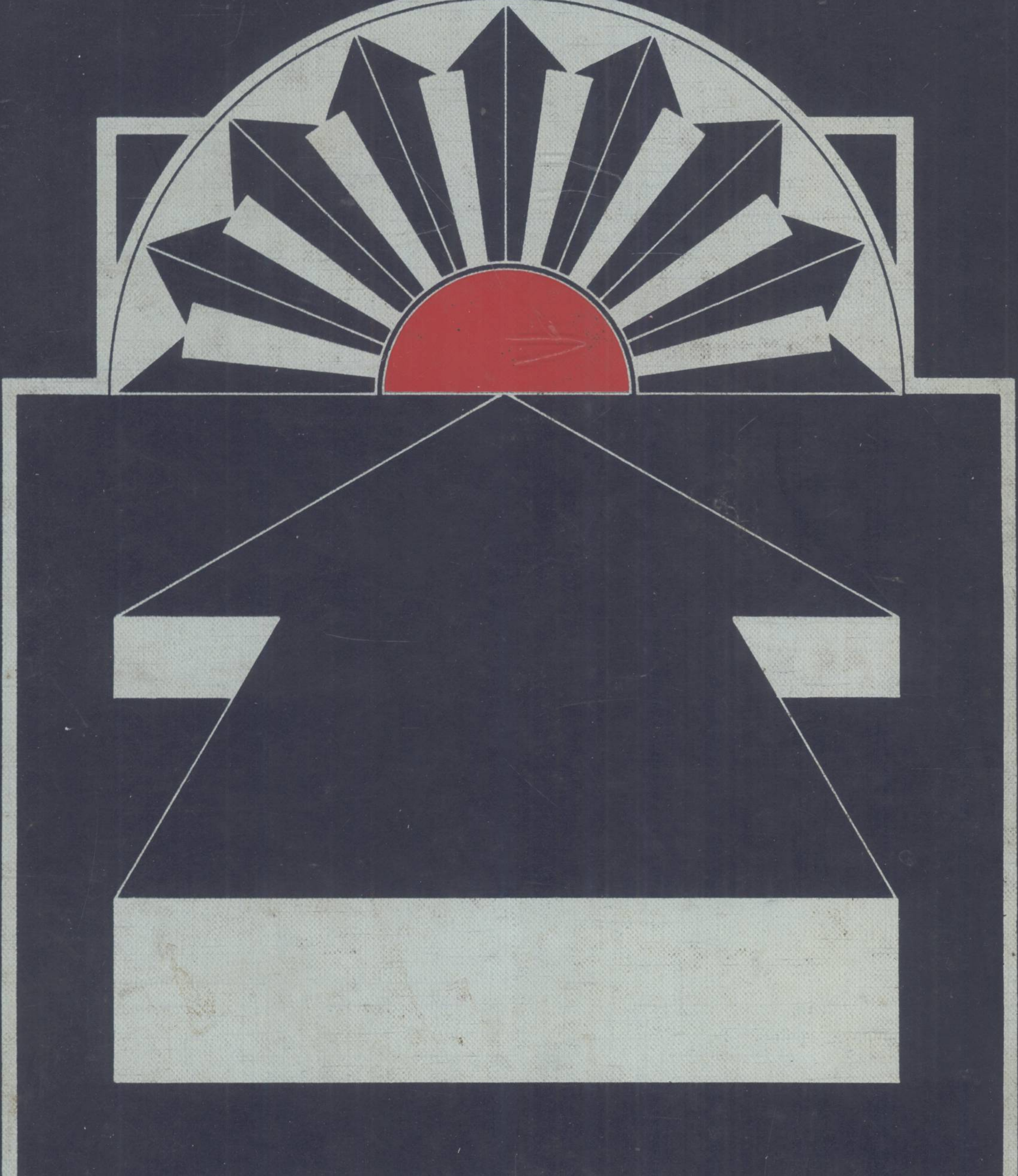
Third Edition

MODERN CONTROL SYSTEMS

Richard C. Dorf



THIRD EDITION

Modern Control Systems

RICHARD C. DORF
UNIVERSITY OF CALIFORNIA, DAVIS

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Modern Control Systems

Man cannot inherit the past; he has to recreate it.

