信号与线性系统

学习指南与题解

张渭滨 编著

Xinhao yu Xianxingxitong
Xuexi Zhinan Yu Tijie

知识产权出版社

信号与线性系统学习指南与题解

张渭滨　编著

知识产权出版社

图书在版编目(CIP)数据

信号与线性系统学习指南与题解/张渭滨编著. —北京:
知识产权出版社,2005.8

ISBN 7-80198-433-1

Ⅰ.信… Ⅱ.张… Ⅲ.①信号理论—自学参考资料
②线性系统—自学参考资料 Ⅳ.TV911.6

中国版本图书馆 CIP 数据核字(2005)第 097326 号

内容提要

本书是为高等学校通信、电子信息类专业《信号与线性系统》课程配套的习题教材.全书共分六章,第一章至第四章讨论连续时间系统,第五章讨论离散时间系统,第六章讨论状态方程与状态变量分析法.本书收录了相关内容的重要公式,精选了211个习题,全部习题均有详尽的解法.

本书可作高校无线电技术、通信、计算机、自动控制与电气工程等专业《信号与系统》课程学习的参考教材,也可供广大的自学者与相关的科技人员参考.

信号与线性系统学习指南与题解

编　　著:张渭滨

责任编辑:石红华

出版发行:知识产权出版社

社　址:北京市海淀区马甸南村1号	邮　编:100088
网　址:http://www.cnipr.com	邮　箱:zscq-bjb@126.com
电　话:010-82000890　82000860 转 8130	传　真:010-82000890
印　刷:知识产权出版社电子制印中心	经　销:新华书店及相关销售网点
开　本:850mm×1168mm　1/32	印　张:8.875
版　次:2005年10月第一版	印　次:2005年10月第一次印刷
字　数:220千字	定　价:30.00元

ISBN 7-80198-433-1/G·199

如有印装质量问题,本社负责调换

前　言

《信号与线性系统》课程是通信、电子、计算机、自动控制以及光学、地学等有关学科的专业基础课，是现代信息处理技术的重要基础课．为了掌握这门课程中解决问题的方法，在学习过程中解答一定数量的习题是至关重要的．

本书是《信号与线性系统》全书所有习题的解答，有的解答还提供了不同的方法．学习这门课程的学生或自修这门课程的读者，在独立思考和独立解题的基础上，可把解题思路、结果与本习题解进行比较，以总结自己解题方法的优缺点，开拓自己的思路．但读者切忌依赖习题解学习《信号与线性系统》这门课程，因为分析问题与解决问题能力的培养绝对不可能来自翻阅习题解，那毕竟是别人思考的结果．只有实践（在课程学习过程中，解习题就是一种实践）才能提高自己．尤其是学生，若依赖习题解来应付老师的作业，那就完全背离了编写这本习题解的初衷，于教于学都是有损无益的．

在编写习题解的过程中，作者力图使解题方法新颖、结果正确，但由于水平有限，错误与不足在所难免，因此作者衷心地欢迎读者指正．

目　　录

第一章　绪　论

一、基本公式

1. 典型连续信号

1.1　复指数信号

$$\begin{aligned} f(t) &= A\mathrm{e}^{st} = A\mathrm{e}^{(\sigma+\mathrm{j}\omega)t} \\ &= A\mathrm{e}^{\sigma t}\mathrm{e}^{\mathrm{j}\omega t} \\ &= A\mathrm{e}^{\sigma t}(\cos\omega t + \mathrm{j}\sin\omega t) \end{aligned} \tag{1-1}$$

当 $\sigma = 0, \omega = 0$

$$f(t) = A \quad \text{直统信号} \tag{1-2}$$

当 $\sigma = 0 \quad \omega \neq 0$

$$f(t) = A(\cos\omega t + \mathrm{j}\sin\omega t) \quad \text{等幅振荡信号} \tag{1-3}$$

当 $\sigma < 0 \quad \omega \neq 0$

$$f(t) = A\mathrm{e}^{\sigma t}\cos\omega t + \mathrm{j}A\mathrm{e}^{\sigma t}\sin\omega t \quad \text{衰减振荡信号} \tag{1-4}$$

