World Scientific Lecture Notes in Complex Systems – Vol. 4

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Lecture Notes on

Turbulence and Coherent Structures in Fluids, Plasmas and Nonlinear Media

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TURBULENCE AND COHERENT STRUCTURES IN FLUIDS, PLASMAS AND NONLINEAR MEDIA

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WORLD SCIENTIFIC LECTURE NOTES IN COMPLEX SYSTEMS

AIMS AND SCOPE

The aim of this new interdisciplinary series is to promote the exchange of information between scientists working in different fields, who are involved in the study of complex systems, and to foster education and training of young scientists entering this rapidly developing research area.

The scope of the series is broad and will include: Statistical physics of large nonequilibrium systems; problems of nonlinear pattern formation in chemistry; complex organization of intracellular processes and biochemical networks of a living cell; various aspects of cell-to-cell communication; behaviour of bacterial colonies; neural networks; functioning and organization of animal populations and large ecological systems; modeling complex social phenomena; applications of statistical mechanics to studies of economics and financial markets; multi-agent robotics and collective intelligence; the emergence and evolution of large-scale communication networks; general mathematical studies of complex cooperative behaviour in large systems.

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Vol. 1 Nonlinear Dynamics: From Lasers to Butterflies
Vol. 2 Emergence of Dynamical Order: Synchronization Phenomena in Complex Systems
Vol. 3 Networks of Interacting Machines

Preface

The problem of turbulence and coherent structures is of key importance in many fields of science and engineering. It is an area which is vigorously researched across a diverse range of disciplines such as theoretical physics, oceanography, atmospheric science, magnetically confined plasma, nonlinear optics, etc. Modern studies in turbulence and coherent structures are based on a variety of theoretical concepts, numerical simulation techniques and experimental methods, which cannot be reviewed effectively by a single expert. The main goal of these lecture notes is to introduce state-of-the-art turbulence research in a variety of approaches (theoretical, numerical simulations and experiments) and applications (fluids, plasmas, geophysics, nonlinear optical media) by several experts.

This book is based on the lectures delivered at the 19th Canberra International Physics Summer School held at the Australian National University in Canberra (Australia) from 16-20 January 2006. The Summer School was sponsored by the Australian Research Council's Complex Open Systems Research Network (COSNet).

The lecturers aimed at (1) giving a smooth introduction to a subject to readers who are not familiar with the field, while (2) reviewing the most recent advances in the area. This collection of lectures will provide a useful review for both postgraduate students and researchers new to the advancements in this field, as well as specialists seeking to expand their knowledge across different areas of turbulence research.

The material covered in this book includes introductions to the theory of developed turbulence (G. Falkovich) and statistical and renormalization methods (D. McComb). The role of turbulence in ocean energy balance is addressed in a review by H. Dijkstra. A comprehensive introduction to the complex area of the theory of turbulence in plasma (J. Krommes) is complemented by a review of experimental methods in plasma turbulence (M. Shats and H. Xia). An introduction to the main ideas and modern capabilities of numerical simulation of turbulence is given by J. Jimenez.

Experimental methods in fluid turbulence studies are illustrated in the lectures by J. Soria describing the particle image velocimetry. Finally, the relatively new field of the physics of vortex flows in optical fields is reviewed by A. Desyatnikov.

The Summer School in Canberra was accompanied by a workshop on the same topic. The Workshop Proceedings (editors J. Denier and J. Frederiksen) will also be published by World Scientific under the same title as these Lecture Notes ("Turbulence and Coherent Structures in Fluids, Plasmas and Nonlinear Media"). References in this book to the Workshop papers are given as "I. Jones, Workshop Proceedings".

Michael Shats
Convenor of the 19th Canberra International Physics Summer School
Canberra, August 2006

Contents

Chapter 1

Introduction to Developed Turbulence

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This is a short course on developed turbulence, weak and strong. The main emphasis is on fundamental properties like universality and symmetries. Two main notions are explained: i) fluxes of dynamical integrals of motion, ii) statistical integrals of motion.