To Christine and Renée
as they seek to create

Preface

Man cannot inherit the past; he has to recreate it.* The most important and productive approach to learning is for the reader to rediscover and recreate anew the answers and methods of the past. Thus, the ideal is to present the student with a series of problems and questions and point to some of the answers that have been obtained over the past decades. The traditional method of confronting the student not with the problem but with the finished solution means depriving him or her of all excitement, to shut off the creative impulse, to reduce the adventure of humankind to a dusty heap of theorems. The issue, then, is to present some of the unanswered and important problems which we continue to confront. For it may be asserted that what we have truly learned and understood we discovered ourselves.

The purpose of this book is to present the structure of feedback control theory and to provide a sequence of exciting discoveries as one proceeds through the text and problems. In that this book is able to assist the student in discovering feedback control system theory and practice, it will have succeeded.

The book is organized around the concepts of control system theory as they have been developed in the frequency- and time-domain. A real attempt has been made to make the selection of topics, as well as the systems discussed in the examples and problems, modern in the best sense. Therefore, one will find a discussion of sensitivity, performance indices, state variables, and computer control systems, to name a few. However, a valiant attempt has been made to retain the classical topics of control theory which have proven to be so very useful in practice.

The text is written in an integrated form so that one should proceed from the first to the last chapter. However, it is not necessary to include all the sections of a given chapter in any given course, and there appears to be quite a large number of combinations of sequences of the sections for study. The book is designed for an introductory undergraduate course in control systems for engineering students. There is very little demarcation between electrical, mechanical, chemical, and industrial engineering in control system practice; therefore, this text is written with-

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* A. Koestler, *The Act of Creation*, Hutchinson, London, 1964; p. 266.

out any conscious bias toward one discipline. Thus, it is hoped that this book will equally be useful for all engineering disciplines and, perhaps, assist in illustrating the unity of control engineering. The problems and examples are chosen from all fields, and the examples of the sociological, biological, ecological, and economic control systems are intended to provide the reader with an awareness of the general applicability of control theory to many facets of life.

The book is primarily concerned with linear, constant parameter control systems. This is a deliberate limitation, since the author believes that for an introduction to control systems, it is wisest to initially consider linear systems. Nevertheless, several nonlinear systems are introduced and discussed where it is appropriate.

Chapter 1 provides an introduction and basic history of control theory. Chapter 2 is concerned with developing mathematical models of these systems. With the models available, we are able to describe the characteristics of feedback control systems in Chapter 3 and illustrate why feedback is introduced in a control system. In Chapter 4 we examine the performance of control systems, and in Chapter 5 we investigate the stability of feedback systems. Chapter 6 is concerned with the s-plane representation of the characteristic equation of a system and the root locus. Chapters 7 and 8 treat the frequency response of a system and the investigation of stability using the Nyquist criterion. Chapter 9 develops the time-domain concepts in terms of the state variables of a system. Chapter 10 describes and develops several approaches to designing and compensating a control system. Finally, Chapter 11 discusses digital computer control systems.

This book is suitable for an introductory course in control systems. The text, in its first and second editions, has been used for a senior level course for engineering students at over one hundred colleges and universities. Also, it has been used for a course for engineering graduate students with no previous background in control system theory.

The text presumes a reasonable familiarity with the Laplace transformation and transfer functions as developed in a first course in linear system analysis or network analysis. These concepts are discussed in Chapter 2 and are used to develop mathematical models for control system components. Answers to selected problems are provided at the end of the book.

This material has been developed with the assistance of many individuals to whom I wish to express my sincere appreciation. Among those to whom I owe a particular debt of gratitude are Professors L. Gould, G. Thaler, and S. Weissenberger, and Mr. S. Mori. I wish to acknowledge the unflagging assistance of my secretaries, Mrs. M. McKenna, Mrs. M. Mahaffey, Mrs. B. Moore, and Mrs. P. Needle. Finally, I can only partially acknowledge the encouragement and patience of my wife, Joy, who helped to make this book possible.

Davis, California
January 1980

R.C.D.