当 $\sigma \neq 0, \omega = 0$

$$f(t) = A\mathrm{e}^{\sigma t} \quad \text{实指数信号} \tag{1-5}$$

1.2　抽样信号

$$S_{\mathrm{a}}(t) = \frac{\sin t}{t} \tag{1-6}$$

1.3　高斯信号

$$f(t) = E\mathrm{e}^{-(t/\tau)^2} \tag{1-7}$$

1.4　单位斜变信号

$$f(t) = \begin{cases} t & (t \geqslant 0) \\ 0 & (t < 0) \end{cases} \tag{1-8}$$

2. 奇异信号

2.1　单位阶跃信号

$$u(t) = \begin{cases} 1 & (t > 0) \\ 0 & (t < 0) \end{cases} \tag{1-9}$$

2.2 矩形脉冲信号

$$G(t) = u(t) - u(t - t_0) \tag{1-10}$$

2.3 符号信号

$$\mathrm{sgn}(t) = \begin{cases} 1 & (t > 0) \\ -1 & (t < 0) \end{cases} \tag{1-11}$$

显然有

$$u(t) = \frac{1}{2} + \frac{1}{2}\mathrm{sgn}(t) \tag{1-12}$$

$$\mathrm{sgn}(t) = 2u(t) - 1 \tag{1-13}$$

2.4 单位冲激信号

定义 $$\int_{-\infty}^{\infty} \delta(t)\,\mathrm{d}t = \int_{0-}^{0+} \delta(t)\,\mathrm{d}t = 1 \qquad \delta(t) = 0 \quad (t \neq 0) \tag{1-14}$$

$\delta(t)$ 与 $u(t)$ 关系

$$\frac{\mathrm{d}u(t)}{\mathrm{d}t} = \delta(t) \tag{1-15}$$

$$u(t) = \int_{-\infty}^{t} \delta(t)\,\mathrm{d}t \tag{1-16}$$

$\delta(t)$ 的抽样性质

$$f(t)\delta(t) = f(0)\delta(t) \tag{1-17}$$

$$\int_{-\infty}^{\infty} f(t)\delta(t)\,\mathrm{d}t = f(0) \tag{1-18}$$

$$f(t)\delta(t - t_0) = f(t_0)\delta(t - t_0) \tag{1-19}$$

$$\int_{-\infty}^{\infty} f(t)\delta(t - t_0)\,\mathrm{d}t = f(t_0) \tag{1-20}$$

对称性

$$\delta(t) = \delta(-t) \tag{1-21}$$

标度变换

$$\delta(at) = \frac{1}{|a|}\delta(t) \tag{1-22}$$

2.5 冲激偶函数

定义

$$\frac{\mathrm{d}\delta(t)}{\mathrm{d}t}=\delta'(t) \tag{1-23}$$

$$\int_{-\infty}^{t}\delta'(\tau)\mathrm{d}\tau=\delta(t) \tag{1-24}$$

性质

$$\int_{-\infty}^{\infty}f(t)\delta'(t)\mathrm{d}t=-f'(0) \tag{1-25}$$

$$\int_{-\infty}^{\infty}f(t)\delta'(t-t_0)\mathrm{d}t=-f'(t_0) \tag{1-26}$$

$$\int_{-\infty}^{\infty}\delta'(t-t_0)\mathrm{d}t=0 \tag{1-27}$$

$$\delta'[-(t-t_0)]=-\delta'(t-t_0) \tag{1-28}$$

$$\int_{-\infty}^{\infty}f(t)\delta^{(n)}(t-t_0)\mathrm{d}t=(-1)^n f^{(n)}(t_0) \tag{1-29}$$

3. 线性时不变系统性质

3.1 线性

设 $T[e_1(t)]=r_1(t),\quad T[e_2(t)]=r_2(t)$

若有 $$T[c_1e_1(t)+c_2e_2(t)]=c_1r_1(t)+c_2r_2(t) \tag{1-30}$$

则称系统为线性系统,式中算符 T 代表系统的作用,c_1、c_2 为任意常数.

3.2 时不变性

设 $T[e(t)]=r(t)$,若有

$$T[e(t-t_0)]=r(t-t_0), \tag{1-31}$$

则称系统为时不变系统.

3.3 线性时不变系统

同时具有线性与时不变性的系统.线性时不变系统具有微分特性与积分特性.

