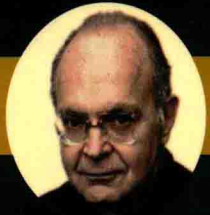
第1卷 基本算法

The Art of Computer Programming

Volume 1: Fundamental Algorithms, Third Edition

（英文版·第3版）

(美) Donald E. Knuth 斯坦福大学 著



机械工业出版社
China Machine Press

计算机程序设计艺术

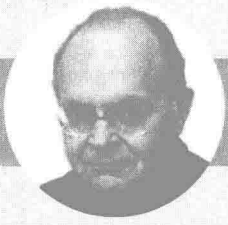
第1卷 基本算法

The Art of Computer Programming

Volume 1: Fundamental Algorithms, Third Edition

（英文版·第3版）

(美) Donald E. Knuth
斯坦福大学 著



机械工业出版社
China Machine Press

本书版权登记号：图字：01-2007-2438

图书在版编目（CIP）数据

计算机程序设计艺术　第1卷：基本算法（英文版·第3版）/（美）克努特（Knuth, D. E.）著. -北京：机械工业出版社，2008. 1

书名原文：The Art of Computer Programming, Volume 1: Fundamental Algorithms, Third Edition

ISBN 978-7-111-22709-0

Ⅰ. 计…　Ⅱ. 克…　Ⅲ. 程序设计-英文　Ⅳ. TP311.1

中国版本图书馆CIP数据核字（2007）第169310号

机械工业出版社（北京市西城区百万庄大街22号　邮政编码　100037）

责任编辑：迟振春

北京京北制版厂印刷·新华书店北京发行所发行

2008年1月第1版第1次印刷

170mm×242mm　·　42.25印张

定价：95.00元

凡购本书，如有倒页、脱页、缺页，由本社发行部调换

本社购书热线：（010）68326294

出版者的话

文艺复兴以降，源远流长的科学精神和逐步形成的学术规范，使西方国家在自然科学的各个领域取得了垄断性的优势；也正是这样的传统，使美国在信息技术发展的六十多年间名家辈出、独领风骚。在商业化的进程中，美国的产业界与教育界越来越紧密地结合，计算机学科中的许多泰山北斗同时身处科研和教学的最前线，由此而产生的经典科学著作，不仅擘划了研究的范畴，还揭橥了学术的源变，既遵循学术规范，又自有学者个性，其价值并不会因年月的流逝而减退。

近年，在全球信息化大潮的推动下，我国的计算机产业发展迅猛，对专业人才的需求日益迫切。这对计算机教育界和出版界都既是机遇，也是挑战；而专业教材的建设在教育战略上显得举足轻重。在我国信息技术发展时间较短、从业人员较少的现状下，美国等发达国家在其计算机科学发展的几十年间积淀的经典教材仍有许多值得借鉴之处。因此，引进一批国外优秀计算机教材将对我国计算机教育事业的发展起积极的推动作用，也是与世界接轨、建设真正的世界一流大学的必由之路。

机械工业出版社华章图文信息有限公司较早意识到“出版要为教育服务”。自1998年开始，华章公司就将工作重点放在了遴选、移译国外优秀教材上。经过几年的不懈努力，我们与Prentice Hall，Addison-Wesley，McGraw-Hill，Morgan Kaufmann等世界著名出版公司建立了良好的合作关系，从它们现有的数百种教材中甄选出Tanenbaum，Stroustrup，Kernighan，Jim Gray等大师名家的一批经典作品，以“计算机科学丛书”为总称出版，供读者学习、研究及庋藏。大理石纹理的封面，也正体现了这套丛书的品位和格调。

“计算机科学丛书”的出版工作得到了国内外学者的鼎力襄助，国内的专家不仅提供了中肯的选题指导，还不辞劳苦地担任了翻译和审校的工作；而原书的作者也相当关注其作品在中国的传播，有的还专程为其书的中译本作序。迄今，“计算机科学丛书”已经出版了近260个品种，这些书籍在读者中树立了良好的口碑，并被许多高校采用为正式教材和参考书籍，为进一步推广与发展打下了坚实的基础。