Contents

1 Introduction to Control Systems

1.1 Introduction 1
1.2 History of automatic control 3
1.3 Control engineering practice 6
1.4 Examples of modern control systems 6

2 Mathematical Models of Systems

2.1 Introduction 23
2.2 Differential equations of physical systems 23
2.3 Linear approximations of physical systems 28
2.4 The Laplace transform 31
2.5 The transfer function of linear systems 38
2.6 Block diagram models 49
2.7 Signal flow graph models 55
2.8 Simulation of control systems 60
2.9 Summary 63

3 Feedback Control System Characteristics

3.1 Open- and closed-loop control systems 81
3.2 Sensitivity of control systems to parameter variations 83
3.3 Control of the transient response of control systems 87
3.4 Disturbance signals in a feedback control system 90
3.5 Steady-state error 94
3.6 The cost of feedback 97
3.7 Summary 99

4 The Performance of Feedback Control Systems

4.1 Introduction 109
4.2 Time-domain performance specifications 110
4.3 The s-plane root location and the transient response 119
4.4 The steady-state error of feedback control systems 119
4.5 Performance indices 124
4.6 The simplification of linear systems 133
4.7 Summary 135

5 The Stability of Linear Feedback Systems

5.1 The concept of stability 145
5.2 The Routh-Hurwitz stability criterion 148
5.3 The relative stability of feedback control systems 154
5.4 The determination of root locations in the s-plane 156
5.5 Summary 159

6 The Root Locus Method

6.1 Introduction 165
6.2 The root locus concept 165
6.3 The root locus procedure 170
6.4 An example of a control system analysis and design utilizing the root locus method 184
6.5 Parameter design by the root locus method 188
6.6 Sensitivity and the root locus 193
6.7 Summary 202

7 Frequency Response Methods

7.1 Introduction 215
7.2 Frequency response plots 217
7.3 Example of drawing the Bode diagram 234
7.4 Frequency response measurements 237
7.5 Performance specifications in the frequency domain 241
7.6 Log magnitude and phase diagrams 243
7.7 Summary 246

8 Stability in the Frequency Domain

8.1 Introduction 255
8.2 Mapping of contours in the s-plane 256

8.3 The Nyquist criterion 262
8.4 Relative stability and the Nyquist criterion 271
8.5 The closed-loop frequency response 278
8.6 The stability of control systems with time delays 286
8.7 Summary 288

9 Time-Domain Analysis of Control Systems

9.1 Introduction 311
9.2 The state variables of a dynamic system 312
9.3 The state vector differential equation 315
9.4 Signal flow graph state models 317
9.5 The stability of systems in the time domain 324
9.6 The time response and the transition matrix 331
9.7 A discrete-time evaluation of the time response 335
9.8 Summary 341

10 The Design and Compensation of Feedback Control Systems

10.1 Introduction 357
10.2 Approaches to compensation 359
10.3 Cascade compensation networks 361
10.4 System compensation on the Bode diagram using the phase-lead network 366
10.5 Compensation on the *s*-plane using the phase-lead network 372
10.6 System compensation using integration networks 379
10.7 Compensation on the *s*-plane using a phase-lag network 383
10.8 Compensation on the Bode diagram using a phase-lag network 388
10.9 Compensation on the Bode diagram using analytical and computer methods 392
10.10 The design of control systems in the time domain 394
10.11 State-variable feedback 405
10.12 Summary 408

11 Digital Control Systems

11.1 Introduction 421
11.2 Digital computer control system applications 421
11.3 Automatic assembly and robots 426
11.4 Sampled-data systems 427
11.5 Computer programs for control 432
11.6 The *z*-transform 433
11.7 Stability analysis in the *z*-plane 439

11.8 Performance of a sampled-data second-order system 440
11.9 Closed-loop systems with digital computer compensation 443
11.10 Summary 444

Appendix A

Laplace transform pairs 451

Appendix B

Table B.1 Symbols and units 453
Table B.2 Conversion factors 454

Appendix C

An introduction to matrix algebra 455

Appendix D

The evaluation of the transition matrix of a linear time-invariant system by means of a computational algorithm 465

Appendix E

Program transition 469

Appendix F

Decibel conversion 475

Answers to Selected Problems 479

Index 489

1 / Introduction to Control Systems

1.1 INTRODUCTION

Engineering is concerned with understanding and controlling the materials and forces of nature for the benefit of mankind. Control system engineers are concerned with understanding and controlling segments of their environment, often called *systems,* in order to provide useful economic products for society. The twin goals of understanding and control are complementary since, in order to control more effectively, the systems under control must be understood and modeled. Furthermore, control engineering often must consider the control of poorly understood systems such as chemical process systems. The present challenge to control engineers is the modeling and control of modern, complex, interrelated systems such as traffic-control systems, chemical processes, and economic regulation systems. However, simultaneously, the fortunate engineer has the opportunity to control many very useful and interesting industrial automation systems. Perhaps the most characteristic quality of control engineering is the opportunity to control machines, and industrial and economic processes for the benefit of society.

