DIFFUSIONS, MARKOV PROCESSES AND MARTINGALES

Volume 1
FOUNDATIONS

扩散 马尔可夫过程和鞅
第 1 卷

L. C. G. Rogers & D. Williams

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Diffusions, Markov Processes, and Martingales

Volume 1: FOUNDATIONS

2nd Edition

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For our parents

From the Original (1979) Preface

Long ago (or so it seems today), Chung wrote on page 196 of his book [1]: 'One wonders if the present theory of stochastic processes is not still too difficult for applications.' Advances in the theory since that time have been phenomenal, but these have been accompanied by an increase in the technical difficulty of the subject so bewildering as to give a quaint charm to Chung's use of the word 'still'. Meyer writes in the preface to his definitive account of stochastic integral theory: '*...il faut... un cours de six mois sur les définitions. Que peut on y faire?*'

I have thought up as intuitive a picture of the subject as I can, written it down at speed, and refused to be lured back by piety (or even by wit!) to cancel half a line. 'First' intuition, which is what you need when you are learning the subject, is raw, rough and ready; and, as you have guessed, I make the excuse that it demands a compatible style and lack of polish.

Note that I wrote '*first intuition*'. Consider an example. Meyer's concept of a *right process* is exactly right for Markov process theory, but the concept is the result of a long evolution. To understand it properly, you need a highly developed intuition, and that takes time to acquire. The difficulty with the best advanced literature is that its authors have too *much* intuition; never make the mistake of thinking otherwise.

My aim then is to sharpen your intuition to a point where the advanced abstract literature becomes accessible, enjoyable and 'relevant'. Like my expository article [1], this is a missionary tract not a theological treatise. (Those of you who have read my article [1] will see that this book often follows it very closely, except that now I have the time and the duty to be more obviously appreciative of the abstract theory!)

I believe that, in the end, it is *applications* which justify mathematics. The 'artistic' justification of pure mathematics in terms of intrinsic qualities like elegance and generality rings rather hollow in my ears when I compare the best mathematics with the greatest music. Many applied workers will regard this book as extremely 'pure', but I see it as *one stage in shunting pure theory over towards applications*. The shunting is not always necessary: time and again, one finds 'applied' papers which 'solve' problems long since solved for 'purely theoretical' purposes. Moral: the pure/applied division of probability theory (as of mathematics in general) is a nonsense.

Acknowledgements. This is an appropriate place at which to thank David Kendall and Harry Reuter for teaching me probability theory and for giving me an enthusiasm for the subject which is wearing well. My best way to thank them is to try to share that enthusiasm.

I have to say another huge 'thank you' to David Kendall for the immense amount of work he has done in making editorial comments on the original manuscript. I now see that my determination to convey a sense of adventure did need to be tempered by a greater concern for the reader's sense of security. So I have acceded to many of David Kendall's requests for 'more details'; and as a result, you will learn more techniques of calculation and have a clearer idea of several concepts. (But I still see it as part of my job to keep you on your toes!)

I am very grateful to Ronald Getoor and André Meyer for clearing up some confusions.

I have been extremely fortunate in having been able to rely on the superb typing skills of Sheila Campbell, Eileen Jenkins and Gladys Maddocks; my thanks and best wishes to them.

I thank Springer-Verlag and the authors for granting me permission to quote from Chung [1] in Section III.44, from Getoor [1] in Section III.54, and from Chung [1] and Meyer [1] earlier in ths preface.

Finally, I have to thank James Cameron and Wiley for encouragement and great patience; and subeditors, copy-editors, and printers, whose skills have much impressed me.

David Williams
Swansea, 1978

Preface to the Second Edition

This second edition differs profoundly from the first—and not only in having two authors rather than one. We retain the Gallic tradition of dividing the volume into three massive chapters: Chapter I, which says why the subject is worth studying; Chapter II, which provides background; and Chapter III, which presents an account of Markov processes. Chapter I is now much more extensive and wide-ranging, and covers much work done since the first edition appeared. Chapter II is now a highly systematic account, with detailed proofs, of what every young probabilist must know. It is rather unashamedly a sequel to DW's *Probability with Martingales*, Cambridge University Press having been very generous in allowing us to follow that account closely (but without many proofs, without the examples, etc.). *It is perfectly possible to read Chapter II before Chapter I if you so wish.* We would suggest however that you try things in the order 'heuristics then rigour':

