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Compact Complex Surfaces

Second Enlarged Edition

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Preface to the First Edition

> Par une belle matinée du mois de mai,
> une élégante amazone parcourait, sur
> une superbe jument alezane, les allées
> fleuries de Bois de Boulogne.
> (A. Camus, La Peste)

Early versions of parts of this work date back to the mid-sixties, when the third author started to write a book on surfaces. But for several reasons, in particular the appearance of Šafarevič's book, he postponed the projects. It was revived about ten years later, when all three authors were in Leiden. It is impossible to cover in one book the vast and rapidly developing theory of surfaces. Choices have to be made, with respect to content as well as to presentation. We have chosen for a complex-analytic point of view; this distinguishes our text from most of the existing treatments. Relations with the case of characteristic p are not discussed.

We hope to have succeeded in writing a readable book; a book that can be used by non-specialists. The specialist will find very little that is new to him anyhow.

As to acknowledgements, the authors certainly have to thank the Koninklijke Shellprijs, awarded to the third author in 1964. The numerous contacts with colleagues from other countries made possible by that award have had a very favourable influence on this book. Our thanks are furthermore due to G. Angermüller, G. Barthel, G. Fischer, G. van der Geer, N. Hitchin, D. Husemoller, M. Reid, T. A. Springer, D. Zagier and S. Zucker. Each of them has read some part of the manuscript and has made valuable suggestions.

Editor and printer have done an excellent job, and the Springer-Verlag has been very generous in fulfilling all of our last-minute wishes.

We are also indebted to Mrs. W. M. Van de Ven who not only typed the better part of the book, but also helped in preparing it for the printer, and to Mrs. H. Dohrman who carefully typed many pages. Finally the authors want to thank their wives for all their patience and endurance.

Erlangen/Leiden, February 1984

W. Barth
C. Peters
A. Van de Ven

Preface to the Second Edition

In the 19 years which passed since the first edition was published, several important developments have taken place in the theory of surfaces. The most sensational one concerns the differentiable structure of surfaces. Twenty years ago very little was known about differentiable structures on 4-manifolds, but in the meantime Donaldson on the one hand and Seiberg and Witten on the other hand, have found, inspired by gauge theory, totally new invariants. Strikingly, together with the theory explained in this book these invariants yield a wealth of new results about the differentiable structure of algebraic surfaces.

Other developments include the systematic use of nef-divisors (in accordance with the progress made in the classification of higher dimensional algebraic varieties), a better understanding of Kähler structures on surfaces, and Reider's new approach to adjoint mappings.

All these developments have been incorporated in the present edition, though the Donaldson and Seiberg-Witten theory only by way of examples. Of course we use the opportunity to correct some minor mistakes, which we ether have discovered ourselves or which were communicated to us by careful readers to whom we are much obliged.

We gratefully acknowledge the support of various bodies which helped us prepare this new edition; in particular the following grants and institutions: EAGER (European Algebraic Geometry Research Network) and the DFG (Deutsche Forschungsgemeinschaft) as well as the universities of Essen, Grenoble, Hannover and Leiden for the hospitality we were offered at various occasions. Our thanks go to those who have read and commented on parts of the manuscript: R. Eckert, C. Erdenberger, M. Friedland, A. Gathmann, M. Lönne, K. Ludwig, John D. McCarthy, M. Schütt, J. Spandaw and H. Verrill. We are in particular grateful to J.-P. Demailly, L. Bonavero and A. Teleman for all the advice they offered which helped us to understand some of the hard analysis needed in various new parts of the book.

Special thanks also to Mme. A. Guttin-Lombard, who efficiently prepared a major part of the book, and to Mrs. S. Guttner for the careful typing of several chapters.

Grenoble/Erlangen/Hannover/Leiden, July 2003

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Introduction

Historical Note

This book is mainly concerned with the classification of smooth compact complex surfaces, i.e., of compact 2-dimensional complex manifolds, which in the introduction we shall always assume to be connected *).