Control engineering is based on the foundations of feedback theory and linear system analysis, and integrates the concepts of network theory and communication theory. Therefore, control engineering is not limited to any engineering discipline but is equally applicable for aeronautical, chemical, mechanical, environmental, civil, and electrical engineering. For example, quite often a control system includes electrical, mechanical, and chemical components. Furthermore, as the understanding of the dynamics of business, social, and political systems increases, the ability to control these systems will increase also.

A *control system* is an interconnection of components forming a system configuration which will provide a desired system response. The basis for analysis of a system is the foundation provided by linear system theory which assumes a cause-

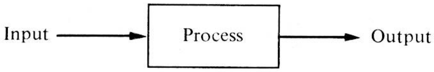

Fig. 1.1. Process to be controlled

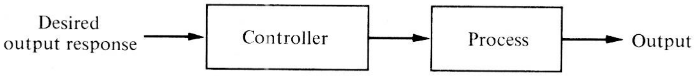

Fig. 1.2. Open-loop control system.

effect relationship for the components of a system. Therefore, a component or process to be controlled can be represented by a block as shown in Fig. 1.1. The input-output relation represents the cause and effect relationship of the process, which in turn represents a processing of the input signal to provide an output signal variable, often with a power amplification. An *open-loop* control system utilizes a controller or control actuator in order to obtain the desired response as shown in Fig. 1.2.

In contrast to an open-loop control system, a closed-loop control system utilizes an additional measure of the actual output in order to compare the actual output with the desired output response. A simple *closed-loop feedback control system* is shown in Fig. 1.3. A standard definition of a feedback control system is as follows: A feedback control system is a control system which tends to maintain a prescribed relationship of one system variable to another by comparing functions of these variables and using the difference as a means of control.

A feedback control system often uses a function of a prescribed relationship between the output and reference input to control the process. Often the difference between the output of the process under control and the reference input is amplified and used to control the process so that the difference is continually reduced. The feedback concept has been the foundation for control system analysis and design.

Due to the increasing complexity of the system under control and the interest in achieving optimum performance, the importance of control system engineering has grown in this decade. Furthermore, as the systems become more complex, the interrelationship of many controlled variables must be considered in the control scheme. A block diagram depicting a *multivariable control system* is shown in Fig. 1.4. A humorous example of a closed-loop feedback system is shown in Fig. 1.5.

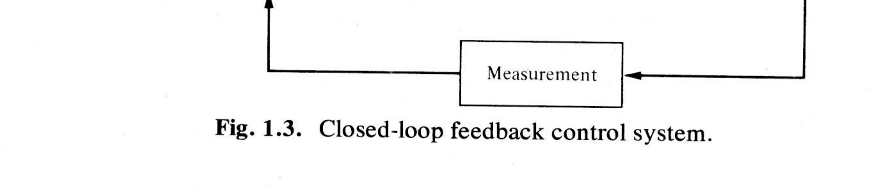

Fig. 1.3. Closed-loop feedback control system.

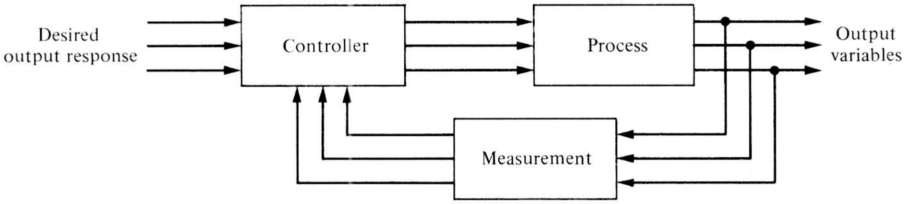

Fig. 1.4. Multivariable control system.

1.2 HISTORY OF AUTOMATIC CONTROL

The use of feedback in order to control a system has had a fascinating history. The first applications of feedback control rest in the development of float regulator mechanisms in Greece in the period 300 to 1 B.C. [1, 2]. The waterclock of Ktesibios used a float regulator (refer to Problem 1.11). An oil lamp devised by Philon in approximately 250 B.C. used a float regulator in an oil lamp for maintaining a constant level of fuel oil. Heron of Alexandria, who lived in the first century A.D., published a book entitled *Pneumatica* which outlined several forms of water level mechanisms using float regulators [1].

