Mathematics Monograph Series 30

The Elements of Financial Econometrics

Jianqing Fan Qiwei Yao (范剑青 姚琦伟)

(计量金融精要)

SCIENCE PRESS
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Editorial Board

Preface to Mathematics Monograph Series

Science Press asked me to write a preface for their series of books called "Mathematics Monograph Series". They told me that the Press had published nearly 30 mathematical monographs in this series since 2006. This reminded me that, also in 2006, I received an email message from the Editor in Chief of "Sugaku Tushin" ("Mathematical Communications", the membership magazine of the Mathematical Society of Japan). The Editor in Chief told me that they were planning to have a special section on "Recent development in Chinese mathematical community", and invited me to write an article for the special section. As a result, I published in their magazine an article called "Some Aspects of Mathematical Community in China". Among other things, in the article I demonstrated that, with the favorable environment, the Chinese mathematical community had made great progress since the late 1970s (when China started to implement "Reform and Opening-up Policy"): A large number of publications (including articles and monographs) have been written by Chinese mathematicians, there have been always Chinese mathematicians presenting their speeches at various international academic conferences or workshops, many Chinese mathematicians have served as editors of international academic journals, or as members in various academic organizations. All these reveal that the Chinese mathematical community which has been growing rapidly has exerted more and more influence in the world.

Indeed, the series of mathematical monographs published by Science Press reflects partly more and more influence of Chinese mathematical community in the world. Chinese people are good at mathematics. In the past, Science Press published many high level mathematical monographs and textbooks. Among them some were written in Chinese and some were written in English. Some monographs which appeared originally in Chinese have been purchased by international publishers who then re-published them abroad in English, and this has gained influence in corresponding areas of the international mathematical community.

In recent years most Chinese mathematicians have mastered good English. In accordance with this situation, Science Press has decided to publish "Mathematics Monograph Series" —— a series in which high level mathematical monographs and textbooks are written directly in English. The goal of this series is to provide further good service for Chinese mathematicians and to enhance further the influence of the mathematics study in China in the international mathematical community.

I would like to conclude this short preface with the following wish which I expressed also at the end of the afore mentioned article "Some Aspects of Mathematical Community in China":

The Chinese mathematical community will continuously make its effort to work hard, and to strengthen its international exchanges and collaborations, so as to make more contributions to the study and development of mathematics in the world.

Zhi-Ming Ma
March 15, 2015

Preface

This is an introductory textbook for financial econometrics at the master's level. Readers are assumed to have some background in calculus, linear algebra, statistics, and probability, all at undergraduate level. Knowledge in economics and finance is beneficial but not essential.

This book grew out of the lecture notes for the "Financial Econometrics" course taught by Jianqing Fan for Master in Finance students at Princeton University since 2003 and for Master in Financial Engineering students at Fudan University since 2011. The audiences are always very broad with diverse background in mathematics, physics, computer science, economics, or finance. Those talented students wish to learn fundamentals of financial econometrics in order to work in financial industry or to pursue a Ph.D. degree in related fields. The challenges are to give all of them sufficient financial econometrics knowledge upon the completion of the class. This text book is written with the aim to achieve such an ambitious goal.

We trust that the book will be of interest to those coming to the area for the first time, to readers who already have economics and finance background but would like to further sharpen their quantitative skills, and also to those who have strong quantitative skills and would like to learn how to apply them to finance. Application-oriented analysts will also find this book useful, as it focuses on methodology and includes numerous case studies with real data sets. We purposely keep the level of mathematics moderate, as the power of the quantitative methods can be understood without sophisticated technical details. We also avoid the cook book style of writing, as good understanding of statistical and econometric principles in finance enable readers to apply the knowledge far beyond the problems stated in the book. Numerical illustration with real financial data is throughout the book. We also indicate, whenever possible, where to find the relevant `R` codes to implement the various methods. Due to the nature of the subject, it is inevitable that we occasionally step into more sophisticated techniques which rely on more advanced mathematics; such sections are marked with "$*$" and can be ignored for beginners. Most technical arguments are collected in a "Complements" section at the end of some chapters,

but key ideas are left within the main body of the text.