随着学科建设的初步完善和教材改革的逐渐深化，教育界对国外计算机教材的需求和应用都步入一个新的阶段。为此，华章公司将加大引进教材的力度，除“计算机科学丛书”之外，对影印版的教材，则单独开辟出“经典原版书库”。为了保证这两套丛书的权威性，同时也为了更好地为学校和老师们服务，华章

公司聘请了中国科学院、北京大学、清华大学、国防科技大学、复旦大学、上海交通大学、南京大学、浙江大学、中国科技大学、哈尔滨工业大学、西安交通大学、中国人民大学、北京航空航天大学、北京邮电大学、中山大学、解放军理工大学、郑州大学、湖北工学院、中国国家信息安全测评认证中心等国内重点大学和科研机构在计算机的各个领域的著名学者组成“专家指导委员会”，为我们提供选题意见和出版监督。

这两套丛书是响应教育部提出的使用外版教材的号召，为国内高校的计算机及相关专业的教学度身订造的。其中许多教材均已为M. I. T.，Stanford，U.C. Berkeley，C. M. U. 等世界名牌大学所采用。不仅涵盖了程序设计、数据结构、操作系统、计算机体系结构、数据库、编译原理、软件工程、图形学、通信与网络、离散数学等国内大学计算机专业普遍开设的核心课程，而且各具特色——有的出自语言设计者之手、有的历经三十年而不衰、有的已被全世界的几百所高校采用。在这些圆熟通博的名师大作的指引之下，读者必将在计算机科学的宫殿中由登堂而入室。

权威的作者、经典的教材、一流的译者、严格的审校、精细的编辑，这些因素使我们的图书有了质量的保证，但我们的目标是尽善尽美，而反馈的意见正是我们达到这一终极目标的重要帮助。教材的出版只是我们的后续服务的起点。华章公司欢迎老师和读者对我们的工作提出建议或给予指正，我们的联系方法如下：

电子邮件：hzjsj@hzbook.com
联系电话：(010) 68995264
联系地址：北京市西城区百万庄南街1号
邮政编码：100037

专家指导委员会

（按姓氏笔画顺序）

尤晋元	王　珊	冯博琴	史忠植	史美林
石教英	吕　建	孙玉芳	吴世忠	吴时霖
张立昂	李伟琴	李师贤	李建中	杨冬青
邵维忠	陆丽娜	陆鑫达	陈向群	周伯生
周克定	周傲英	孟小峰	岳丽华	范　明
郑国梁	施伯乐	钟玉琢	唐世渭	袁崇义
高传善	梅　宏	程　旭	程时端	谢希仁
裘宗燕	戴　葵			

PREFACE

Here is your book, the one your thousands of letters have asked us to publish. It has taken us years to do, checking and rechecking countless recipes to bring you only the best, only the interesting, only the perfect. Now we can say, without a shadow of a doubt, that every single one of them, if you follow the directions to the letter, will work for you exactly as well as it did for us, even if you have never cooked before.

— *McCall's Cookbook* (1963)

THE PROCESS of preparing programs for a digital computer is especially attractive, not only because it can be economically and scientifically rewarding, but also because it can be an aesthetic experience much like composing poetry or music. This book is the first volume of a multi-volume set of books that has been designed to train the reader in various skills that go into a programmer's craft.

The following chapters are *not* meant to serve as an introduction to computer programming; the reader is supposed to have had some previous experience. The prerequisites are actually very simple, but a beginner requires time and practice in order to understand the concept of a digital computer. The reader should possess:

a) Some idea of how a stored-program digital computer works; not necessarily the electronics, rather the manner in which instructions can be kept in the machine's memory and successively executed.

b) An ability to put the solutions to problems into such explicit terms that a computer can "understand" them. (These machines have no common sense; they do exactly as they are told, no more and no less. This fact is the hardest concept to grasp when one first tries to use a computer.)

c) Some knowledge of the most elementary computer techniques, such as looping (performing a set of instructions repeatedly), the use of subroutines, and the use of indexed variables.

d) A little knowledge of common computer jargon — "memory," "registers," "bits," "floating point," "overflow," "software." Most words not defined in the text are given brief definitions in the index at the close of each volume.

These four prerequisites can perhaps be summed up into the single requirement that the reader should have already written and tested at least, say, four programs for at least one computer.