Surface theory has its roots on the one hand in projective geometry and on the other hand in Riemann's theory of algebraic functions of a single variable. As to projective geometry, around the middle of the 19th century an extensive study was made of (smooth as well as singular) low-degree surfaces in complex-projective 3-space $\mathbb{P}_3$. The twenty-seven lines on a smooth cubic and names such as Cayley cubic and Steiner quartic remind us of that period. The extension of Riemann's work, in geometric form, will always be associated with mathematicians like Clebsch and M. Noether, whereas the topological and transcendental approach is linked to Poincaré and others, in particular Picard.

Soon attention was focused on a classification of algebraic surfaces with respect to birational equivalence. The classical geometers clearly had in mind something similar to what was known for curves: a coarse classification according to the value of some numerical invariants, and then a finer classification. At the beginning of the 20th century Castelnuovo, Enriques and many others had succeeded in creating an impressive, essentially geometric theory of birational classification of smooth algebraic surfaces. (This was in fact a birational classification of *all* algebraic surfaces, smooth or not, since every algebraic surface is birationally equivalent to a smooth one; but a rigorous proof of this fact was given for the first time by R. Walker in 1935.) Among the main birational invariants, it was discovered, are the irregularity or, equivalently, the first Betti number $b_1(X)$ and the plurigenera $P_n(X)$ of a smooth algebraic surface X. For any $n \geq 1$ the n-th plurigenus $P_n(V)$ of a smooth algebraic variety V is defined as the dimension of the space of sections $\Gamma(V, \mathcal{K}_V^{\otimes n})$, where $\mathcal{K}_V = \bigwedge^n \mathcal{T}_V^\vee$ is the canonical bundle of V. Traditionally the first plurigenus $P_1(V)$ is denoted by $p_g(V)$, and called the geometric genus of V.

*) From Chap. II on the meaning of the word "surface" in a given chapter is defined at the very beginning of that chapter. In Chap. I there is no danger of confusion.

Given any smooth algebraic surface X, there a four possibilities:

1) all P_n vanish;
2) not all P_n vanish, but are all either 0 or 1;
3) P_n grows linearly in n;
4) P_n grows quadratically in n.

Nowadays this fact is expressed by saying that the Kodaira dimension $\text{kod}(X)$ of X is either $-\infty$, 0, 1, or 2. (For a precise definition of this concept, due to Iitaka, we refer to Chap. I, Sect.7.) For curves the corresponding classification is the division into the rational curve $\mathbb{P}_1$ (Kodaira dimension $-\infty$), elliptic curves (Kodaira dimension 0), and curves of higher genus (Kodaira dimension 1).

It was known at the time which surfaces are in class 1), namely those surfaces which are birationally equivalent to the product of $\mathbb{P}_1$ and another curve. This includes in particular the rational surfaces, i.e., those birationally equivalent to $\mathbb{P}_2$. A key stone of the proof was Castelnuovo's criterion: a smooth surface X is rational if and only if its first Betti number $b_1(X)$ and its bi-genus $P_2(X)$ vanish.

As to class 2), the classical geometers knew that there is a subdivision into four types, distinguished by the values of the first Betti number and the plurigenera, namely into surfaces, birationally equivalent to respectively algebraic tori, bi-elliptic surfaces, algebraic K 3-surfaces and Enriques surfaces (the names are the modern ones). The precise classification of the first two types was known, but not that of the last two types.

It had also been established that all surfaces in class 3) are elliptic (i.e., admitting a map onto a curve such that all but a finite number of fibres are elliptic curves); however, not much was known about their further classification.

Finally, the surfaces in class 4), which are analogous to curves of genus $\geq$ 2, were (and still are) called surfaces of general type. The classical geometers certainly had the right idea how to classify them, but – contrary, say, to the case of Castelnuovo's criterion or the case of bi-elliptic surfaces – they never arrived at precise results or even precise statements. Before we explain a little bit the present state of this classification, we first have to make a few remarks of a more general nature.