若 $T[e(t)]=r(t)$,则有

$$T\left[\frac{\mathrm{d}e(t)}{\mathrm{d}t}\right]=\frac{\mathrm{d}r(t)}{\mathrm{d}t} \tag{1-32}$$

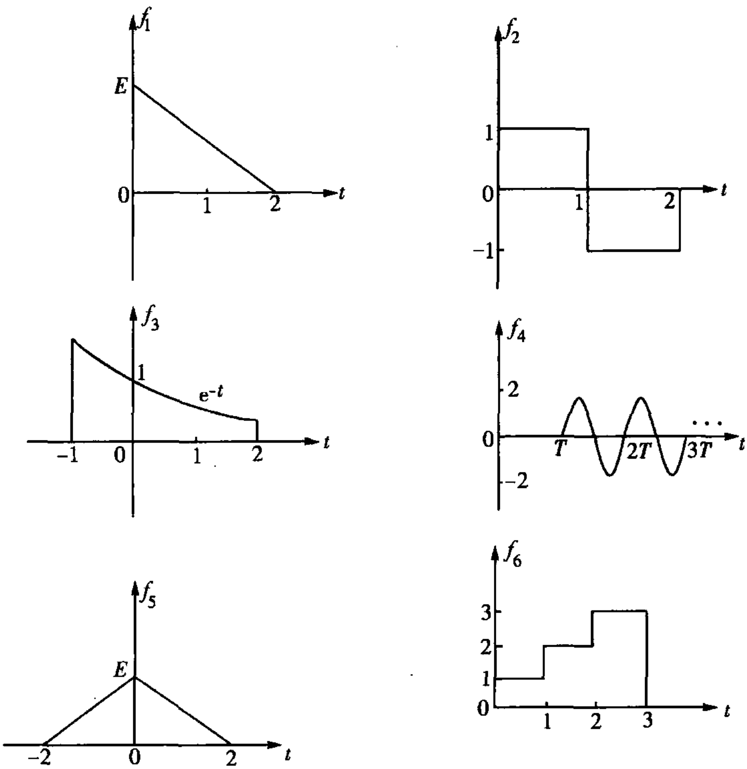
$$T\left[\int_{-\infty}^{t} e(\tau)\,\mathrm{d}\,\tau\right] = \int_{-\infty}^{t} r(\tau)\,\mathrm{d}\,\tau \tag{1-33}$$

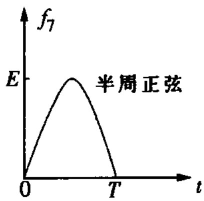
二、本章重点

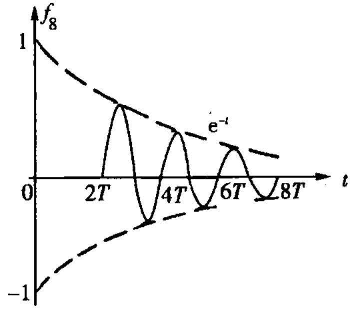
1. 信号的函数表示与图形表示 .
2. 单位冲激信号与单位阶跃信号 .
3. 线性时不变系统的性质 .
4. 正交函数族以及 n 维信号空间的概念 .

三、习题解答

1.1　写出题图 1-1 所示信号的函数式 .







题图 1-1

解 $f_1(t) = E\left(1 - \dfrac{t}{2}\right)[u(t) - u(t-2)]$

$$f_2(t) = u(t) - 2u(t-1) + u(t-2)$$

$$f_3(t) = e^{-t}[u(t+1) - u(t-2)]$$

$$f_4(t) = 2\sin\frac{2\pi}{T}(t-T)u(t-T) = 2\sin\frac{2\pi}{T}tu(t-T)$$

$$f_5(t) = \frac{E}{2}(t+2)[u(t+2) - u(t)] +$$

$$\frac{E}{2}(2-t)[u(t) - u(t-2)]$$

$$= \frac{E}{2}(2-|t|)[u(t+2) - u(t)] +$$

$$\frac{E}{2}(2-|t|)[u(t) - u(t-2)]$$

$$= E\left(1 - \frac{|t|}{2}\right)[u(t+2) - u(t-2)]$$

$$f_6(t) = u(t) + u(t-1) + u(t-2) - 3u(t-3)$$

$$f_7(t) = E\sin\frac{2\pi}{2T}t[u(t) - u(t-T)]$$

$$= E\sin\frac{\pi}{T}t[u(t) - u(t-T)]$$

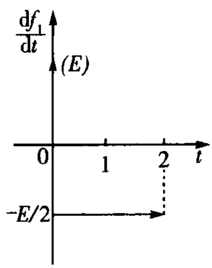
$$f_8(t) = e^{-t}\sin\frac{2\pi}{2T}t[u(t-2T)] = e^{-t}\sin\frac{\pi}{T}tu(t-2T)$$

