ESSENTIALS *of* MECHANICS

ESSENTIALS OF MECHANICS
A Unified First Course

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Preface

Although essentially all engineering curricula require some course work in mechanics, the needs of the various engineering disciplines vary widely. This book is intended as a text for a first course in mechanics for those students who require only a one- or two-semester course. It is the belief of the authors that the needs of these students can best be met by a broad, unified treatment of mechanics rather than an in-depth study of one or two specialized areas. The material contained in this book is an expanded version of Young's *Introduction to Applied Mechanics.* Young's book was written primarily for nonengineering students, whereas the present text is designed primarily for engineering students.

In the selection of specific subject matter, emphasis has been placed on topics which engineers with limited training in mechanics are most likely to find of value. Although the coverage is broad, an in-depth, rigorous treatment of the selected topics has been attempted and the material contained in the book is not intended as simply a "survey of mechanics."

The text is organized into five major parts, each containing one or more chapters, as follows:

Part I—Review
Part II—Equilibrium
Part III—Motion and Deformation
Part IV—Rigid-Body Dynamics and Vibrations
Part V—Engineering Materials and Applications

Part I contains material normally found in calculus and physics courses which are presumed as prerequisites for a first course in mechanics. However, it has been the authors' experience that ideas associated with forces and moments and topics such as elementary particle dynamics, centroids, moments of inertia, units, etc. need to be reviewed to reinforce the students' understanding of these elementary but important facets of mechanics.

Part II emphasizes the fundamental nature of equilibrium by considering not only rigid bodies but equilibrium concepts applied to a continuum through the introduction of the concept of stress. Equilibrium of fluids is also treated in this section.

Kinematics of particles, rigid bodies in plane motion, and deformable bodies are included in Part III. The concept of strain is introduced, and the pertinent equations relating to strain transformations and the measurement of strain are included. This section is followed in Part IV with a discussion of rigid body dynamics, with emphasis on plane motion, and an introduction to the vibratory motion of single degree of freedom systems.

In the first four parts of the book material properties play a minimal role. Only the ideas of a rigid body and a fluid at rest are introduced. Thus the student is able to develop general concepts and equations which are applicable to broad classes of problems. In Part V important characteristics of materials are discussed and related to applications in solid and fluid mechanics. The final chapter provides an introduction to dimensional analysis and model theory.

Although the English gravitational system of units (pound-slug-second-foot) is used in the majority of examples the International System of Units (newton-kilogram-second-metre), commonly called SI, is also used in a number of examples. With the increasing usage of SI in engineering disciplines the need for engineers to become familiar with this metric system of units is apparent.

Students are encouraged to carefully study all examples, since the solutions frequently contain important points associated with practical aspects of problem-solving techniques. All data given in examples and problems are assumed to be accurate to at least three significant figures, consistent with the use of the slide rule for obtaining numerical answers. Over 350 problems are included with answers supplied to odd-numbered problems.

This text is organized so that it may be used in a number of ways. It is particularly well suited for a three-quarter (9-hour) or two-semester (6-hour) sequence, and essentially the entire text can be covered in this period of time. In two quarters (6 hours) an integrated course can be developed from the first ten chapters to include the topics of rigid body equilibrium, stress, fluid statics, kinematics, rigid body dynamics, vibrations, strain, materials, and applications in solid mechanics. The material in these chapters has been arranged so that only portions of a given chapter need be covered without loss of continuity with succeeding topics.

The authors gratefully acknowledge the help of many colleagues and students in the development of this book. We especially appreciate the support and encouragement of Dr. H. J. Weiss, Head, Department of Engineering Science and Mechanics at Iowa State University.

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REVIEW

chapter 1

Introduction to Mechanics

1.1 Scope of Mechanics

When a collection of matter is acted on by a system of forces, this action will in general induce resisting forces, internal stresses, deformation, and motion. The science of mechanics consists of the study and analysis of these factors and their interrelationships.

The scope and range of topics normally considered within the province of mechanics is indeed broad. The following list of topics and descriptive phrases is suggestive, but not all-inclusive, of the numerous classical subdivisions of mechanics:

1. Statics—rigid bodies in equilibrium.
2. Dynamics—rigid bodies in motion.
3. Mechanics of materials—stresses and deformations in solids.
4. Fluid mechanics—behavior of liquids and gases at rest and in motion.
5. Vibrations—periodic and transient motion of machines, structures, and systems.
6. Elasticity—mathematical analysis of stress and deformation in elastic systems.
7. Rheology—flow and deformation of materials.

Applications for each of these topics can be found not only in the fields of engineering and physics but also in agriculture, biology, geology, medicine, oceanography, etc.