Today it is standard to look at the above classification of algebraic surfaces (the Enriques classification for algebraic surfaces) in a slightly different way. The basic idea: first a classification according to Kodaira dimension, and then a finer classification, remains the same, but it is seen as a *biregular* classification of *minimal* smooth algebraic surfaces, i.e., surfaces, which cannot be obtained from another *smooth* algebraic surface by blowing up a point. Every smooth surface X can be obtained from such a surface by successive blow-ups. At first sight it might seem that classifying only minimal surfaces is not very satisfactory, because one and the same surface X might be obtained by blowing up different minimal surfaces Y. However, always $\text{kod}(X) = \text{kod}(Y)$ and if $\text{kod}(X) \geq 0$, then Y is determined by X up to an isomorphism. So even

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from the biregular point of view it is sufficient to classify minimal surfaces, at least in the case of non-negative Kodaira dimension. (If $\text{kod}(X) = -\infty$, then different Y can give the same X, but this case is rather easy to handle.) Furthermore, a birational transformation between minimal surfaces of non-negative Kodaira dimension is always an isomorphism. In other words, for Kodaira dimension ≥ 0 birational classification of all surfaces amounts to biregular classification of minimal surfaces. And, most importantly, whereas from the birational point of view good moduli spaces never exist, they do exist for many of the finer classes in the case of minimal surfaces.

Now let us return to surfaces of general type. We consider minimal ones X with given Chern numbers $c_1^2(X) = p$, $c_2(X) = q$. It turns out that for $n \geq 5$, any n-canonical map (given by the ratios $\gamma_1 : \ldots : \gamma_{N+1}$, where $\gamma_1, \ldots, \gamma_{N+1}$ is a basis for $\Gamma(X, \mathcal{K}_X^{\otimes n})$) is everywhere defined on X and maps this surface birationally onto a surface X' of degree n^2p in $\mathbb{P}_N$ with N depending only on n, p and q. Choosing a different basis for $\Gamma(X, \mathcal{K}_X^{\otimes n})$ yields a surface which is projectively equivalent to X' in $\mathbb{P}_N$. In this way minimal surfaces of general type with fixed c_1^2 and c_2 correspond one-to-one to the points of the quotient of a Zariski-open subset in a Chow variety (or Hilbert scheme) by a projective-linear group. Of course, one wants this quotient to be a variety of moduli (coarse, at least) for the surfaces under consideration. A theorem of Gieseker (1977), based on geometric invariant theory (due to Hilbert and Mumford) says that for n large enough this is indeed the case.

Perhaps it should be mentioned at this point, that the results obtained by the classical geometers, their importance notwithstanding, were in many ways built on sand, for the foundations of algebraic geometry were lacking.

The years 1910–1950 did not bring too much change as far as the classification of surfaces is concerned. In the first two of these decades we see continuing great progress in the general theory of algebraic varieties, from the geometric point of view (Severi) as well as from the transcendental point of view (Lefschetz). But for both directions, a solid basis was still not available. Such a basis was laid in the thirties and forties, on the one hand for geometry by van der Waerden, Zariski, Weil and on the other hand for much of the topological and transcendental theory by de Rham and Hodge.

Once the foundations were present, some of the classification questions were taken up again and also considered for other ground fields: minimal models, Castelnuovo's criterion (Zariski), Enriques surfaces (M. Artin).

Decisive progress came only after the second revolution, i.e., after sheaf theory had been developed, and applied by Serre, Hirzebruch, Grothendieck and many others to analytic and algebraic geometry.

On this basis Kodaira not only extended the classical results on algebraic surfaces in an essential way, but also treated non-algebraic surfaces. For these surfaces the plurigenera and Kodaira dimension can be defined in the same way as for algebraic surfaces, and thus the Enriques classification is extended to the Enriques-Kodaira classification of all compact, complex surfaces.

As to the algebraic surfaces, it will hardly surprise anybody that Kodaira gave the Enriques classification the necessary precision and solid basis. But he went further in many directions. For example, he did the first step towards the classification of K 3-surfaces. A K 3-surface is a compact complex surface with $b_1(X) = 0$ and $\mathcal{K}_X$ trivial. As we have mentioned, the classification of the algebraic ones among them (which form a minority) was already an important problem in older times. Since the fifties they have been studied intensively, the main goal being to prove a conjecture, independently due to Andreotti and Weil about their classification (compare the comments at the end of Chap. VIII). Kodaira verified part of this conjecture, namely that all K 3-surfaces are complex-analytic deformations of each other. The deformation theory of complex manifolds, which Kodaira created together with Spencer, realized at least part of an old ideal of Riemann and Noether: to have a theory of moduli for curves and surfaces. Another contribution of Kodaira of far reaching significance and influence, was his extensive study of (algebraic and non-algebraic) elliptic surfaces, something that had definitely been lacking in the work of the Italian geometers.