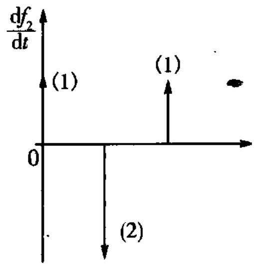
1.2　求题 1.1 中各信号的微分并画出其波形图.

解

$$\frac{df_1(t)}{dt} = E\delta(t) - \frac{E}{2}[u(t) - u(t-2)]$$

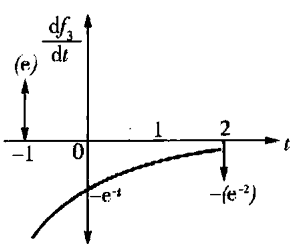
$$\frac{df_2(t)}{dt} = \delta(t) - 2\delta(t-1) + \delta(t-2)$$

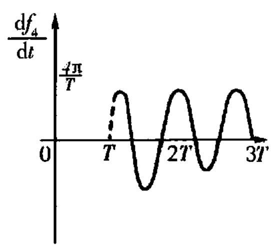




$$\frac{df_3(t)}{dt} = e\cdot\delta(t+1) - e^{-2}\delta(t-2) - e^{-t}[u(t+1) - u(t-2)]$$

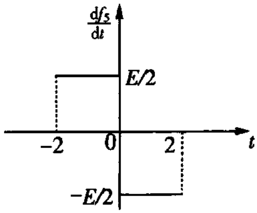
$$\frac{df_4(t)}{dt} = \frac{4\pi}{T}\cos\frac{2\pi}{T}tu(t-T)$$

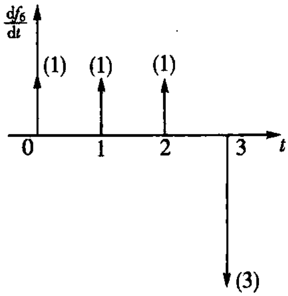




$$\frac{df_5(t)}{dt} = \frac{E}{2}[u(t+2) - u(t)] - \frac{E}{2}[u(t) - u(t-2)]$$

$$\frac{df_6(t)}{dt} = \delta(t) + \delta(t-1) + \delta(t-2) - 3\delta(t-3)$$





$$\frac{df_7(t)}{dt} = \frac{E\pi}{T}\cos\frac{\pi}{T}t[u(t) - u(t-T)]$$

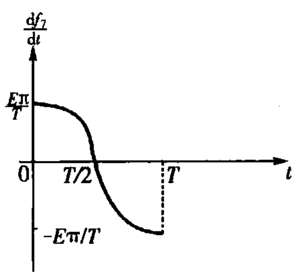
$$\frac{df_8(t)}{dt} = \left[-e^{-t}\sin\frac{\pi t}{T} + \frac{\pi}{T}e^{-t}\cos\frac{\pi t}{T}\right]u(t-2T) +$$

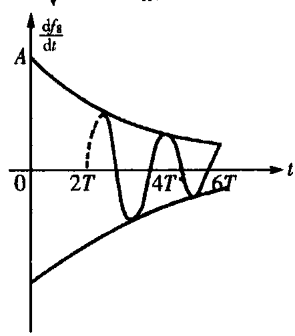
$$e^{-t}\sin\frac{\pi}{T}t\delta(t-2T)$$

$$= e^{-t}\left[\frac{\pi}{T}\cos\frac{\pi t}{T} - \sin\frac{\pi t}{T}\right]u(t-2T)$$

$$= e^{-t}\left[A\cos\left(\frac{\pi}{T}t + \varphi\right)\right]u(t-2T)$$

$$A = \sqrt{1 + (\frac{\pi}{T})^2}, \quad \varphi = \sin^{-1}\left(\frac{\pi/T}{\sqrt{1+(\pi/T)^2}}\right)$$