Contents

1.1. Introduction

Turbulence is a state of a physical system with many interacting degrees of freedom deviated far from equilibrium. This state is irregular both in time and in space. Turbulence can be maintained by some external influence or it can be decaying turbulence on the way of relaxation to equilibrium. As the term suggests, it first appeared in fluid mechanics and was later generalized for far-from-equilibrium states in solids and plasma. For example, obstacle of size L placed into fluid moving with velocity V provides for a turbulent wake if the Reynolds number is large: $Re = VL/\nu \gg 1$. Here ν is the kinematic viscosity. At large Re, flow perturbations produced at the scale L have their viscous dissipation small compared to the nonlinear effects.

Nonlinearity produces motions of smaller and smaller scales until viscous dissipation stops this at a scale much smaller than L so that there is a wide (so-called inertial) interval of scales where viscosity is negligible and nonlinearity dominates. Another example is the system of waves excited on a fluid surface by wind or moving bodies and in plasma and solids by external electromagnetic fields. The state of such system is called wave turbulence when the wavelength of the waves excited strongly differs from the wavelength of the waves that effectively dissipate. Nonlinear interaction excites waves in the interval of wavelengths (called transparency window, or inertial interval) between the injection and dissipation scales.

The ensuing complicated and irregular dynamics calls for a statistical description based on averaging either over regions of space or intervals of time. Here we focus on a single-time statistics of steady turbulence that is on the spatial structure of fluctuations. Because of the conceptual simplicity of the inertial range, it is natural to ask if our expectation of universality—that is, freedom from the details of external forcing and internal friction—is true at the level of a physical law. Another facet of the universality problem concerns features that are common to different turbulent systems. This quest for universality is motivated by the hope of being able to distinguish general principles that govern far-from-equilibrium systems, similar in scope to the variational principles that govern thermal equilibrium.

Constraints on dynamics are imposed by conservation laws, and therefore conserved quantities must play an essential role in turbulence. The conservation laws are broken by pumping and dissipation, but both factors do not act in the inertial interval. For example, in the incompressible turbulence, the kinetic energy is pumped by external forcing and is dissipated by viscosity. According to the idea suggested by Richardson in 1921, the kinetic energy flows throughout the inertial interval of scales in a cascade-like process. The cascade idea explains the basic macroscopic manifestation of turbulence: the rate of dissipation of the dynamical integral of motion has a finite limit when the dissipation coefficient tends to zero. For example, the mean rate of the viscous energy dissipation does not depend on viscosity at large Reynolds numbers. That means that a symmetry of the inviscid equation (here, time-reversal invariance) is broken by the presence of the viscous term, even though the latter might have been expected to become negligible in the limit $Re \to \infty$.

The cascade idea fixes only the mean flux of the respective integral of motion demanding it to be constant across the inertial interval of scales. We shall see that flux constancy determines the system completely only for weakly nonlinear system (where the statistics is close to Gaussian). To describe an entire turbulence statistics of strongly interacting systems, one has to solve problems on a case-by-case basis with most cases still unsolved. Particularly difficult (and interesting) are the cases with broken scale invariance where knowledge of flux does not allow one to predict even the order of magnitude of high moments. We describe the new concept of statistical integrals of motion which allows for the description of system with broken scale invariance.

1.2. Weak wave turbulence

From a theoretical point of view, the simplest case is the turbulence of weakly interacting waves. Examples include waves on the water surface, waves in plasma with and without magnetic field, spin waves in magnetics. We assume spatial homogeneity and denote a_k the amplitude of the wave with the wavevector **k**. When the amplitude is small, it satisfies the linear equation

$$\partial a_k/\partial t = -i\omega_k a_k + f_k(t) - \gamma_k a_k \,. \tag{1.1}$$