Though the concept of an n-dimensional complex manifold had been known implicitly for a long time (certainly since Weyl's *Die Idee der Riemannschen Fläche*), it appeared explicitly only around 1945, in the work of Ehresmann and H. Hopf. In particular Hopf constructed an entirely new class of compact complex surfaces (the first example of what now are called the Hopf surfaces) which are topologically very different from any algebraic surface. A Hopf surface has first Betti number 1 and second Betti number 0, whereas a smooth algebraic surface always has an even first Betti number and a strictly positive second Betti number. So, contrary to tori and K 3-surfaces, a Hopf surface can never be deformed into an algebraic one.

This example shows that there is much more to non-algebraic surfaces than deforming some algebraic ones. It was again Kodaira who started with the classification of non-algebraic surfaces in general, and he completed this task to a considerable degree. In his papers he uses Atiyah-Singer's Riemann-Roch theorem in an essential way.

As to non-algebraic deformations of algebraic surfaces, their significance for a better understanding of algebraic surfaces arises clearly from the Andreotti-Weil conjecture (mentioned before) and Kodaira's work. This point is already obvious from the case of tori, but it gains more weight if K 3-surfaces and elliptic surfaces are taken into consideration.

It would be wrong to think that with Kodaira the theory of surfaces more or less came to its end. On the contrary, the interest in surfaces has only been increasing since the days that Kodaira produced most of his results in the late 1950s and 1960s.

One of the main centres of interest has already been mentioned: the conjecture of Andreotti and Weil on the classification of K 3-surfaces. After important contributions by many mathematicians, in particular Šafarevich and Piateckii-Shapiro, the most important parts of the conjecture could finally be proved, but only with the help of S.-T. Yau's deep differential-geometric results on the Calabi conjecture.

Another centre of attention was the classification of (minimal) surfaces of general type. We spoke already about Gieseker's theorem, saying that for each ordered pair (p, q) of integers there is a (possibly empty) coarse moduli scheme parametrising minimal surfaces of general type X with $c_1^2(X) = p$, $c_2(X) = q$. The next question is of course: when is this scheme non-empty? In this direction an important result was obtained in 1976 by S.-T. Yau and Miyaoka, who independently proved an older conjecture of Van de Ven, saying that for every surface X of general type the inequality $c_1^2(X) \leq 3c_2(X)$ holds. Yau obtained this inequality as a consequence of his famous work on the Calabi conjecture. Miyaoka was very much inspired by Bogomolov, who only proved the weaker inequality $c_1^2(X) \leq 4c_2(X)$, but linked the question in an exciting way to the theory of stable vector bundles. Establishing these inequalities is the first step towards the "geography" of surfaces of general type (in addition to the existence problem this asks the question how special values of the numerical invariants influence the geometry of the surface, such as the existence of special fibrations). In this area Horikawa and Persson were the pioneers, but despite several extensions of their work, it is still not known whether every pair of integers allowed by the preceding inequalities can actually be realized by a minimal surface of general type.

Much research was also done on surfaces which are special, mostly for low values of c_1^2. For example, already many years ago Severi had raised the question, whether there exist surfaces which are homeomorphic, but not algebraically isomorphic to $\mathbf{P}_2$. It took a long time before the final answer was given by S.-T. Yau who proved that these do not exist. However, as was shown by Mumford using p-adic geometry, there does exist at least one "fake projective plane", i.e., a surface different from $\mathbb{P}_2$ but with the same Betti numbers. In spite of many efforts, a direct geometric construction (within the framework of complex algebraic geometry) of such a fake projective plane is still lacking.