1.3 画出下列各时间函数的波形图

(1) $f_1(t) = e^{-t+1}u(t-1)$

(2) $f_2(t) = e^{-t+1}u(t)$

(3) $f_3(t) = e^{-t}[u(t) - u(t-1)]$

(4) $f_4(t) = \dfrac{\sin\omega(t-t_0)}{\omega(t-t_0)}$

(5) $f_5(t) = (1+\cos\pi t)[u(t) - u(t-4)]$

(6) $f_6(t) = \sin\omega t u(t)$

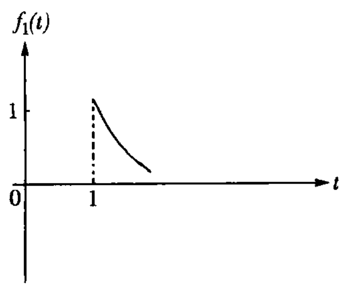
(7) $f_7(t) = \sin(t-1)u(t-1)$

(8) $f_8(t) = \sin\omega(t-t_0)u(t)$

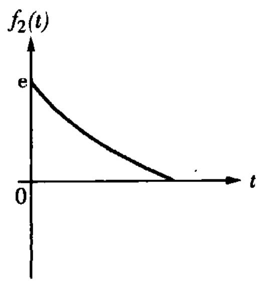
(9) $f_9(t) = \sin\omega t u(t-t_0)$

(10) $f_{10}(t) = \sin\omega(t-t_0)u(t-t_0)$

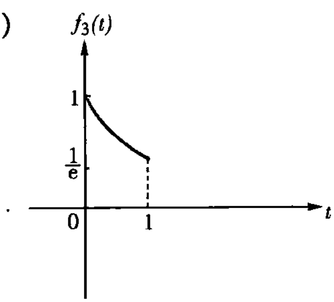
解 (1)



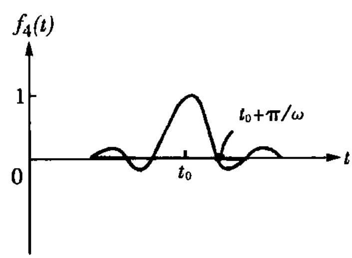
(2)



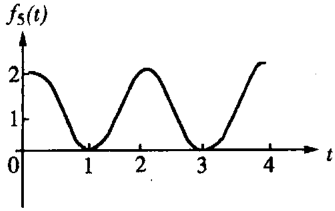
(3)



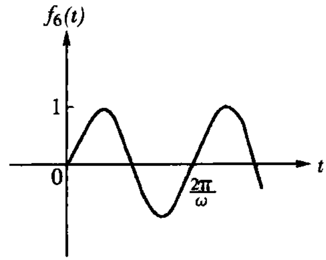
(4)



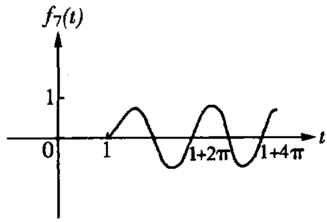
(5)



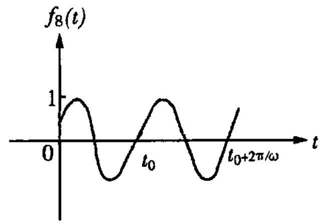
(6)



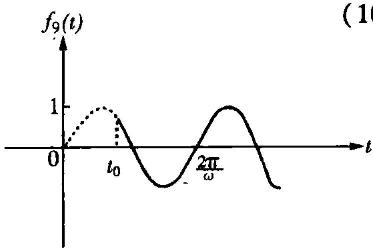
(7)



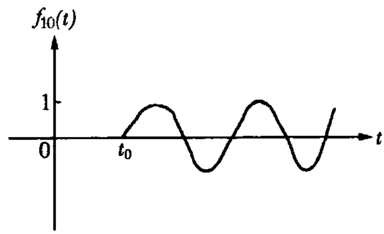
(8)



(9)



(10)



1.4 画出下列信号的波形图.