I have tried to write this set of books in such a way that it will fill several needs. In the first place, these books are reference works that summarize the

knowledge that has been acquired in several important fields. In the second place, they can be used as textbooks for self-study or for college courses in the computer and information sciences. To meet both of these objectives, I have incorporated a large number of exercises into the text and have furnished answers for most of them. I have also made an effort to fill the pages with facts rather than with vague, general commentary.

This set of books is intended for people who will be more than just casually interested in computers, yet it is by no means only for the computer specialist. Indeed, one of my main goals has been to make these programming techniques more accessible to the many people working in other fields who can make fruitful use of computers, yet who cannot afford the time to locate all of the necessary information that is buried in technical journals.

We might call the subject of these books "nonnumerical analysis." Computers have traditionally been associated with the solution of numerical problems such as the calculation of the roots of an equation, numerical interpolation and integration, etc., but such topics are not treated here except in passing. Numerical computer programming is an extremely interesting and rapidly expanding field, and many books have been written about it. Since the early 1960s, however, computers have been used even more often for problems in which numbers occur only by coincidence; the computer's decision-making capabilities are being used, rather than its ability to do arithmetic. We have some use for addition and subtraction in nonnumerical problems, but we rarely feel any need for multiplication and division. Of course, even a person who is primarily concerned with numerical computer programming will benefit from a study of the nonnumerical techniques, for they are present in the background of numerical programs as well.

The results of research in nonnumerical analysis are scattered throughout numerous technical journals. My approach has been to try to distill this vast literature by studying the techniques that are most basic, in the sense that they can be applied to many types of programming situations. I have attempted to coordinate the ideas into more or less of a "theory," as well as to show how the theory applies to a wide variety of practical problems.

Of course, "nonnumerical analysis" is a terribly negative name for this field of study; it is much better to have a positive, descriptive term that characterizes the subject. "Information processing" is too broad a designation for the material I am considering, and "programming techniques" is too narrow. Therefore I wish to propose *analysis of algorithms* as an appropriate name for the subject matter covered in these books. This name is meant to imply "the theory of the properties of particular computer algorithms."

The complete set of books, entitled *The Art of Computer Programming*, has the following general outline:

Volume 2. Seminumerical Algorithms

Chapter 3. Random Numbers
Chapter 4. Arithmetic

Volume 3. Sorting and Searching

Chapter 5. Sorting
Chapter 6. Searching

Volume 4. Combinatorial Algorithms

Chapter 7. Combinatorial Searching
Chapter 8. Recursion

Volume 5. Syntactical Algorithms

Chapter 9. Lexical Scanning
Chapter 10. Parsing

Volume 4 deals with such a large topic, it actually represents three separate books (Volumes 4A, 4B, and 4C). Two additional volumes on more specialized topics are also planned: Volume 6, *The Theory of Languages* (Chapter 11); Volume 7, *Compilers* (Chapter 12).

I started out in 1962 to write a single book with this sequence of chapters, but I soon found that it was more important to treat the subjects in depth rather than to skim over them lightly. The resulting length of the text has meant that each chapter by itself contains more than enough material for a one-semester college course; so it has become sensible to publish the series in separate volumes. I know that it is strange to have only one or two chapters in an entire book, but I have decided to retain the original chapter numbering in order to facilitate cross-references. A shorter version of Volumes 1 through 5 is planned, intended specifically to serve as a more general reference and/or text for undergraduate computer courses; its contents will be a subset of the material in these books, with the more specialized information omitted. The same chapter numbering will be used in the abridged edition as in the complete work.

The present volume may be considered as the "intersection" of the entire set, in the sense that it contains basic material that is used in all the other books. Volumes 2 through 5, on the other hand, may be read independently of each other. Volume 1 is not only a reference book to be used in connection with the remaining volumes; it may also be used in college courses or for self-study as a text on the subject of *data structures* (emphasizing the material of Chapter 2), or as a text on the subject of *discrete mathematics* (emphasizing the material of Sections 1.1, 1.2, 1.3.3, and 2.3.4), or as a text on the subject of *machine-language programming* (emphasizing the material of Sections 1.3 and 1.4).

The point of view I have adopted while writing these chapters differs from that taken in most contemporary books about computer programming in that I am not trying to teach the reader how to use somebody else's software. I am concerned rather with teaching people how to write better software themselves.

