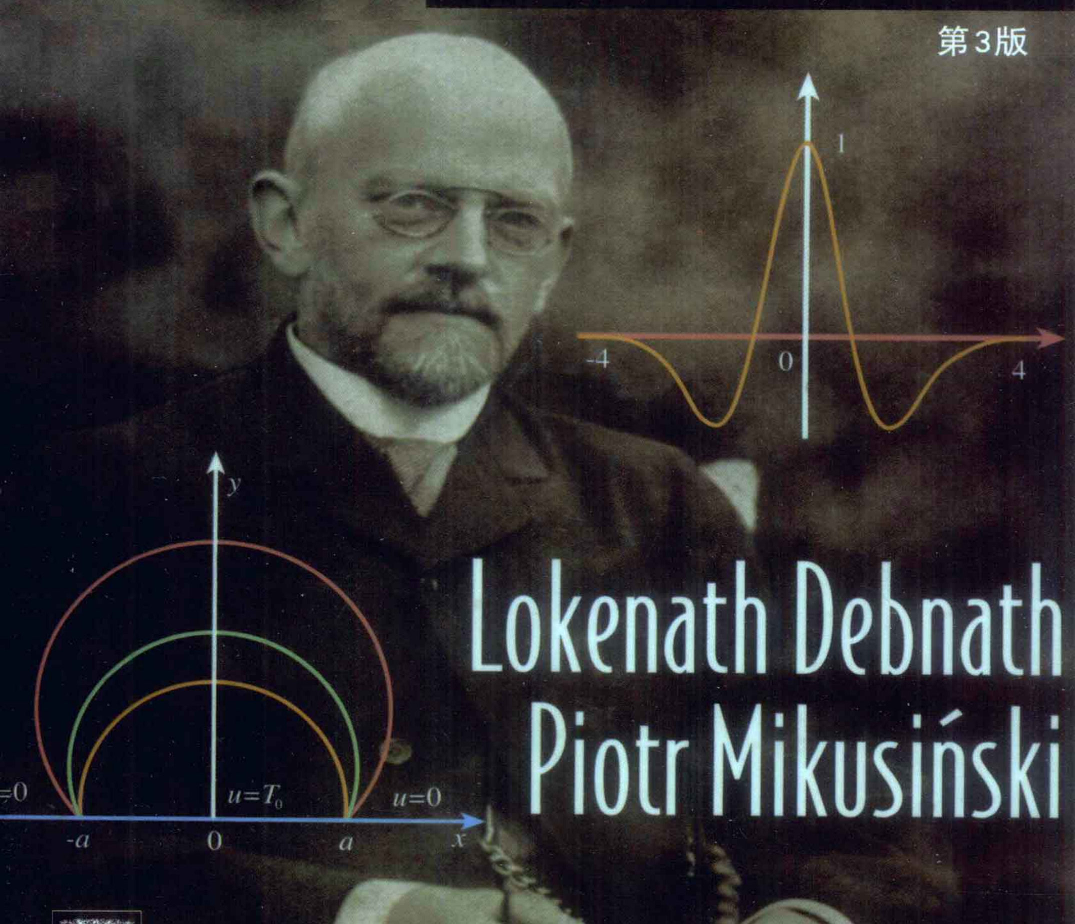
3rd Edition

INTRODUCTION TO HILBERT SPACES with APPLICATIONS

希尔伯特空间及其应用导论

第3版



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ELSEVIER ACADEMIC PRESS
Elsevier (Singapore) Pte Ltd.

世界图书出版公司
www.wpcbj.com.cn

Hilbert Spaces with Applications

Third Edition

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ELSEVIER
ACADEMIC PRESS

AMSTERDAM • BOSTON • HEIDELBERG • LONDON • NEW YORK • OXFORD • PARIS • SAN DIEGO • SAN FRANCISCO • SINGAPORE • SYDNEY • TOKYO

图书在版编目(CIP)数据

希尔伯特空间及其应用导论:第3版 = Introduction to Hilbert Spaces with Applications 3rd ed.:英文/(美)德布纳斯(Debnath,L.)著.—影印本.—北京:世界图书出版公司北京公司,2011.12