(1) $f_1(t) = tu(t)$

(2) $f_2(t) = (t-1)u(t)$

(3) $f_3(t) = (t+1)u(t)$

(4) $f_4(t) = (t+1)u(t+1)$

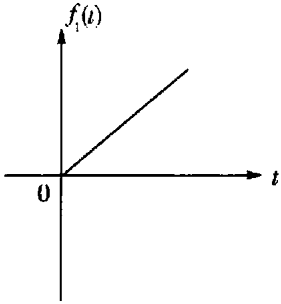
(5) $f_5(t) = t[u(t) - u(t-1)]$

(6) $f_6(t) = (t-2)[u(t-2) - u(t-3)]$

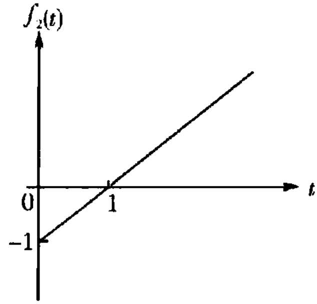
(7) $f_7(t) = tu(t) - (t-1)u(t)$

(8) $f_8(t) = t[u(t) - u(t-1)] + u(t-1)$

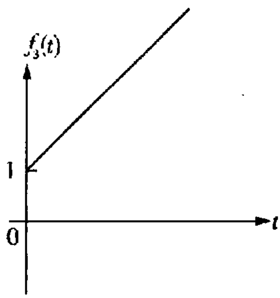
解 (1)


$f_1(t)$
0
t

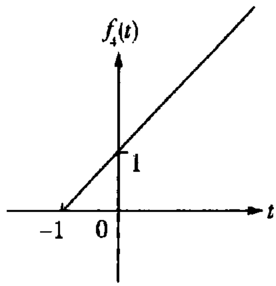
(2)


$f_2(t)$
0
1
−1
t

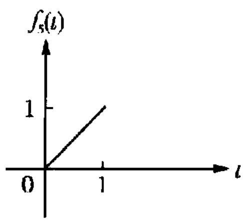
(3)


$f_3(t)$
1
0
t

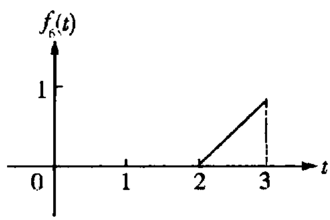
(4)


$f_4(t)$
1
−1
0
t

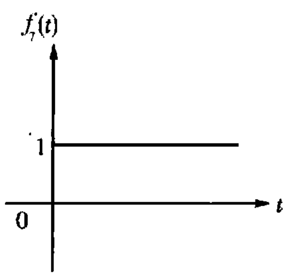
(5)


$f_5(t)$
1
0
1
t

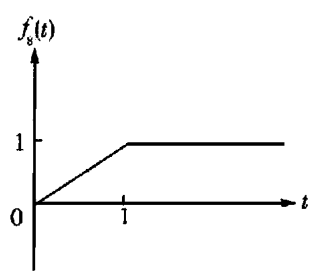
(6)


$f_6(t)$
1
0
1
2
3
t

(7)


$f_7(t)$
1
0
t

(8)


$f_8(t)$
1
0
1
t

1.5　应用冲激信号的抽样特性计算以下各式.

(1) $\int_{-\infty}^{\infty} f(t+t_0)\delta(t-t_0)\mathrm{d}t = f(t_0+t_0) = f(2t_0)$

(2) $\int_{-4}^{2} \mathrm{e}^t\delta(t+3)\mathrm{d}t = \int_{-4}^{2}\mathrm{e}^t\delta[t-(-3)]\mathrm{d}t = \mathrm{e}^{-3}$

(3) $\int_{-\infty}^{\infty} \mathrm{e}^{-t}\sin t\delta(t+1)\mathrm{d}t = \mathrm{e}^{-(-1)}\sin(-1) = -\mathrm{e}\cdot\sin 1$

(4) $\int_{-\infty}^{\infty} f(t-t_0)\delta(t)\mathrm{d}t = f(-t_0)$

(5) $\int_{-\infty}^{\infty} f(t_0-t)\delta(t)\mathrm{d}t = f(t_0)$

(6) $\int_{-\infty}^{\infty} \delta(t-t_0)u(t-t_0/2)\mathrm{d}t = u\left(\frac{t_0}{2}\right) = \begin{cases}1 & t_0>0\\0 & t_0<0\end{cases}$

(7) $\int_{-\infty}^{\infty} \delta(t-t_0)u(t-2t_0)\mathrm{d}t = u(-t_0) = \begin{cases}0 & t_0>0\\1 & t_0<0\end{cases}$

(8) $\int_{-\infty}^{\infty} (\mathrm{e}^{-t}+t)\delta(t+2)\mathrm{d}t = \mathrm{e}^{-(-2)}+(-2) = \mathrm{e}^2-2$

(9) $\int_{-\infty}^{\infty} (t+\sin t)\delta(t-\pi/6)\mathrm{d}t = \frac{\pi}{6}+\sin\frac{\pi}{6} = \frac{\pi}{6}+1/2$

(10) $\int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{j}\omega t}[\delta(t)-\delta(t-t_0)]\mathrm{d}t = 1-\mathrm{e}^{-\mathrm{j}\omega t_0}$

1.6　判断下列系统是否为线性系统(设输入信号为 $e(t)$，输出为 $r(t)$)，说明理由.