My original goal was to bring readers to the frontiers of knowledge in every subject that was treated. But it is extremely difficult to keep up with a field that is economically profitable, and the rapid rise of computer science has made such a dream impossible. The subject has become a vast tapestry with tens of thousands of subtle results contributed by tens of thousands of talented people all over the world. Therefore my new goal has been to concentrate on "classic" techniques that are likely to remain important for many more decades, and to describe them as well as I can. In particular, I have tried to trace the history of each subject, and to provide a solid foundation for future progress. I have attempted to choose terminology that is concise and consistent with current usage. I have tried to include all of the known ideas about sequential computer programming that are both beautiful and easy to state.

A few words are in order about the mathematical content of this set of books. The material has been organized so that persons with no more than a knowledge of high-school algebra may read it, skimming briefly over the more mathematical portions; yet a reader who is mathematically inclined will learn about many interesting mathematical techniques related to discrete mathematics. This dual level of presentation has been achieved in part by assigning ratings to each of the exercises so that the primarily mathematical ones are marked specifically as such, and also by arranging most sections so that the main mathematical results are stated *before* their proofs. The proofs are either left as exercises (with answers to be found in a separate section) or they are given at the end of a section.

A reader who is interested primarily in programming rather than in the associated mathematics may stop reading most sections as soon as the mathematics becomes recognizably difficult. On the other hand, a mathematically oriented reader will find a wealth of interesting material collected here. Much of the published mathematics about computer programming has been faulty, and one of the purposes of this book is to instruct readers in proper mathematical approaches to this subject. Since I profess to be a mathematician, it is my duty to maintain mathematical integrity as well as I can.

A knowledge of elementary calculus will suffice for most of the mathematics in these books, since most of the other theory that is needed is developed herein. However, I do need to use deeper theorems of complex variable theory, probability theory, number theory, etc., at times, and in such cases I refer to appropriate textbooks where those subjects are developed.

The hardest decision that I had to make while preparing these books concerned the manner in which to present the various techniques. The advantages of flow charts and of an informal step-by-step description of an algorithm are well known; for a discussion of this, see the article "Computer-Drawn Flowcharts" in the ACM *Communications*, Vol. 6 (September 1963), pages 555–563. Yet a formal, precise language is also necessary to specify any computer algorithm, and I needed to decide whether to use an algebraic language, such as ALGOL or FORTRAN, or to use a machine-oriented language for this purpose. Perhaps many of today's computer experts will disagree with my decision to use a

machine-oriented language, but I have become convinced that it was definitely the correct choice, for the following reasons:

a) A programmer is greatly influenced by the language in which programs are written; there is an overwhelming tendency to prefer constructions that are simplest in that language, rather than those that are best for the machine. By understanding a machine-oriented language, the programmer will tend to use a much more efficient method; it is much closer to reality.

b) The programs we require are, with a few exceptions, all rather short, so with a suitable computer there will be no trouble understanding the programs.

c) High-level languages are inadequate for discussing important low-level details such as coroutine linkage, random number generation, multi-precision arithmetic, and many problems involving the efficient usage of memory.

d) A person who is more than casually interested in computers should be well schooled in machine language, since it is a fundamental part of a computer.

e) Some machine language would be necessary anyway as output of the software programs described in many of the examples.

f) New algebraic languages go in and out of fashion every five years or so, while I am trying to emphasize concepts that are timeless.

From the other point of view, I admit that it is somewhat easier to write programs in higher-level programming languages, and it is considerably easier to debug the programs. Indeed, I have rarely used low-level machine language for my own programs since 1970, now that computers are so large and so fast. Many of the problems of interest to us in this book, however, are those for which the programmer's art is most important. For example, some combinatorial calculations need to be repeated a trillion times, and we save about 11.6 days of computation for every microsecond we can squeeze out of their inner loop. Similarly, it is worthwhile to put an additional effort into the writing of software that will be used many times each day in many computer installations, since the software needs to be written only once.

Given the decision to use a machine-oriented language, which language should be used? I could have chosen the language of a particular machine X, but then those people who do not possess machine X would think this book is only for X-people. Furthermore, machine X probably has a lot of idiosyncrasies that are completely irrelevant to the material in this book yet which must be explained; and in two years the manufacturer of machine X will put out machine $X+1$ or machine $10X$, and machine X will no longer be of interest to anyone.