> *'Our doubts are traitors,*
> *And make us lose the good we oft might win,*
> *Through fearing to attempt.*

(W. Shakespeare, *Measure for Measure.*)

Chapter III seems to have been regarded as the most successful part of the original; and it is reproduced here without much modification (except that some of the functional analysis is given fuller treatment). It was always intended as a missionary tract on Markov processes. The full theory may be found in Sharpe [1] and in the final two volumes of the probabilist's bible, Dellacherie and Meyer [1]. All kinds of important developments are ignored in Chapter III: they would require another complete volume, and will be, or are, covered by greater experts. Dawson's eagerly awaited treatment [1] of measure-valued processes has now appeared; Mark Davis has a very nice new book [4] on piecewise-deterministic Markov processes; and so on. You can access the huge literature on measure-valued processes via Dawson's account.

The musical allusions in the first edition have been excised. Apparently many people found them annoying. 'Would David Williams like a book on mathematics filled with references to baseball?', they say. (To which the answer is, of course, 'Yes.') So, this is Mathematics all the way from A to Zzzz—or from Ω on, if you want to be rigorous.

Our thanks to Sue Collins and Wolfgang Stummer, and to other colleagues at Bath, Cambridge, and Queen Mary and Westfield College, London. Our thanks too to Helen Ramsey and other Wiley staff for suggesting this new version; and the copy-editor and printer whose skills have impressed us.

Chris Rogers
David Williams
November 1993

Some Frequently Used Notation

We use ':=' to mean 'is defined to equal'. This Pascal notation can also be used in reverse. We define

$$\mathbb{Z}^+ := \{0, 1, 2, \ldots\} \supseteq \{1, 2, 3, \ldots\} =: \mathbb{N},$$

$$\mathbb{R}^+ := [0, \infty), \qquad \mathbb{R}^{++} := (0, \infty), \qquad \mathbb{Q}^+ := \mathbb{Q} \cap \mathbb{R}^+.$$

We neaten layout, and make things easier for our printers, by the use of alternative notations:

$$X(t_1, \omega) \text{ for } X_{t_1}(\omega), \quad f_{n(1)} \text{ for } f_{n_1}, \quad \mathscr{F}(T_1) \text{ for } \mathscr{F}_{T_1}, \quad P_t f(x) \text{ for } (P_t f)(x),$$

etc. Once things are underway, such switches in notation will be made without comment. The composition notation

$$f \circ g(t) := f(g(t))$$

will often be used for tidiness.

If f and g are real numbers or real-valued functions, we define

$$f \vee g := \max(f, g), \quad f \wedge g := \min(f, g), \quad f^+ := f \vee 0, \quad f^- := (-f) \vee 0;$$

hence $f = f^+ - f^-$ and $|f| = f^+ + f^-$.

If $\mathscr{H}$ is a set of real-valued functions, we write

$\mathscr{H}^+$ for the set of non-negative elements of $\mathscr{H}$,

$b\mathscr{H}^+$ for the set of bounded elements in $\mathscr{H}$.

If Σ is a σ-algebra, we write

$m\Sigma$ for the set of real-valued (or perhaps $[\infty, \infty]$-valued)

Σ-measurable functions,

$b\Sigma$ for the space of bounded Σ-measurable functions.

If S is a topological space, we write

$C(S)$ for the space of *all* continuous functions from S to $\mathbb{R}$.
$C_b(S)$ for the space of all *bounded* continuous functions from S to $\mathbb{R}$.

Monotone convergence. We write '$s \uparrow t$' to signify that $s \to t, s \leqslant t$; and '$s \uparrow\uparrow t$' to signify that $s \to t, s < t$. If (s_n) is a sequence then '$s_n \uparrow t$' signifies that $s_n \to t, s_n \leqslant s_{n+1} \leqslant t$; while '$s_n \uparrow\uparrow t$' signifies that $s_n \to t, s_n \leqslant s_{n+1} < t$. If f_n and f are real-valued functions then (for example) $f := \uparrow \lim f_n$ signifies that $f_n \uparrow f$ pointwise.

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