The first feedback system to be invented in modern Europe was the temperature regulator of Cornelis Drebbel (1572–1633) of Holland [1]. Dennis Papin [1647–1712] invented the first pressure regulator for steam boilers in 1681. Papin's pressure regulator was a form of safety regulator similar to a pressure-cooker valve.

The first automatic feedback controller used in an industrial process is generally agreed to be James Watt's flyball governor developed in 1769 for controlling the speed of a steam engine [1, 2]. The all-mechanical device, as shown in Fig. 1.6, measured the speed of the output shaft and utilized the movement of the flyball with

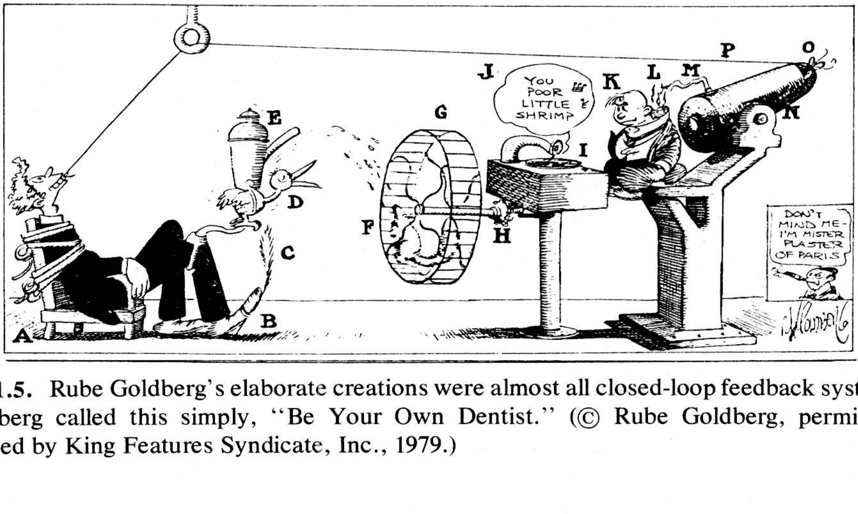

Fig. 1.5. Rube Goldberg's elaborate creations were almost all closed-loop feedback systems. Goldberg called this simply, "Be Your Own Dentist." (© Rube Goldberg, permission granted by King Features Syndicate, Inc., 1979.)

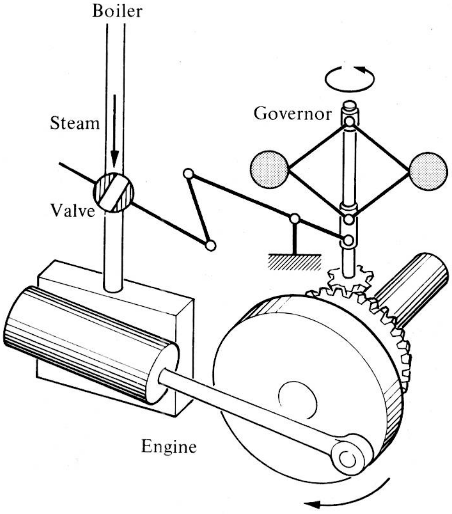

Fig. 1.6. Watt flyball governor.

speed to control the valve and therefore the amount of steam entering the engine. As the speed increases, the ball weights rise and move away from the shaft axis thus closing the valve. The flyweights require power from the engine in order to turn and therefore make the speed measurement less accurate.

The first historical feedback system claimed by the Soviet Union is the water level float regulator said to have been invented by I. Polzunov in 1765 [4]. The level regulator system is shown in Fig. 1.7. The float detects the water level and controls the valve which covers the water inlet in the boiler.

The period preceding 1868 was characterized by the development of automatic control systems by intuitive invention. Efforts to increase the accuracy of the con-

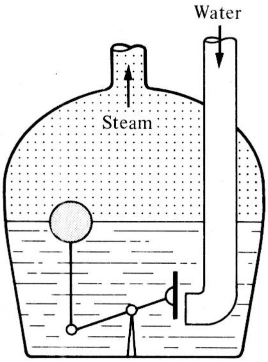

Fig. 1.7. Water-level float regulator.

trol system led to slower attenuation of the transient oscillations and even to unstable systems. It then became imperative to develop a theory of automatic control. J. C. Maxwell formulated a mathematical theory related to control theory using a differential equation model of a governor [5]. Maxwell's study was concerned with the effect various system parameters had on the system performance. During the same period, I. A. Vyshnegradskii formulated a mathematical theory of regulators [6].

