E. F. NOGOTOV

Applications of Numerical Heat Transfer

SERIES IN THERMAL AND FLUIDS ENGINEERING

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APPLICATIONS OF NUMERICAL HEAT TRANSFER

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Preface

Ideas for the practical application of numerical methods are finding an increasingly receptive audience among progressive engineers around the world. In the present work, numerical methods are applied to an important engineering problem, because it is believed that, in general, engineers will prefer a demonstration of this kind to an abstract mathematical presentation.

Engineers working in the energy field, in particular in the field of heat and mass transfer, will find in these pages the information they need for the employment of modern numerical methods in their daily practice. Most of the problems involved in heat transfer are reducible to the solution of partial differential equations. As a general rule, however, these are complicated and their solution in the form of final formulas is possible in only the simplest cases. The most important group of approximate methods of solution of these equations is composed of numerical ones and, of these, the finite-difference technique is certainly the most universal and widely used.

A complete and systematic treatment of the subject, from basic concepts of finite-difference methods to sophisticated finite-difference schemes, is provided in this work. General recommendations for the application of the finite-difference technique are given, as well as examples of the solution of particular problems in heat transfer.

This book has been written within the framework of Unesco's program in technological research and higher education. Unesco wishes to thank the Byelorussian National Commission for Unesco for having proposed the author, and Mr. Nogotov himself for having carried out this valuable work. The author, who is an expert with wide experience in the application of numerical methods to heat-transfer problems, is responsible for the choice and presentation of materials and the opinions expressed in this study.

Contents

Chapter 1

Introduction

Many problems involving heat and mass transfer are reducible to the solution of partial differential equations. The differential equations that govern real physical processes are generally of a very complicated nature, and their closed-form solution is possible only in the simplest cases.

Approximate methods therefore become very useful for the solution of such problems. The methods generally are divided into two categories. The first category covers those methods that allow an analytical expression, say, a part of a certain series, as the approximate solution of the problem. It should be emphasized that in most cases such a solution has a complicated structure, as it contains integrals, special functions, etc., and is hardly convenient.

The second category of approximate methods is composed of numerical techniques that allow the determination of a table of approximate values of the desired solution. In this category are such approaches as the finite-difference method, straight-line method, large-particle method, and Monte Carlo method.

Of the numerical methods, the finite-difference technique is certainly the most universal and most widely used. The essence of the method involves substitution, for the differential operators in the initial differential equations, of approximate values expressed in terms of differences of the functions at discrete points of the differencing grid. The substitution results in algebraic systems of equations with the function values at the grid points being the unknowns. The method has a universal appeal because of its generality and its relative simplicity of adaptation to a computer.

The straight-line method is closely associated with the finite-difference technique. In this method, the solution of the partial differential equation is sought along a certain straight-line family; the equation is reduced to a system of ordinary differential equations, which may often be solved by a finite-difference procedure.

Multidimensional equations of mathematical physics can often be solved with a large-particle method suggested recently by Harlow. In this method, the hydrodynamic equations are reduced to two simpler systems in each time step, on the basis of some weak approximation. The first system describes the interaction of hydrodynamic fields, with transfer effects neglected, and it is integrated by ordinary means using a fixed Eulerian grid. The second system describes transfer effects. For its solution, a simplified continuum model is used, with a set of particles substituted for each Eulerian mesh. The net balance of the mass, momentum, and energy of particles in a mesh is identical to that in a continuum. As soon as a particle that "carries" a certain mass crosses a mesh boundary in following its path, the mass, momentum, and energy of the particle are subtracted from the mesh it has left, and are added to the mesh that now contains the particle.

Harlow's system is based on the explicit solutions of the first and second stages and is generally assumed quite stable as a whole. No absolutely stable schemes for large-particle methods are available as yet, but significant progress may be expected sometime in the near future.

Recently, O. M. Belotserkovsky, Y. M. Davydov, N. N. Yanenko, and others have presented some modifications of the method that essentially reduce the velocity and pressure fluctuations and increase the stability.

It should be noted that the structure of the method is rather complicated. Moreover, computations based on the method always require a large amount of computer time and storage, thus making the range of application of the method rather narrow.

During the last two decades, another method, known as the *Monte Carlo* method, has been actively developed. This method is most effective when used with high-speed computers, since it requires a large number of statistical tests to reduce the mean-square error of the result.

Recently, the method has been improved by the use of conditional probabilities of the processes and statistical weights based on the solution of the conjugate equations. In particular cases, this approach reduces the error dispersion by about an order, thus also decreasing the computation time by an order. Much more work is still required in this regard.

The wide use of numerical methods for the different problems of mathematical physics, including those of heat transfer, is closely tied to the current development of computers. Modern high-speed computers allow the numerical investigation of many important practical problems that are formulated in the most general terms.

The strict formulation of a problem with a small number of assumptions makes numerical investigation comparable to the best physical experiments. Moreover, numerical experiments possess a number of advantages such as relative simplicity and low cost and provide the possibility of accounting for most of the effects involved in the process being considered. Furthermore, numerical experiments permit the variation of problem parameters and boundary conditions over wide ranges, and they provide complete information on the process under investigation, which is practically impossible under laboratory conditions.

The increased availability of computers and associated algorithms, which considerably simplify programming, has made computer techniques accessible to a large number of investigators, and has accelerated the use of mathematical techniques in science and technology. As a result, the number of workers requiring a knowledge of the fundamentals of mathematics is rapidly increasing. For these workers, the basic methods of computation must be presented in an accessible form. It is hoped that this book will serve that purpose.