Here the dispersion law ω_k describes wave propagation, γ_k is the linear damping rate and f_k describes pumping. For the linear system, a_k is different from zero only in the regions of **k**-space where f_k is nonzero. To describe wave turbulence which involves wavenumbers outside the pumping region, one must account for the interaction between different waves. Considering for a moment wave system as closed (that is, without external pumping and dissipation) one can describe it as a Hamiltonian system using wave amplitudes as normal canonical variables (see, for instance, the monograph[1]). At small amplitudes, the Hamiltonian can be written as an expansion over a_k, where the second-order term describes non-interacting waves and high-order terms determine the interaction:

$$\begin{aligned} H &= \int \omega_k |a_k|^2 \, d\mathbf{k} \\ &+ \int \Big(V_{123} a_1 a_2^* a_3^* + c.c.\Big) \delta(\mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3) \, d\mathbf{k}_1 d\mathbf{k}_2 d\mathbf{k}_3 + \mathrm{O}(a^4) \,. \end{aligned} \tag{1.2}$$

Here $V_{123} = V(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ is the interaction vertex and c.c. means complex conjugation. In the Hamiltonian expansion, we presume every subsequent term smaller than the previous one, in particular, $\xi_k = |V_{kkk} a_k| k^d / \omega_k \ll 1$. Wave turbulence that satisfies this condition is called weak turbulence. Here d is the space dimensionality which can be 1, 2 or 3.

The dynamic equation which accounts for pumping, damping, wave propagation and interaction has thus the following form:

$$\partial a_k/\partial t = -i\delta H/\delta a_k^* + f_k(t) - \gamma_k a_k \,. \tag{1.3}$$

It is likely that the statistics of the weak turbulence at $k \gg k_f$ is close to Gaussian for wide classes of pumping statistics (that has not been shown rigorously though). This is definitely the case for the random force with the statistics not very much different from Gaussian. We consider here and below a pumping by a Gaussian random force statistically isotropic and homogeneous in space, and white in time:

$$\langle f_k(t) f^*_{k'}(t') \rangle = F(k)\delta(\mathbf{k}+\mathbf{k}')\delta(t-t') \ . \tag{1.4}$$

Angular brackets mean spatial average. We assume $\gamma_k \ll \omega_k$ (for waves to be well defined) and that $F(k)$ is nonzero only around some k_f.

Since the dynamic equation (1.3) contains a quadratic nonlinearity then the statistical description in terms of moments encounters the closure problem: the time derivative of the second moment is expressed via the third one, the time derivative of the third moment ix expressed via the fourth one etc. Fortunately, weak turbulence in the inertial interval is expected to have the statistics close to Gaussian so one can express the fourth moment as the product of two second ones. As a result one gets a closed equation for the single-time pair correlation function[1] $\langle a_k a_{k'} \rangle = n_k \delta(\mathbf{k}+\mathbf{k}')$

$$\begin{aligned}
\frac{\partial n_k}{\partial t} &= F_k - \gamma_k n_k + I_k^{(3)}, \\
I_k^{(3)} &= \int \left(U_{k12} - U_{1k2} - U_{2k1}\right) d\mathbf{k}_1 d\mathbf{k}_2, \qquad (1.5)\\
U_{123} &= \pi \left[n_2 n_3 - n_1(n_2+n_3)\right] |V_{123}|^2 \delta(\mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3)\delta(\omega_1 - \omega_2 - \omega_3) \ .
\end{aligned}$$

It is called kinetic equation for waves. The collision integral $I_k^{(3)}$ results from the cubic terms in the Hamiltonian i.e. from the quadratic terms in the equations for amplitudes. It can be *interpreted* as describing three-wave interactions: the first term in the integral (1.5) corresponds to a decay of a given wave while the second and third ones correspond to a confluence with other wave.

One can estimate from (1.5) the inverse time of nonlinear interaction at a given k as $|V(k,k,k)|^2 n(k) k^d/\omega(k)$. We define k_d as the wavenumber where this inverse time is comparable to $\gamma(k)$ and assume that nonlinearity dominates over dissipation at $k \ll k_d$. As has been noted, wave turbulence appears when there is a wide (inertial) interval of scales where both pumping and damping are negligible, which requires $k_d \gg k_f$, the condition analogous to $Re \gg 1$. This is schematically shown in Fig. 1.