ISBN 978-7-5100-4066-5

Ⅰ.①希… Ⅱ.①德… Ⅲ.①希尔伯特空间—英文 Ⅳ.①O177.1

中国版本图书馆CIP数据核字(2011)第217072号

书　　名:	Introduction to Hilbert Spaces with Applications 3rd ed.
作　　者:	Lokenath Debnath, Piotr Mikusiński
中 译 名:	希尔伯特空间及其应用导论 第3版
责任编辑:	高蓉　刘慧
出 版 者:	世界图书出版公司北京公司
印 刷 者:	三河市国英印务有限公司
发　　行:	世界图书出版公司北京公司(北京朝内大街137号　100010)
联系电话:	010-64021602,010-64015659
电子信箱:	kjb@wpcbj.com.cn
开　　本:	24开
印　　张:	25
版　　次:	2012年01月
版权登记:	图字:01-2011-2981
书　　号:	978-7-5100-4066-5/O·918　　**定　　价:** 69.00元

Introduction to Hibert Spaces with Applications 3rd ed.
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ISBN:978 - 0 - 12 - 208438 - 6

Elsevier(Singapore) Pte Ltd.
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Tel:(65)6349 - 0200
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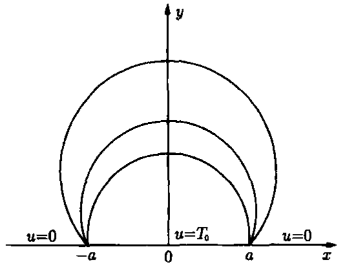
First Published 2012
2012 年初版

This book is dedicated to the memory of our fathers:
JOGESH CHANDRA DEBNATH and JAN MIKUSIŃSKI

Preface to the Third Edition



The previous two editions of our book were very well received. This new edition preserves the basic content and style of the earlier editions. It is a graduate-level text for students and a research reference for professionals in mathematics, science, and engineering. The theoretical foundations are presented in as simple a way as possible, but without sacrificing the mathematical rigor. In the part devoted to applications, we present a wide variety of topics, from classical applications to some recent developments. While the treatment of those applications is rather brief, our hope is that we present enough to stimulate interest that will encourage readers to further studies in those areas.

We have received various comments and suggestions from our colleagues, readers, and graduate students, from the United States and abroad. Those comments have been very helpful in writing this edition. We have made some additions and changes in order to modernize the contents. An effort to improve clarity of presentation and to correct a number of typographical errors was made. New examples and exercises were added. We have also taken the opportunity to entirely rewrite and reorganize several sections in an appropriate manner and to update the bibliography. Some of the major changes and additions include the following:

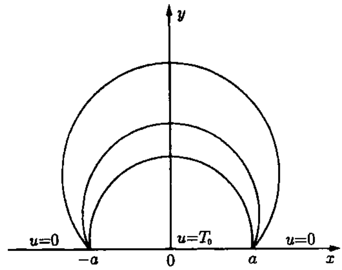
- Chapter 1 has been reorganized, some sections were combined, and the order of material has been modified.
- A complete characterization of finite dimensional normed spaces has been added.
- A new section on L^p spaces has been added in Chapter 2.

- The section on spectral properties of operators in Chapter 4 has been expanded.
- The presentation of the Fourier transform has been moved from Chapter 4 to Chapter 5.
- A new section on Sobolev spaces has been added to Chapter 6.
- Chapter 8 on wavelets and wavelet transforms has been revised and new material added, including a new section on orthonormal wavelets.

We would like to take this opportunity to thank all those who helped us improve the book by reading parts of the manuscript and sharing their comments with us, including Andras Balogh, Cezary Ferens, Ziad Musslimani, Zuhair Nashed, and Vladimir Varlamov. In spite of the best efforts of everyone involved, some typographical errors doubtless remain. Special thanks to June Wingler who helped us with the preparation of the LaTeX files. Finally, we wish to express our grateful thanks to Tom Singer, assistant editor, and staff of Elsevier Academic Press for their help and cooperation.

Lokenath Debnath, *University of Texas — Pan American*
Piotr Mikusiński, *University of Central Florida*
January 2005

Preface
to the Second Edition



When the first edition of this book was published in 1990, it was well received, and we found the comments and criticisms of graduate students and faculty members from the United States and abroad to be helpful, beneficial, and encouraging. This second edition is the result of that input.

We have taken advantage of this new edition to update the bibliography and correct typographical errors, to include additional topics, examples, exercises, comments, and observations, and, in some cases, to entirely rewrite whole sections. The most significant difference from the first edition is the inclusion of a completely new chapter on wavelets.