Although each of the topics noted above represents a highly developed specialized area in mechanics, principles and concepts common to all provide the necessary framework for the development of the specialty. In this book we will focus on these common ingredients and consider a broad range of topics, each topic logically building on the preceding one. This approach will provide an integrated, overall view of the science of mechanics, with a detailed working knowledge of the aspects that commonly arise in many engineering and interdisciplinary applications.

1.2 Continuum Model

In the analysis of a given problem, regardless of the particular application, certain idealizations are made so that a tractable model can be established. It is hoped that the model used will be a satisfactory representation of the physical system of interest. The type and number of idealizations required depend to a certain extent on what information is desired. An important concept, which may be considered as an idealization of a real system, is the *continuum model.* Although we recognize that matter consists of a collection of discrete particles at the molecular level, in most instances we are concerned with a system whose dimensions are very large in comparison with molecular dimensions. Thus for all practical purposes we may assume that matter is continuously distributed over the region of interest. When such an assumption is made, we are utilizing a continuum model, a basic assumption made throughout this text. However, it should be recognized that as the size of the system of interest approaches molecular dimensions, as may be the case in certain biological problems, the continuum model must be discarded. Alternatively, as the distance between molecules becomes large in comparison with the dimensions of the system, the continuum model is no longer applicable. This situation may be encountered in the high-altitude flight of missiles and spacecraft.

A special type of continuum model is the *rigid body.* As the name implies, a rigid body is one that does not deform under the action of external forces. Since all material will deform under loading to some extent, the rigid body concept is an approximation. However, this idealization is frequently adequate for the analysis of certain types of problems. A *particle* is defined as a small quantity of matter whose dimensions are large compared with molecular dimensions, but negligibly small when compared with the dimensions of the overall system of interest.

1.3 Forces

If an object such as a block (Fig. 1.1a) is placed on a plane surface and we push on it, the block may remain at rest or start to slide along the surface. We say that we are applying a "force" to the block. In addition, the block is exerting a "force" on the supporting surface. Since forces always occur in equal and opposite pairs, the surface also exerts a resultant force on the block (Fig. 1.1b). Thus a *force* can be defined as the action of one body on another. In this

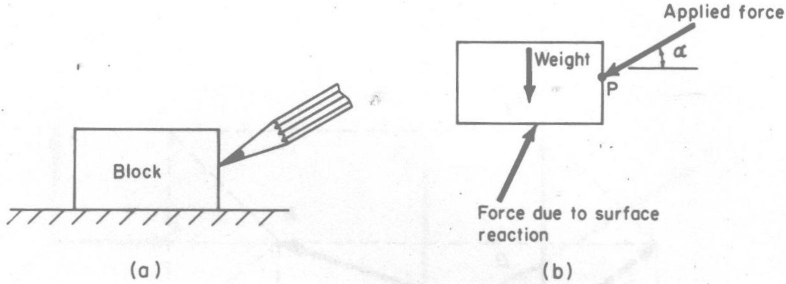

Fig. 1.1

particular example the complete system of interest consists of four "bodies"—the pencil, the block, the object (such as a table) on which the block rests, and the earth that develops the pull of gravity (weight).

The important characteristics of a force are (1) magnitude, (2) direction, and (3) location of line of action. It can also be demonstrated experimentally that the applied force of Fig. 1.1b can be broken down into components in accordance with the parallelogram law (Fig. 1.2). If these components pass through the application point P or any point along the line of action of the force, they would have the same external effect on the body as the original force. The *external effect* of a force either accelerates the body or develops reactions on the body opposing the motion.

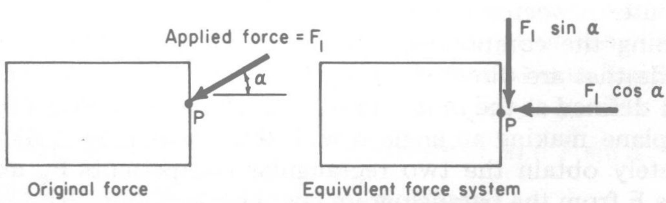

Fig. 1.2

By definition a *vector* is a quantity that has both direction and magnitude and conforms to the parallelogram law of addition. Since these are precisely the characteristics of a force, we arrive at the important conclusion that a force is a vector quantity and must be treated accordingly.

A common requirement in the analysis of elementary mechanics problems is the *resolution* of a force into its components. The force **A** in Fig. 1.3 can first be resolved in accordance with the parallelogram law into a component along the x axis and a component in the