The presence of frequency delta-function in $I_k^{(3)}$ means that in the first order of perturbation theory in wave interaction we account only for resonant processes which conserve the quadratic part of the energy $E = \int \omega_k n_k \, d\mathbf{k} = \int E_k dk$. For the cascade picture to be valid, the collision integral has to converge in the inertial interval which means that energy exchange is small between motions of vastly different scales, the property called interaction locality in k-space. Consider now a statistical steady state established under the action of pumping and dissipation. Let us multiply (1.5) by ω_k and integrate it over either interior or exterior of the ball with radius k. Taking $k_f \ll k \ll k_d$, one sees that the energy flux through any spherical surface (Ω is a solid angle), is constant in the inertial interval and is equal to the energy production/dissipation rate ϵ:

$$P_k = \int_0^k k^{d-1} dk \int d\Omega\, \omega_k I_k^{(3)} = \int \omega_k F_k\, d\mathbf{k} = \int \gamma_k E_k\, dk = \epsilon\ . \qquad (1.6)$$

This (integral) equation determines n_k. Let us assume now that the

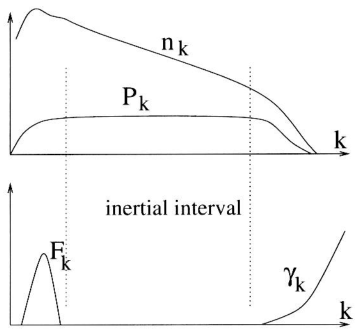

Fig. 1.1. A schematic picture of the cascade.

medium (characterized by the Hamiltonian coefficients) can be considered isotropic at the scales in the inertial interval. In addition, for scales much larger or much smaller than a typical scale (like Debye radius in plasma or the depth of the water) the medium is usually scale invariant: $\omega(k) = ck^\alpha$ and $|V(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2)|^2 = V_0^2 k^{2m} \chi(\mathbf{k}_1/k, \mathbf{k}_2/k)$ with $\chi \simeq 1$. Remind that we presumed statistically isotropic force. In this case, the pair correlation function that describes a steady cascade is also isotropic and scale invariant:

$$n_k \simeq \epsilon^{1/2} V_0^{-1} k^{-m-d}\ . \qquad (1.7)$$

One can show that (1.7), called Zakharov spectrum, turns $I_k^{(3)}$ into zero.[1]

If the dispersion relation $\omega(k)$ does not allow for the resonance condition $\omega(k_1) + \omega(k_2) = \omega(|\mathbf{k}_1 + \mathbf{k}_2|)$ then the three-wave collision integral is zero and one has to account for four-wave scattering which is always resonant. In other words, whatever the $\omega(k)$ relationship is, one can always find four wavevectors that satisfy $\omega(k_1)+\omega(k_2) = \omega(k_3)+\omega(k_4)$ and $\mathbf{k}_1+\mathbf{k}_2 = \mathbf{k}_3+\mathbf{k}_4$.

The collision integral that describes scattering,

$$I_k^{(4)} = \frac{\pi}{2}\int |T_{k123}|^2\big[n_2n_3(n_1+n_k)-n_1n_k(n_2+n_3)\big]\delta(\mathbf{k}+\mathbf{k}_1-\mathbf{k}_2-\mathbf{k}_3)$$
$$\times\delta(\omega_k+\omega_1-\omega_2-\omega_2)\,d\mathbf{k}_1d\mathbf{k}_2d\mathbf{k}_3\ , \tag{1.8}$$