To avoid this dilemma, I have attempted to design an "ideal" computer with very simple rules of operation (requiring, say, only an hour to learn), which also resembles actual machines very closely. There is no reason why a student should be afraid of learning the characteristics of more than one computer; once one machine language has been mastered, others are easily assimilated. Indeed, serious programmers may expect to meet many different machine languages in the course of their careers. So the only remaining disadvantage of a mythical

machine is the difficulty of executing any programs written for it. Fortunately, that is not really a problem, because many volunteers have come forward to write simulators for the hypothetical machine. Such simulators are ideal for instructional purposes, since they are even easier to use than a real computer would be.

I have attempted to cite the best early papers in each subject, together with a sampling of more recent work. When referring to the literature, I use standard abbreviations for the names of periodicals, except that the most commonly cited journals are abbreviated as follows:

CACM = Communications of the Association for Computing Machinery

JACM = Journal of the Association for Computing Machinery

Comp. J. = The Computer Journal (British Computer Society)

Math. Comp. = Mathematics of Computation

AMM = American Mathematical Monthly

SICOMP = SIAM Journal on Computing

FOCS = IEEE Symposium on Foundations of Computer Science

SODA = ACM–SIAM Symposium on Discrete Algorithms

STOC = ACM Symposium on Theory of Computing

Crelle = Journal für die reine und angewandte Mathematik

As an example, "*CACM* **6** (1963), 555–563" stands for the reference given in a preceding paragraph of this preface. I also use "*CMath*" to stand for the book *Concrete Mathematics*, which is cited in the introduction to Section 1.2.

Much of the technical content of these books appears in the exercises. When the idea behind a nontrivial exercise is not my own, I have attempted to give credit to the person who originated that idea. Corresponding references to the literature are usually given in the accompanying text of that section, or in the answer to that exercise, but in many cases the exercises are based on unpublished material for which no further reference can be given.

I have, of course, received assistance from a great many people during the years I have been preparing these books, and for this I am extremely thankful. Acknowledgments are due, first, to my wife, Jill, for her infinite patience, for preparing several of the illustrations, and for untold further assistance of all kinds; secondly, to Robert W. Floyd, who contributed a great deal of his time towards the enhancement of this material during the 1960s. Thousands of other people have also provided significant help — it would take another book just to list their names! Many of them have kindly allowed me to make use of hitherto unpublished work. My research at Caltech and Stanford was generously supported for many years by the National Science Foundation and the Office of Naval Research. Addison–Wesley has provided excellent assistance and cooperation ever since I began this project in 1962. The best way I know how to thank everyone is to demonstrate by this publication that their input has led to books that resemble what I think they wanted me to write.

Preface to the Third Edition

After having spent ten years developing the TEX and METAFONT systems for computer typesetting, I am now able to fulfill the dream that I had when I began that work, by applying those systems to *The Art of Computer Programming*. At last the entire text of this book has been captured inside my personal computer, in an electronic form that will make it readily adaptable to future changes in printing and display technology. The new setup has allowed me to make literally thousands of improvements that I have been wanting to incorporate for a long time.