As a last example of a classification problem that saw much progress in the years 1970–1980 we mention the case of (minimal) surface without non-constant meromorphic functions. Kodaira had already classified these surfaces, except for surfaces with $b_1 = 1$, $b_2 = 0$ on which there are no curves, and surfaces with $b_1 = 1$, $b_2 \geq 1$. For years no example of either class was known. In 1971 Inoue found some examples of surfaces in the first class, and three years later he showed that also the second one is not empty. Since then Inoue, Bombieri, Kato and Enoki have produced many of these surfaces and have started the classification. This is an active area of research; over the last two decades Bogomolov, Dloussky and several other people have made substantial progress. We refer to the historical remarks at the end of Sect. V.20.

We could go on in this way, but we only wanted to indicate the progress made possible by the introduction of sheaf-theoretic methods and the use of new results in other fields.

As to important developments since the appearance of the first edition of this book (in 1984), we first of all want to mention the spectacular developments concerning the (differential) topology of compact complex surfaces. This started around 1985 with Donaldson's example of two algebraic surfaces (one elliptic, the other rational) which are homeomorphic but not diffeomorphic (the fact that the two surfaces are homeomorphic follows from deep results on the topology of compact 4-manifolds which Freedman had obtained just a few years before). Subsequently Donaldson introduced differentiable invariants, now called the Donaldson polynomials, which can be calculated by algebro-geometric means. The use of these invariants enabled Freedman and Qin to prove the "Van de Ven conjecture": the Kodaira dimension is a differentiable invariant. Differential topology in dimension 4 underwent a second revolution through the work of Seiberg and Witten who produced a new set of invariants whose calculation requires much less algebraic geometry and with which one could even prove more than the Van de Ven conjecture, namely that the plurigenera themselves are differentiable invariants.

A second important development is "Reider's method" for dealing with pluricanonical maps. It simplifies and extends Bombieri's treatment which was based on connectedness properties of pluricanonical divisors. For surfaces of general type much work has also been done on the geography as we already mentioned. But also many moduli spaces have been studied in great detail, mainly by Catanese and his students. This work shows how complicated the behaviour of such moduli spaces can be; for instance the number of components although finite, can be arbitrarily large.

Thirdly we want to mention that recently a direct proof has been found of the fact that a surface with even first Betti number is kählerian. This is due to Lamari and Buchdahl (independent of each other) who use Demailly's deep results on the regularisation of positive currents.

There were, of course, many other developments in the theory of surfaces in the last two decades which we are not able to discuss in this book, such as results obtained by projective methods, the possible number of double points of surfaces in $\mathbb{P}_3$ and their configurations, and the classification of smooth surfaces in $\mathbb{P}_4$ of low degree. For surfaces with many double points, see for instance [Bar] and the references cited there, and [Chm]. As to smooth surfaces in $\mathbb{P}_4$, there exists now, due to the combined effort of many authors, a fairly complete classification up to and including degree 10 with further partial results up to degree 15 (see [D-E-S], [D-S] and the references given there). In particular we know by a result of Ellingsrud and Peskine [E-P] that the degree of any smooth surface in $\mathbb{P}_4$ which is not of general type is bounded. Schreyer conjectures this bound to be 15. It cannot be smaller, since there exist (non-minimal) smooth abelian surfaces and bi-elliptic surfaces of degree 15 in $\mathbb{P}_4$. Braun and Fløysted [B-Fl] proved that this bound is smaller than or equal to 105. This was brought down to 76 by Cook [Ck]. There are abelian surfaces of degree 10 in $\mathbb{P}_4$. These have attracted special attention since they give rise to the Horrocks-Mumford bundle, so far still essentially the only known indecomposable rank 2 bundle on $\mathbb{P}_4$. For a survey article on the rich geometry associated to this bundle see [Hu].

To finish, we mention two developments which do not belong to our subject, but are closely related to it. First, the extension of the Enriques classification to characteristic p by Bombieri and Mumford (some results having been obtained previously by Zariski). In characteristics $\neq 2, 3$ the classification is identical to the complex-algebraic case, but in characteristics 2 and 3 certain 'non-classical' surfaces appear. About the finer classification much less is known than in the complex case, but Cossec and Dolgachev extended many results concerning Enriques surfaces to all characteristics. The structure of the pluricanonical map for surfaces of general type has been studied by Ekedahl and Shepherd-Barron. The results are roughly the same as for the complex-algebraic case.