In this book, some aspects of the development and the application of numerical methods to heat- and mass-transfer problems are considered. Methods are limited to those that provide computer solutions to the types of problems most frequently encountered in engineering practice.

Since, for the practical engineer, knowledge of the solution algorithm and its accuracy is of primary importance, strict mathematical proofs are sometimes omitted here; the essence of a particular method and the procedure for estimating the error of the solution are presented only in their final form. The types of problems for which the method is applicable and the process parameters for which the method gives a meaningful solution are generally outlined.

The reader who wishes to become expert in computer work must carefully study the literature, which gives a deeper insight into the fundamentals of computational mathematics. A list of references is provided, for that purpose, at the end of the book. It is hoped that the mathematical methods contained in the book will allow the successful solution of a wide range of heat-transfer problems.

It is expected that readers will have different specialities and levels of mathematical knowledge. Therefore, the presentation will not follow the very formal style traditional for mathematical literature; nor will all the material be given on a "physical" level. The appeal will be to the engineering intuition and common sense, rather than to mathematical education.

Since heat-transfer problems are mainly formulated in terms of partial differential equations, and finite-difference techniques comprise the most convenient and general method for solving such problems, a large part of the book is devoted to the description of these techniques.

A considerable amount of experience has been accumulated on the solution of heat-transfer problems by finite-difference techniques. Unfortunately, the descriptions of particular methods are contained in many different journals, or in special literature concerned with numerical methods, where they are presented in a form hardly accessible to nonspecialists in numerical analysis. This book thus presents, in a convenient form, the important numerical algorithms for the solution of heat-transfer problems.

It is not the author's aim to describe all the available finite-difference schemes, a task that is almost impossible. The book contains only the simplest and most effective finite-difference techniques applicable to typical heat- and mass-transfer problems. In some cases, effective but complicated methods have been omitted.

To help the reader choose and apply particular finite-difference schemes, Chap. 2 presents some basic concepts of the finite-difference technique. Details are given on how to select a particular differencing grid and how to construct the corresponding finite-difference formulas. Definitions of approximation, stability, and convergence are also given.

Chapter 3 is devoted entirely to the solution of heat-conduction problems. It contains various finite-difference schemes for the solution of heat-conduction problems and, in addition, a detailed description of the fractional-step method for solving multidimensional problems. The most interesting modifications of this method are presented.

In Chap. 4, numerical procedures for studying convective heat transfer are considered. Recommended finite-difference schemes, which are applicable in particular to high Rayleigh numbers, are presented.

Chapter 5 gives general recommendations concerning the choice of a finite-difference scheme to solve a particular problem, arrangement for the computation procedure, experimental determination of the stability of a difference problem and the accuracy of the resultant solution, and how to determine local and integral characteristics of the problem.

The book is based on lectures delivered by the author at the International School on Mathematical Aspects of Heat and Mass Transfer during the conference on Heat and Mass Transfer in 1974 (Minsk, U.S.S.R.), which was sponsored by the International Centre of Academies of Socialist Countries. The final text of the book was considerably improved through numerous discussions with colleagues, to whom the author is extremely grateful.

Chapter 2

Basic Concepts of the Finite-Difference Method

2.1 PRELIMINARY REMARKS

To be solved on a computer, a problem must be formulated numerically in terms of some suitable arithmetic operations. Generally, the problem to be solved is first formulated in terms of ordinary mathematical equations, functions, differential operators, etc. Therefore, computer solution first involves an approximation of the problem substitution in terms of numbers and arithmetic operations. For example, special functions are ordinarily expressed in terms of finite series; finite sums are substituted for integrals; and differential operators are usually approximated by difference relations.

The method of solution of differential problems in which differential operators are replaced by their approximate values expressed in terms of functions at individual discrete points is called the *finite-difference* method or, alternatively, the *grid* method.

The substitution reduces the problem to the solution of a set of algebraic equations. Although the set of equations may involve a considerable number of unknown quantities, its solution is mathematically easier than that of the original problem.

The essence of the finite-difference method will be illustrated with a simple example. The heat-conduction equation for a homogeneous thin rod of unit length ($0 \leqslant x \leqslant 1$) with internal distributed heat sources is to be solved:

$$\frac{\partial u}{\partial t} - \kappa \frac{\partial^2 u}{\partial x^2} = Q \tag{2.1}$$

where $u = u(x, t)$ is the temperature, $Q = Q(x, t)$ is the distribution function of the heat sources, $\kappa = \text{const} > 0$ is the thermal conductivity of the material, and t is the time.

Let the temperature distribution in the rod at some initial time $t = 0$ be

$$u(x, 0) = T(x) \tag{2.2}$$

and the temperatures at the ends of the rod be

$$u(0, t) = T_0(t) \qquad u(1, t) = T_1(t) \tag{2.3}$$

We seek the temperature distribution $u(x, t)$ in the rod at any time $t > 0$.

For simplicity and for greater clarity, the problem will be written in an operator form as

$$Lu = f \tag{2.4}$$

with the notations

$$Lu \equiv \begin{cases} \dfrac{\partial u}{\partial t} - \kappa \dfrac{\partial^2 u}{\partial x^2} \\ u(x, 0) \\ u(0, t), u(1, t) \end{cases} \qquad f \equiv \begin{cases} Q(x, t) \\ T(x) \\ T_0(t), T_1(t) \end{cases}$$

Here L is the differential operator, and f is the right-hand side. The differential problem (2.4) is defined in the region $0 \leqslant x \leqslant 1$, $t \geqslant 0$, which will be denoted by G. A continuous solution $u(x, t)$ in the region G is assumed to exist. To find the solution by the finite-difference method, the