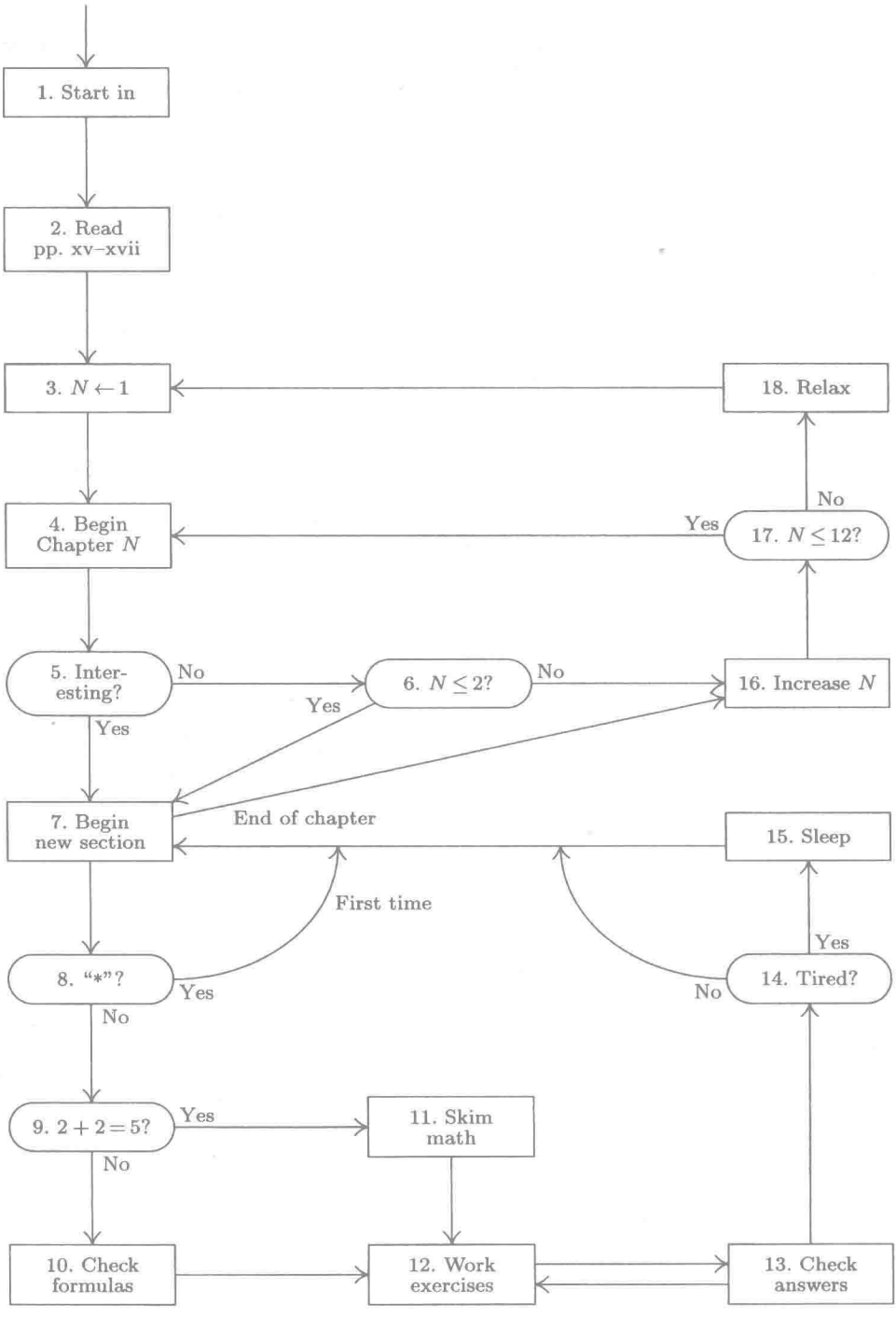
In this new edition I have gone over every word of the text, trying to retain the youthful exuberance of my original sentences while perhaps adding some more mature judgment. Dozens of new exercises have been added; dozens of old exercises have been given new and improved answers.

The Art of Computer Programming is, however, still a work in progress. Therefore some parts of this book are headed by an "under construction" icon, to apologize for the fact that the material is not up-to-date. My files are bursting with important material that I plan to include in the final, glorious, fourth edition of Volume 1, perhaps 15 years from now; but I must finish Volumes 4 and 5 first, and I do not want to delay their publication any more than absolutely necessary.

Most of the hard work of preparing the new edition was accomplished by Phyllis Winkler and Silvio Levy, who expertly keyboarded and edited the text of the second edition, and by Jeffrey Oldham, who converted nearly all of the original illustrations to METAPOST format. I have corrected every error that alert readers detected in the second edition (as well as some mistakes that, alas, nobody noticed); and I have tried to avoid introducing new errors in the new material. However, I suppose some defects still remain, and I want to fix them as soon as possible. Therefore I will cheerfully pay \$2.56 to the first finder of each technical, typographical, or historical error. The webpage cited on page iv contains a current listing of all corrections that have been reported to me.

Stanford, California D. E. K.
April 1997

Things have changed in the past two decades.
— BILL GATES (1995)



Flow chart for reading this set of books.