We have, however, tried to preserve the character of the first edition. We intend the book to be a source of classical and modern topics dealing fully with the basic ideas and results of Hilbert space theory and functional analysis, and we also intend it to be an introduction to various methods of solution of differential and integral equations. Some of the highlights include the following:

- The book offers a detailed and clear explanation of every concept and method that is introduced, accompanied by carefully selected worked examples, with special emphasis being given to those topics in which students experience difficulty.
- A wide variety of modern examples of applications has been selected from areas of integral and ordinary differential equations, wavelets, generalized functions and partial differential equations, control theory, quantum mechanics, fluid dynamics and solid mechanics, optimization, calculus of variations, variational inequalities, approximation theory, linear and nonlinear stability analysis, and bifurcation theory.

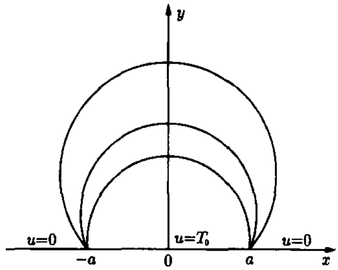
- The book is organized with sufficient flexibility to enable instructors to select chapters appropriate to courses of differing lengths, emphases, and levels of difficulty.
- A wide spectrum of exercises has been carefully chosen and included at the end of each chapter so the reader may further develop both rigorous skills in the theory and applications of functional analysis and a deeper insight into the subject. Answers and hints to selected exercises are provided at the end of the book to provide additional help to students.

It is our pleasure to express our gratitude to those who offered their generous help at different stages of the preparation of this book. Our special thanks are due to Professor Michael Taylor, who read most of the manuscript and suggested many corrections and improvements. Professors Ahmed Zayed and Kit Chan read parts of the manuscript and offered various criticisms and suggestions that have improved the book. June Wingler, with unflagging industry and exemplary patience, typed parts of the manuscript. Finally, we wish to express our grateful thanks to Mr. Charles Glaser, executive editor, and the staff of Academic Press for their help and cooperation. Needless to say, the authors take responsibility for any remaining errors.

The final text was typeset using AmSTeX, and the figures were prepared with the aid of Adobe Illustrator 7.0.

Lokenath Debnath, Piotr Mikusiński
Orlando, January 1998

Preface to the First Edition



Functional analysis is one of the central areas of modern mathematics, and the theory of Hilbert spaces is the core around which functional analysis has developed. Hilbert spaces have a rich geometric structure because they are endowed with an inner product that allows the introduction of the concept of orthogonality of vectors. We believe functional analysis is best approached through a sound knowledge of Hilbert space theory. Our belief led us to prepare an earlier manuscript, which was used as class notes for courses on Hilbert space theory at the University of Central Florida and Georgia Institute of Technology. This book is essentially based on those notes.

One of the main impulses for the development of functional analysis was the study of differential and integral equations arising in applied mathematics, mathematical physics, and engineering; it was in this setting that Hilbert space methods arose and achieved their early successes. With ever greater demands for mathematical tools to provide both theory and applications for science and engineering, the utility and interest of functional analysis and Hilbert space theory seems more clearly established than ever. Keeping these things in mind, our main goal in this book has been to provide both a systematic exposition of the basic ideas and results of Hilbert space theory and functional analysis, and an introduction to various methods of solution of differential and integral equations. In addition, Hilbert space formalism is used to develop the foundations of quantum mechanics and Hilbert space methods are applied to optimization, variational and control problems, and to problems in approximation theory, nonlinear stability, and bifurcation.

One of the most important examples of a Hilbert space is the space of the Lebesgue square integrable functions. Thus, in a study of Hilbert spaces, the Lebesgue integral cannot be avoided. In several books on Hilbert spaces, the reader is asked to use the Lebesgue integral pretending that it is the Riemann integral. We prefer to include a chapter on the Lebesgue integral to give the motivated reader an opportunity to understand this beautiful and powerful extension of the Riemann integral. The presentation of the Lebesgue integral is based on a method discovered independently by H.M. MacNeille and Jan Mikusiński. The method eliminates the necessity of introducing the measure before the integral. This feature makes the approach more direct and less abstract. Since the main tool is the absolute convergence of numerical series, the theory is accessible for senior undergraduate students.

This book is appropriate for a one-semester course in functional analysis and Hilbert space theory with applications. There are two basic prerequisites for this course: linear algebra and ordinary differential equations. It is hoped that the book will prepare students for further study of advanced functional analysis and its applications. Besides, it is intended to serve as a ready reference to the reader interested in research in various areas of mathematics, physics, and engineering sciences to which the Hilbert space methods can be applied with advantage. A wide selection of examples and exercises is included, in the hope that they will serve as a testing ground for the theory and method. Finally, a special effort is made to present a large and varied number of applications to stimulate interest in the subject.