conserves the energy and the wave action $N = \int n_k\, d\mathbf{k}$ (the number of waves). Pumping generally provides for an input of both E and N. If there are two inertial intervals (at $k \gg k_f$ and $k \ll k_f$), then there should be two cascades. Indeed, if $\omega(k)$ grows with k, then the absorbtion of a finite amount of E at $k_d \to \infty$ corresponds to an absorption of an infinitely small N. It is thus clear that the flux of N has to go in the opposite direction, that is, to large scales. A so-called inverse cascade with the constant flux of N can thus be realized at $k \ll k_f$. A sink at small k can be provided by a wall friction in the container or by long waves leaving the turbulent region in open spaces (like in sea storms). The collision integral $I_k^{(3)}$ involves products of two n_k, so that the flux constancy requires $E_k \propto \epsilon^{1/2}$, while for the four-wave case $I_k^{(4)} \propto n^3$ gives $E_k \propto \epsilon^{1/3}$. In many cases (when there is a complete self-similarity) this knowledge is sufficient to obtain the scaling of E_k from a dimensional reasoning without actually calculating V and T. For example, short waves on a deep water are characterized by the surface tension σ and density ρ, so the dispersion relation must be $\omega_k \sim \sqrt{\sigma k^3/\rho}$, which allows for the three-wave resonance, and thus $E_k \sim \epsilon^{1/2}(\rho\sigma)^{1/4}k^{-7/4}$. For long waves on a deep water, the surface-restoring force is dominated by gravity so that the gravity acceleration g replaces σ as a defining parameter and $\omega_k \sim \sqrt{gk}$. Such dispersion law does not allow for the three-wave resonance so that the dominant interaction is four-wave scattering which permits two cascades. The direct energy cascade corresponds to $E_k \sim \epsilon^{1/3}\rho^{2/3}g^{1/2}k^{-5/2}$. The inverse cascade carries the flux of N which we denote Q, it has the dimensionality $[Q] = [\epsilon]/[\omega_k]$ and corresponds to $E_k \sim Q^{1/3}\rho^{2/3}g^{2/3}k^{-7/3}$.

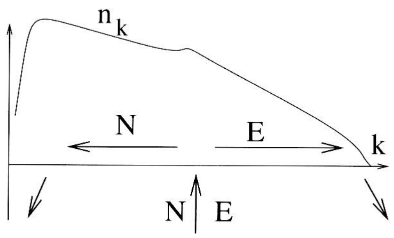

Fig. 1.2. Two cascades under four-wave interaction.

Since the statistics of weak turbulence is near Gaussian, it is completely determined by the pair correlation function, which is in turn determined by the respective flux. We thus conclude that weak turbulence is universal in the inertial interval.

1.3. Strong wave turbulence

One cannot treat wave turbulence as a set of weakly interacting waves when the wave amplitudes are big enough (so that $\xi_k \geq 1$) and also in a particular case of linear (acoustic) dispersion relation $\omega(k) = ck$ for arbitrarily small amplitudes. Indeed, there is no dispersion of wave velocity for acoustic waves so that waves moving in the same direction interact strongly and produce shock waves when viscosity is small. Formally, there is a singularity due to coinciding arguments of delta-functions in (1.5) (and in the higher terms of perturbation expansion for $\partial n_k/\partial t$), which is thus invalid at however small amplitudes. Still, some features of the statistics of acoustic turbulence can be understood even without closed description. We discuss this in a one-dimensional case which pertains, for instance, to sound propagating in long pipes. Since weak shocks are stable with respect to transverse perturbations,[2] quasi one-dimensional perturbations may propagate in 2D and 3D as well. In the frame of reference moving with the sound velocity, the weakly compressible 1d flows ($u \ll c$) are described by the Burgers equation[2,3]

$$u_t + uu_x - \nu u_{xx} = 0 \ . \tag{1.9}$$

Burgers equation has a propagating shock-wave solution $u = 2v\{1 + \exp[v(x - vt)/\nu]\}^{-1}$ with the energy dissipation rate $\nu \int u_x^2 \, dx$ independent of ν. The shock width ν/v is a dissipative scale and we shall consider acoustic turbulence produced by a pumping correlated on much larger scales (for example, pumping a pipe from one end by frequencies much less than cv/ν). After some time, it will develop shocks at random positions. Here we consider the single-time statistics of the Galilean invariant velocity difference $\delta u(x,t) = u(x,t) - u(0,t)$. The moments of δu are called *structure functions* $S_n(x,t) = \langle [u(x,t) - u(0,t)]^n \rangle$. Quadratic nonlinearity makes the time derivative of the second moment to be expressed via the third one:

$$\frac{\partial S_2}{\partial t} = -\frac{\partial S_3}{3\partial x} - 4\epsilon + \nu \frac{\partial^2 S_2}{\partial x^2} \ . \tag{1.10}$$