Secondly, we mention the development by Iitaka, Kawamata, Kollár, Miyaoka, Mori, Reid, Ueno, Viehweg and others of a classification theory for higher dimensional manifolds. Since in this case no unique minimal models exist, it became essential to allow certain singularities and also birational maps such as "flips" and "flops" which are more complicated than blow-ups. The starting point is again a classification according to Kodaira dimension and – at least in dimension 3 – already much is known about the finer division. Here a central role is played by Mori's theorems on the structure of the cone of the so-called nef-divisors (nef is an abbreviation coined by Reid and stands for "numerically eventually free"). A nef-divisor by definition has the property that it has non-negative intersection product with every curve. This concept has indeed become central in modern algebraic geometry. Another major role is played by Iitaka's conjecture $C_{n,m}$: if X and Y are smooth, compact irreducible algebraic varieties, of dimension m and n respectively, and if $f : X \to Y$ is a surjective morphism, then $\mathrm{kod}(X) \geq \mathrm{kod}(Y) + \mathrm{kod}(F)$, where F is a general fibre of f. The conjecture has been proved for several values of m and n, in particular for $m = 2$, $n = 1$ where it follows from the Enriques-Kodaira classification. See also Chap. III where a direct proof is presented.

Some references

Classical results in general: [C-E].

Classical theory of the Enriques classification: [Enr14], [Enr49], [Ge].

Desingularization: [Za71], [Li].

Zariski's work on minimal models and the Castelnuovo criterion: [Za58a], [Za58b].

M. Artin's work on Enriques surfaces can be found in Artin's unpublished thesis [An60]

Enriques classification in characteristic p and subsequent developments: [Mu69], [B-M76], [B-M76], [Co-D], [Ek], [ShB91a].

Classification theory for higher dimensional varieties: [Ue75], [Ue80], [Es], [Cl-K-M], [Vie95].
The other subjects mentioned are treated further on in this book.

The Contents of the Book

As has been explained in the preceding section, the classification of compact, complex surfaces amounts to the classification of minimal surfaces. This is first of all a classification according to Kodaira dimension, which for a surface can assume the values $-\infty$, 0, 1 and 2. A refinement of this very coarse classification is the Enriques-Kodaira classification, a description of which is a first purpose of the book. Some of the classes occurring in the Enriques-Kodaira classification can easily be described in detail, but the others: minimal surfaces of class VII (i.e., minimal surfaces X with $b_1(X) = 0$, $\mathrm{kod}(X) = -\infty$*)), K 3-surfaces, Enriques surfaces, minimal properly elliptic surfaces and minimal surfaces of general type, require further investigation. Apart from the Enriques-Kodaira classification, this book is mainly devoted to a deeper study of some of these classes, namely K 3-surfaces, Enriques surfaces and surfaces of general type. On the other hand, surfaces of class VII and properly elliptic surfaces will not be treated in detail. For elliptic surfaces a number of general properties as well as a classification can be found in Chap. V. We cover only a small part of what is known in this direction, in particular we do not partition elliptic surfaces in families. For this the reader may consult the recent book [F-M94] and the references given there. Surfaces of class VII occur only by way of examples, and neither the beautiful considerations of Kodaira on Hopf surfaces nor most of the work of Enoki, Dloussky, Inoue and Kato on surfaces without non-meromorphic functions can be found in this book.

It goes without saying that in a book like the present one many auxiliary results can only be quoted. As to general theorems on complex and algebraic manifolds or spaces, it has been a difficult question for us to decide whether (special) proofs for the 2-dimensional case should be included or not. Sometimes, when a more elementary treatment is available for the 2-dimensional case, we have explained this in detail. For example, we do not refer to Hironaka for the resolution of surface singularities. The 2-dimensional case is infinitely much simpler and its direct treatment is very rewarding. But at other places we have used a general theorem in spite of the fact that for surfaces an elementary approach exists. For example, in Chapter IV we derive the fundamental projectivity criterion (Theorem IV.6.2) using Grauert's general ampleness theorem, though it would have been possible to avoid this by using a method of Chow and Kodaira. The method we use is shorter, whereas

*) Our definition of a surface of class VII is slightly different from Kodaira's, compare Chap. VI, Sect. 1.

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