The book is divided into two parts: Part I. Theory (Chapters 1–4); Part II. Applications (Chapters 5–8). The reader should be aware that Part II is not always as rigorous as Part I.

The first chapter discusses briefly the basic algebraic concepts of linear algebra and then develops the theory of normed spaces to some extent. This chapter is by no means a replacement for a course on normed spaces. Our intent was to provide the reader who has no previous experience in the theory of normed spaces with enough background for understanding of the theory of Hilbert spaces. In this chapter, we discuss normed spaces, Banach spaces, and bounded linear mappings. A section on the contraction mapping and the fixed point theorem is also included.

In Chapter 2, we discuss the definition of the Lebesgue integral and prove the fundamental convergence theorems. The results are first stated and proved for real valued functions of a single variable, and then they are extended to complex valued functions of several real variables. A discussion of locally integrable functions, measure, and measurable functions is also included. In the last section, we prove some basic properties of convolution.

Inner product spaces, Hilbert spaces, and orthonormal systems are discussed in Chapter 3. This is followed by discussions of strong and weak convergence, orthogonal complements and projection theorems, linear functionals, and the Riesz representation theorem.

Chapter 4 is devoted to the theory of linear operators on Hilbert spaces with special emphasis on different kinds of operators and their basic properties. Bilinear functionals and quadratic forms leading to the Lax–Milgram theorem are discussed. In addition, eigenvalues and eigenvectors of linear operators are studied in some detail. These concepts play a central role in the theory of operators and their applications. The spectral theorem for self-adjoint compact operators and other related results are presented. This is followed by a brief discussion on the Fourier transforms. The last section is a short introduction to unbounded operators in a Hilbert space.

Applications of the theory of Hilbert spaces to integral and differential equations are presented in Chapter 5, and emphasis is placed on basic existence theorems and the solvability of various kinds of integral equations. Ordinary differential equations, differential operators, inverse differential operators, and Green's functions are discussed in some detail. Also included is the theory of Sturm–Liouville systems. The last section contains several examples of applications of Fourier transforms to ordinary differential equations and to integral equations.

Chapter 6 provides a short introduction to distributions and their properties. The major part of this chapter is concerned with applications of Hilbert space methods to partial differential equations. Special emphasis is given to weak solutions of elliptic boundary problems, and the use of Fourier transforms for solving partial differential equations, and, in particular, for calculating Green's functions.

In Chapter 7, the mathematical foundations of quantum mechanics are built upon the theory of Hermitian operators in a Hilbert space. This chapter includes basic concepts and equations of classical mechanics, fundamental ideas and postulates of quantum mechanics, the Heisenberg uncertainty principle, the Schrödinger and the Heisenberg pictures, and the quantum theory of the linear harmonic oscillator and of the angular momentum operators.

The final chapter is devoted to the Hilbert space methods for finding solutions of optimization problems, variational problems and variational inequalities, minimization problems of a quadratic functional, and optimal control problems for dynamical systems. Also included are brief treatments of approximation theory, linear and nonlinear stability problems, and bifurcation theory.

This book contains almost 600 examples and exercises that are either directly associated with applications or phrased in terms of the mathematical, physical, and engineering contexts in which theory arises. The exercises truly complement the text. Answers and hints to some of them are provided at the end of the book. For students and readers wishing to learn more about the subject, important references are listed in the bibliography.

In preparing this book, we have been encouraged by and have benefited from the helpful comments and criticisms of a number of graduate students and faculty members of several universities in the United States and abroad. Professors James V. Herod and Thomas D. Morley have adopted the man-

uscript at Georgia Institute of Technology for a graduate course on Hilbert spaces. We express our grateful thanks to them for their valuable advice and suggestions during the preparation of the book. We also wish to thank Drs. R. Ger and A. Szymański, who have carefully read parts of the manuscript and given some suggestions for improvement. It is our pleasure to acknowledge the encouragement and help of Professor P.K. Ghosh, who has provided several references and books on the subject from his personal library. We also express our grateful thanks to our friends and colleagues, including Drs. Ram N. Mohapatra, Michael D. Taylor, and Carroll A. Webber, for their interest and help during the preparation of the book. Thanks also go to Mrs. Grazyna Mikusiński for drawing all diagrams. In spite of all the best efforts of everyone involved, it is doubtless that there are still typographical errors in the book. We do hope that any remaining errors are both few and obvious and will not create undue confusion. Finally, the authors wish to express their thanks to Mrs. Alice Peters, editor, and the staff of Academic Press for their help and cooperation.

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