Here $\epsilon = \nu \langle u_x^2 \rangle$ is the mean energy dissipation rate. Equation (1.10) describes both a free decay (then ϵ depends on t) and the case of a permanently acting pumping which generates turbulence statistically steady at scales less than the pumping length. In the first case, $\partial S_2/\partial t \simeq S_2 u/L \ll \epsilon \simeq u^3/L$

(where L is a typical distance between shocks) while in the second case $\partial S_2/\partial t = 0$ so that $S_3 = 12\epsilon x + \nu\partial S_2/\partial x$.

Consider now a limit $\nu \to 0$ at fixed x (and t for decaying turbulence). Shock dissipation provides for a finite limit of ϵ at $\nu \to 0$, then

$$S_3 = -12\epsilon x \ . \tag{1.11}$$

This formula is a direct analog of (1.6). Indeed, the Fourier transform of (1.10) describes the energy density $E_k = \langle |u_k|^2\rangle/2$ which satisfies the equation $(\partial_t - \nu k^2)E_k = -\partial P_k/\partial k$ where the k-space flux is

$$P_k = \int_0^k dk' \int_{-\infty}^{\infty} dx S_3(x) k' \sin(k'x)/24 \ .$$

It is thus the flux constancy that fixes $S_3(x)$ which is universal, i.e., it is determined solely by ϵ and depends neither on the initial statistics of decay nor on the pumping for steady turbulence. On the contrary, other structure functions $S_n(x)$ are not given by $(\epsilon x)^{n/3}$. Indeed, the scaling of the structure functions can be readily understood for any dilute set of shocks (when shocks do not cluster in space) which seems to be the case both for smooth initial conditions and a large-scale pumping in Burgers turbulence. In this case, $S_n(x) \sim C_n|x|^n + C'_n|x|$, where the first term comes from the regular (smooth) parts of the velocity (the right x-interval in Fig. 1.3.), while the second term comes from $O(x)$ probability to have a shock in the interval x. The scaling exponents, $\xi_n = d\ln S_n/d\ln x$, thus behave as follows: $\xi_n = n$ for $n \le 1$ and $\xi_n = 1$ for $n > 1$. That means that the probability density function (PDF) of the velocity difference in the inertial interval $P(\delta u, x)$ is not scale-invariant, i.e., the function of the rescaled velocity difference $\delta u/x^a$ cannot be made scale-independent for any a. As one goes to smaller scales, the low-order moments decrease faster than the high-order ones, that means that the smaller the scale, the more probable are large fluctuations. In other words, the level of fluctuations increases with the resolution. When the scaling exponents ξ_n do not lie on a straight line, this is called an anomalous scaling since it is related again to the symmetry (scale invariance) of the PDF broken by pumping and not restored even when $x/L \to 0$. As an alternative to the description in terms of structures (shocks), one can relate the anomalous scaling in Burgers turbulence to additional integrals of motion. Indeed, the integrals $E_n = \int u^{2n}\, dx/2$ are all conserved by the inviscid Burgers equation. Any shock dissipates the finite amount of E_n in the limit of $\nu \to 0$, so that similarly to (1.11), one denotes $\langle \dot{E}_n\rangle = \epsilon_n$ and obtains $S_{2n+1} = -4(2n+1)\epsilon_n x/(2n-1)$ for integer n.

Note that $S_2(x) \propto |x|$ corresponds to $E(k) \propto k^{-2}$, since every shock gives $u_k \propto 1/k$ at $k \ll v/\nu$, such that the energy spectrum is determined by