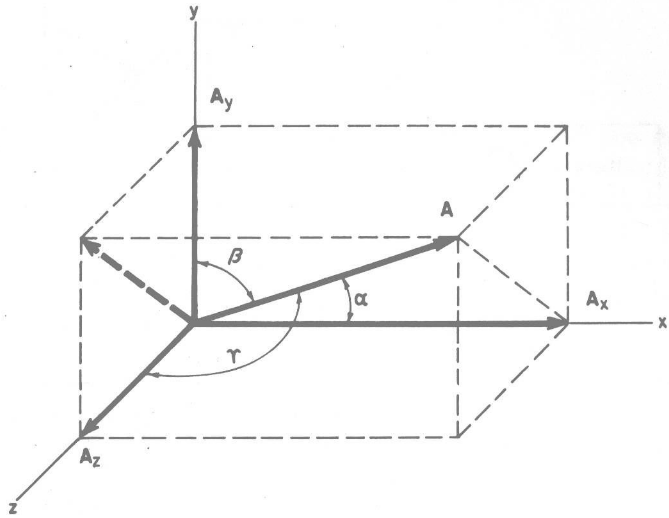

Fig. 1.3

yz plane.[1] This latter component can then be resolved into components along the *y* axis and the *z* axis. By this process the original force **A** has been resolved into three orthogonal components commonly called *rectangular components.* The magnitudes of the three rectangular components are:

$$A_x = A \cos \alpha, \qquad A_y = A \cos \beta, \qquad A_z = A \cos \gamma \tag{1.1}$$

The use of vector algebra provides a very efficient method for determining the components of a force. Consider vectors of unit magnitude that are directed along the positive *x*, *y*, and *z* axes (Fig. 1.4) and defined as the *unit vectors*, **i**, **j**, and **k**. Consider a force **F** in the *xy* plane making an angle α with the *x* axis (Fig. 1.5). We can immediately obtain the two rectangular components $\mathbf{F}_x$ and $\mathbf{F}_y$ of the force **F** from the parallelogram law; i.e.,

$$\mathbf{F}_x = F \cos \alpha \, \mathbf{i}, \qquad \mathbf{F}_y = F \sin \alpha \, \mathbf{j} \tag{1.2}$$

where F is the magnitude of the force **F**. The *dot product* of two vectors **A** and **B** is given by the expression

$$\mathbf{A} \cdot \mathbf{B} = AB \cos \beta \tag{1.3}$$

where β is the angle between the two vectors (Fig. 1.6). Note that the dot product is a *scalar.* From Fig. 1.6 it is observed that $\mathbf{A} \cdot \mathbf{B}$ can be interpreted as the magnitude of the component of **B** in the

1. Vectors will be indicated by boldface symbols.

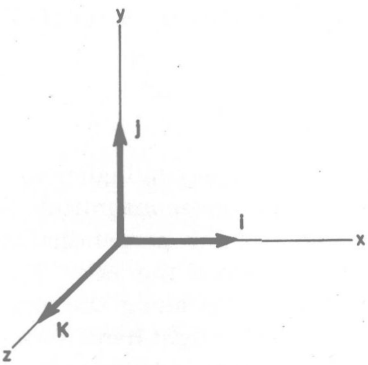

Fig. 1.4

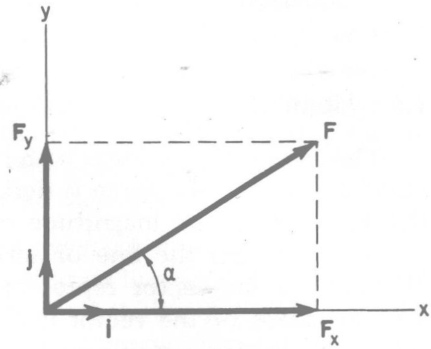

Fig. 1.5

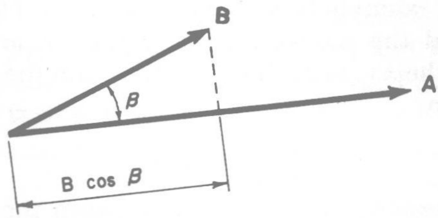

Fig. 1.6

direction of **A** times the magnitude of **A**, or the magnitude of the component of **A** in the direction of **B** times the magnitude of **B**. We see from this that the dot product of a given vector and a unit vector gives the magnitude of the rectangular component of the vector in the direction of the unit vector. Thus for the example in Fig. 1.5 we can write

$$F_x = \mathbf{F} \cdot \mathbf{i} = F \cos \alpha, \qquad F_y = \mathbf{F} \cdot \mathbf{j} = F \cos (90 - \alpha) = F \sin \alpha \tag{1.4}$$

which checks with the results previously obtained.

In certain cases we may be given the components and wish to obtain the resultant. This can be done by the application of the parallelogram law and is known as the process of *composition*.

On a diagram or figure a vector quantity can be designated in various ways; e.g., in Fig. 1.5 the vector component $\mathbf{F}_x$ could be indicated as shown in this figure or as $\rightarrow F_x$ or $\rightarrow F_x\ \mathbf{i}$, where F_x is understood to represent the magnitude of the vector and the arrow indicates that the quantity is a vector. These ways of designating a vector quantity are used interchangeably in this text. However, it is not convenient to use arrows to distinguish between scalars and vectors in equations, and it should be clearly understood that in any