(1) $r(t)=a+be(t)$，即 $H[e(t)]=a+be(t)$，$H[ce(t)]=a+bce(t)\neq cr(t)=ac+bce(t)$，故为非线性系统.

(2) $r'(t)=a+be(t)$

则　$r(t)=\int[a+be(t)]\mathrm{d}t,\ H[e(t)]=\int[a+be(t)]\mathrm{d}t$

$$H[ce(t)]=\int[a+bce(t)]\mathrm{d}t\neq cr(t)$$

故为非线性系统.

(3) $r(t)=a+be'(t)$，即 $H[e(t)]=a+be'(t)$，$H[ce(t)]=a+b(ce(t))'=a+bce'(t)\neq cr(t)$，故为非线性系统.

(4) $r(t)=a+e(t-c)$，即 $H[e(t)]=a+e(t-c)$，而 $H[ce(t)]=a+ce(t-c)\neq cr(t)$，故为非线性系统.

(5) $r(t-c)=a+be(t)$，即 $r(t)=a+be(t+c)$，如(4)为非线

性系统.

1.7 判断下列系统是否为时不变系统:

(1) $r(t) = a + be(t) \qquad r(t-t_0) = a + be(t-t_0)$

而 $H[e(t)] = a + be(t)$,则 $H[e(t-t_0)] = a + be(t-t_0)$

故为时不变系统.

(2) $r(t) = a + be(t-c) \qquad r(t-t_0) = a + be(t-t_0-c)$

而 $H[e(t)] = a + be(t-c)$,则 $H[e(t-t_0)] = a + be(t-t_0-c)$

故为时不变系统.

(3) $r(t) = a + bte(t) \qquad r(t-t_0) = a + b(t-t_0)e(t-t_0)$

而 $H[e(t)] = a + bte(t)$,则 $H[e(t-t_0)] = a + bte(t-t_0)$

故不是时不变系统.

1.8 设 $e(t)$ 与 $r(t)$ 分别为系统的激励与响应,试根据以下的输入与输出关系确定系统是否为线性(假设齐次性成立)与时不变系统.

(1) $r(t) = [e(t)]^2$, $H[e_1(t)] = [e_1(t)]^2 = r_1(t)$

$H[e_2(t)] = [e_2(t)]^2 = r_2(t)$

而 $\quad H[e_1(t) + e_2(t)] = [e_1(t) + e_2(t)]^2 \neq r_1(t) + r_2(t)$

故为非线性系统.

又 $\quad r(t) = [e(t)]^2, \quad r(t-t_0) = [e(t-t_0)]^2$

而 $H[e(t)] = [e(t)]^2$, 则 $H[e(t-t_0)] = [e(t-t_0)]^2$

即 $H[e(t-t_0)] = r(t-t_0)$, 故为时不变系统.

(2) $r(t) = e(t) + e(t-T)$

可加性: $r_1(t) + r_2(t) = e_1(t) + e_1(t-T) + e_2(t) + e_2(t-T)$

而 $\quad H[e_1(t) + e_2(t)] = e_1(t) + e_2(t) + e_1(t-T) + e_2(t-T)$

$$= r_1(t) + r_2(t)$$

故满足可加性,题目已给定均匀性,故为线性系统.

时不变性: $\quad r(t-t_0) = e(t-t_0) + e(t-T-t_0)$

而 $\quad H[e(t-t_0)] = e(t-t_0) + e(t-T-t_0)$

故有时不变性. 系统为线性时不变系统.

(3) $r(t) = \begin{cases} e(t) & e(t) > 0 \\ 0 & e(t) < 0 \end{cases} \qquad r_1(t) = \begin{cases} e_1(t) & e_1(t) > 0 \\ 0 & e_1(t) < 0 \end{cases}$