Procedure for Reading This Set of Books

1. Begin reading this procedure, unless you have already begun to read it. *Continue to follow the steps faithfully.* (The general form of this procedure and its accompanying flow chart will be used throughout this book.)
2. Read the Notes on the Exercises, on pages xv–xvii.
3. Set N equal to 1.
4. Begin reading Chapter N. Do *not* read the quotations that appear at the beginning of the chapter.
5. Is the subject of the chapter interesting to you? If so, go to step 7; if not, go to step 6.
6. Is $N \leq 2$? If not, go to step 16; if so, scan through the chapter anyway. (Chapters 1 and 2 contain important introductory material and also a review of basic programming techniques. You should at least skim over the sections on notation and about `MIX`.)
7. Begin reading the next section of the chapter; if you have already reached the end of the chapter, however, go to step 16.
8. Is section number marked with "*"? If so, you may omit this section on first reading (it covers a rather specialized topic that is interesting but not essential); go back to step 7.
9. Are you mathematically inclined? If math is all Greek to you, go to step 11; otherwise proceed to step 10.
10. Check the mathematical derivations made in this section (and report errors to the author). Go to step 12.
11. If the current section is full of mathematical computations, you had better omit reading the derivations. However, you should become familiar with the basic results of the section; they are usually stated near the beginning, or in *slanted type* right at the very end of the hard parts.
12. Work the recommended exercises in this section in accordance with the hints given in the Notes on the Exercises (which you read in step 2).
13. After you have worked on the exercises to your satisfaction, check your answers with the answer printed in the corresponding answer section at the

rear of the book (if any answer appears for that problem). Also read the answers to the exercises you did not have time to work. *Note:* In most cases it is reasonable to read the answer to exercise n before working on exercise $n+1$, so steps 12–13 are usually done simultaneously.

14. Are you tired? If not, go back to step 7.

15. Go to sleep. Then, wake up, and go back to step 7.

16. Increase N by one. If $N = 3$, 5, 7, 9, 11, or 12, begin the next volume of this set of books.

17. If N is less than or equal to 12, go back to step 4.

18. Congratulations. Now try to get your friends to purchase a copy of Volume 1 and to start reading it. Also, go back to step 3.

Woe be to him that reads but one book.
— GEORGE HERBERT, *Jacula Prudentum*, 1144 (1640)

Le défaut unique de tous les ouvrages
c'est d'être trop longs.
— VAUVENARGUES, *Réflexions*, 628 (1746)

Books are a triviality. Life alone is great.
— THOMAS CARLYLE, *Journal* (1839)

NOTES ON THE EXERCISES

THE EXERCISES in this set of books have been designed for self-study as well as classroom study. It is difficult, if not impossible, for anyone to learn a subject purely by reading about it, without applying the information to specific problems and thereby being encouraged to think about what has been read. Furthermore, we all learn best the things that we have discovered for ourselves. Therefore the exercises form a major part of this work; a definite attempt has been made to keep them as informative as possible and to select problems that are enjoyable as well as instructive.

In many books, easy exercises are found mixed randomly among extremely difficult ones. This is sometimes unfortunate because readers like to know in advance how long a problem ought to take—otherwise they may just skip over all the problems. A classic example of such a situation is the book *Dynamic Programming* by Richard Bellman; this is an important, pioneering work in which a group of problems is collected together at the end of some chapters under the heading "Exercises and Research Problems," with extremely trivial questions appearing in the midst of deep, unsolved problems. It is rumored that someone once asked Dr. Bellman how to tell the exercises apart from the research problems, and he replied, "If you can solve it, it is an exercise; otherwise it's a research problem."

Good arguments can be made for including both research problems and very easy exercises in a book of this kind; therefore, to save the reader from the possible dilemma of determining which are which, *rating numbers* have been provided to indicate the level of difficulty. These numbers have the following general significance:

Rating *Interpretation*

00 An extremely easy exercise that can be answered immediately if the material of the text has been understood; such an exercise can almost always be worked "in your head."

10 A simple problem that makes you think over the material just read, but is by no means difficult. You should be able to do this in one minute at most; pencil and paper may be useful in obtaining the solution.

20 An average problem that tests basic understanding of the text material, but you may need about fifteen or twenty minutes to answer it completely.