Prior to World War II, control theory and practice developed in the United States of America and Western Europe in a different manner than in the U.S.S.R. and Eastern Europe. One main impetus for the use of feedback in the U.S.A. was the development of the telephone system and electronic feedback amplifiers by Bode, Nyquist, and Black at the Bell Telephone Laboratories [7, 8, 9, 10, 12]. The frequency domain was used primarily to describe the operation of the feedback amplifiers in terms of bandwidth and other frequency variables. In contrast, the eminent mathematicians and applied mechanicians in Russia inspired and dominated the field of control theory. Therefore, the Russian theory tended to utilize a time-domain formulation using differential equations.

A large impetus to the theory and practice of automatic control occurred during World War II when it became necessary to design and construct automatic airplane pilots, gun-positioning systems, radar antenna control systems, and other military systems based on the feedback control approach. The complexity and expected performance of these military systems necessitated an extension of the available control techniques and fostered interest in control systems and the development of new insights and methods. Prior to 1940, for most cases, the design of control systems was an art involving a trial and error approach. During the decade of the 1940's, mathematical and analytical methods increased in number and utility, and control engineering became an engineering discipline in its own right [10, 11, 12].

Frequency-domain techniques continued to dominate the field of control following World War II with the increased use of the Laplace transform and the complex frequency plane. During the 1950s, the emphasis in control engineering theory was on the development and use of the s-plane methods and, particularly, the root locus approach. Furthermore, during the 1950s, the utilization of both analog and digital computers for control components became possible. These new controlling elements possessed an ability to calculate rapidly and accurately which was formerly not available to the control engineer. There are now over thirty thousand digital process control computers installed in the United States [13, 14]. These computers are employed especially for process control systems in which many variables are measured and controlled simultaneously by the computer.

With the advent of Sputnik and the space age, another new impetus was imparted to control engineering. It became necessary to design complex, highly accurate control systems for missiles and space probes. Furthermore, the necessity to minimize the weight of satellites and to control them very accurately has spawned the important field of optimal control. Due to these requirements, the time-domain methods due to Liapunov, Minorsky, and others have met with great interest in the last decade [15]. Furthermore, new theories of optimal control have been developed by L. S. Pontryagin in Russia and R. Bellman in the U.S.A. It now appears that control engineering must consider both the time-domain and the frequency-domain approaches simultaneously in the analysis and design of control systems.

1.3 CONTROL ENGINEERING PRACTICE

Control engineering is concerned with the analysis and design of goal-oriented systems. Therefore, the mechanization of goal-oriented policies has grown into a hierarchy of goal-oriented control systems. Modern control theory is concerned with systems with the self-organizing, adaptive, learning, and optimum qualities. This interest has aroused even greater excitement among control engineers.

The control of an industrial process (manufacturing, production, etc.) by automatic rather than human means is often called *automation.* Automation is prevalent in the chemical, electric power, paper, automobile, and steel industries, among others. The concept of automation is central to our industrial society. Automatic machines are used to increase the production of a plant per worker in order to offset rising wages and inflationary costs. Thus, industries are concerned with the productivity per worker of their plant. *Productivity* is defined as the ratio of physical output to physical input. In this case we are referring to labor productivity, which is real output per hour of work. In a study conducted by the U.S. Commerce Department it was determined that labor productivity grew at an average annual rate of 2.8% from 1948 to 1978 [20]. In order to continue these productivity gains, expenditures for factory automation in the United States are expected to double from 1.5 billion dollars in 1978 to 3.0 billion dollars in 1984 [13, 14]. World-wide, expenditures for process control and manufacturing plant control are expected to grow from 4.4 billion dollars in 1976 to 9.5 billion dollars in 1986 [22]. The U.S. manufacturers currently supply approximately one-half of worldwide control equipment.

There are about 120,000 control engineers in the United States and over 100,000 control engineers in the Soviet Union. In the United States alone, the control industry does a business of over eight billion dollars per year! The theory, practice, and application of automatic control is a large, exciting, and extremely useful engineering discipline. One can readily understand the motivation for a study of modern control systems.

1.4 EXAMPLES OF MODERN CONTROL SYSTEMS

Feedback control is a fundamental fact of modern industry and society. Driving an automobile is a pleasant task when the auto responds rapidly to the driver's com-