What is financial econometrics? Broadly speaking, it is an interdisciplinary subject that uses statistical methods and economic theory to address a variety of quantitative problems in finance. These include building financial models, testing financial economics theory, simulating financial systems, volatility estimation, risk management, capital asset pricing, derivative pricing, portfolio allocation, proprietary trading, portfolio and derivative hedging, among others. Financial econometrics is an active field of integration of finance, economics, probability, statistics, and applied mathematics. Financial activities generate many new problems and products, economics provides useful theoretical foundation and guidance, and quantitative methods such as statistics, probability and applied mathematics are essential tools to solve quantitative problems in finance. Professionals in finance now routinely use sophisticated statistical techniques and modern computation power in portfolio management, proprietary trading, derivative pricing, financial consulting, securities regulation, and risk management.

When the class was first taught in 2003, there were very few books on financial econometrics. The books that have strong impact on our preparation of lecture notes are Campbell et al. (1997) and Fan and Yao (2003). With one semester of teaching, we can only cover the important elements of financial econometrics. We use this name as the title of the book, as it also reflects the modulus aspect of the book. This allows us to expand this textbook to cover other fundamental materials in the future. For example, "Simulation methods in finance" and "Econometrics of continuous time finance" are taught at Princeton, but are not in this book nor in Fudan's financial engineering class due to time constraints. Another important topic that we wish to cover is the analysis of high-frequency financial data.

The book consists of two integrated parts: The first four chapters are on time series aspects of financial econometrics while the last five chapters on cross-sectional aspects. The introduction in Chapter 1 sets the scene for the book: using two financial price time series we illustrate the stylized features in financial returns. The efficient markets hypothesis is deliberated together with statistical tests for random walks and white noise. A compact view of linear time series models is given in Chapter 2, including ARMA models, random walks, and inference with trends. We also include a brief introduction on the exponential smoothing based forecasting techniques for trends and momentum, which are widely used in financial industry. Chapter 3 introduces various heteroscedastic volatility models. A compact introduction to state space models including the techniques such as Kalman filter and particle filters is included as an appendix. Chapter 4 contains some selective topics in multivariate time series analysis. Within the context of vector autoregressive models, we also introduce the topics such as Granger causality, impulse response

functions, and cointegration, as they play important roles in economics and finance.

The second begins with Chapter 5, which introduces portfolio theory and derives the celebrated capital asset model. We also introduce statistical techniques to test such a celebrated model and provide extensive empirical studies. Chapter 6 extends the capital asset pricing model to multi-factor pricing model. The applications of the factor models and econometrics tests on the validity of such pricing models are introduced. In addition, principal component analysis and factor analysis are briefly discussed. Chapter 7 touches several practical aspects of portfolio allocation and risk management. The highlights of this chapter include risk assessments of large portfolios, portfolio allocation under gross-exposure constraints, and large volatility matrix estimation using factor models and covariance regularization. Chapter 8 derives the capital asset pricing model from consumption, investment, and saving point of view. This gives students different perspective on where the financial prices come from and a chance to appreciate its differences from pricing financial derivatives. Chapter 9 calculates the prices implied by the models of returns. It gives us an idea of what the fundamental price of a stock is and how the prices are related to the dividend payments and short-term interest rates.

Many people have been of great help to our work on this book. Early drafts of this book has been taught to about over five hundred students and there are our enthusiastic readers. In particular, we are grateful to Yingying Fan, Yue Niu, Jingjing Zhang, Feng Yang, Weijie Gu, Xin Tong, Wei Dai, Jiawei Yao, Xiaofeng Shi, and Weichen Wang for their gracious assistance of teaching Financial Econometrics class at Princeton. Réne Carmona provides us his course outlines on Financial Econometrics that helps the selection of the topics of the course. Many treatments of ARIMA models are inspired by the lecture notes of George Tiao. Alex Furger and Michael Lachans spend a great amount of their precious time to proof-read the final version of the book. We are very grateful to their contributions and generosity. We would like to thank to Yacine Ait-Sahalia and Yazhen Wang for stimulating discussions on the topic and Shaojun Guo for formatting the references of the book. We are indebted to Jiaan Yan for his encouragement and support to publish this book in China and to Xiongwen Lu for inviting us to teach the course in Fudan University. We would also like to thank to Yuzuo Chen for providing various editorial assistance.

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Jianqing Fan, Princeton
Qiwei Yao, London

Contents

List of Figures