S. Marburg
B. Nolte
Editors

Computational Acoustics of Noise Propagation in Fluids

Finite and Boundary Element Methods

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Steffen Marburg · Bodo Nolte
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Computational Acoustics of Noise Propagation in Fluids – Finite and Boundary Element Methods

With 285 Figures and 29 Tables

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Editors:

Steffen Marburg (Ed.)
Institut für Festkörpermechanik
Technische Universität Dresden
Helmholtzstraße 10
01062 Dresden, Germany
marburg@ifkm.mw.tu-dresden.de

Bodo Nolte (Ed.)
Forschungsanstalt der Bundeswehr
für Wasserschall und Geophysik
Klausdorfer Weg 2–24
24148 Kiel, Germany
BodoNolte@bwb.org

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Computational Acoustics of Noise Propagation in Fluids – Finite and Boundary Element Methods

For Mäxi, Jolly & Milan, who was the WDL[1]

[1] das Wunder des Lebens (*The Wonder of life*)

Preface

Low noise constructions receive increasing attention in highly industrialized countries. Consequently, control of noise emission challenges a growing community of engineers. Classically, noise emission is controlled experimentally utilizing the trial and error method and engineering experience. The development of numerical methods such as the finite element and the boundary element method for low frequency acoustic problems and statistic methods for high frequency problems allows simulation of radiation and scattering from arbitrary geometric objects.

For low and medium frequency problems, classical approaches for solution of problems of acoustics favor analytical methods including Fourier series approaches. These approaches are quite powerful and they are still developed further. In particular, if orthogonal eigenfunctions are used as the basis functions of the Fourier series, they converge rapidly. However, if the geometry of the radiator or scatterer becomes more complicated, Fourier series become impractical to use. In these cases, numerical methods can be used more conveniently. The easiest and most straightforward approach consists of the finite difference method. However, finite difference methods suffer from a number of specific problems such as mesh restrictions and dispersion. Alternatively, finite element and boundary element methods use a more complicate mathematical formulation but can be applied in a very general way.

This book deals with finite element and boundary element methods for acoustic problems. Although, the title contains the restriction of the acoustics of fluids, a number of chapters consider solid structures as well. The edition comprises 21 chapters. The first one, i.e. Chapter 0, is a concept chapter. It starts with the derivation of the harmonic wave equation from the fundamental relations of continuum mechanics. It is followed by ten chapters on finite element methods and another ten chapters on boundary element methods. The reader is referred to Chapter 0 and Section 0.6, cf. pages 20–22, to survey the remaining chapters and discuss them related to the formulations which are given in Chapter 0.

This is a book on numerical methods. In the first volume of his series *The Hitchhiker's Guide to the Galaxy*,[2] Douglas Adams formulates "the ultimate answer to life, the universe, and everything." It is a numeric solution: 42, evaluated by the computer Deep Thought. A CPU time of $\approx 2.37 \times 10^{14}$ seconds (7.5 Million years) was required to achieve this interesting result. This book on numerical methods contains contributions written by 42 authors. The number of 42 might indicate that it covers a wide range of topics of computational acoustics. However, the reader should not expect the ultimate answer to the problems of computational acoustics in general. It took the editors $\approx$ 20 months ($\approx 5.2 \times 10^{7}$ seconds) of manual work from the idea to the final version of the book. There are many reasons why this book has been completed much faster than the evaluation of Deep Thought. Maybe this was achieved because the overall content is less general than the ultimate answer to life, the universe, and everything. Probably, the major reason for the successful and efficient completion consists in the willingly collaboration of all authors to supply the editors with their contributions. The editors wish to acknowledge that it has been their great pleasure to work together with all authors.

A number of other persons have been relevant for the successful completion of this edition. First of all, we wish to mention Eva Hestermann–Beyerle of the Springer–Verlag in Heidelberg. It is worth mentioning that she encouraged the editors to start with their editorial work. Moreover, Eva Hestermann–Beyerle has continuously supervised the progress of the edition and provided the editors with numerous valuable advice.

The editors wish to thank their close colleagues at the Institute of Solid Mechanics at Technische Universität Dresden and at the Federal Armed Forces Underwater Acoustics and Marine Geophysics Research Institute for their patience and their support. There are many others who contributed to the successful completion of this work. It seems to be impossible to mention all of them. However, the editors are very thankful for every single assistance in during the preparation of this book.

Dresden and Kiel,
October 2007

Steffen Marburg
Bodo Nolte

[2]Douglas Adams' *The Hitchhiker's Guide to the Galaxy* was originally published in 1979 by *Pan Books Ltd., London.*

List of Contributors

Irfan Ali Friedrich–Alexander–Universität Erlangen–Nürnberg, Lehrstuhl für Strömungsmechanik, Cauerstraße 4, 91058 Erlangen, Germany
irfan@lstm.uni–erlangen.de

R Jeremy Astley Institute of Sound and Vibration Research, University of Southampton, Highfield, Southampton, SO17 1BJ, United Kingdom
rja@isvr.soton.ac.uk

Stefan Becker Friedrich–Alexander–Universität Erlangen–Nürnberg, Lehrstuhl für Strömungsmechanik, Cauerstraße 4, 91058 Erlangen, Germany
sbecker@lstm.uni–erlangen.de

Alfredo Bermúdez Departamento de Matemática Aplicada, Universidade de Santiago de Compostela, 15782 Santiago de Compostela, Spain
mabermud@usc.es

Robert Bernhard Purdue University, Hovde Hall, 610 Purdue Mall, West Lafayette, IN 47907, USA
bernhard@purdue.edu

Haike Brick TFH Berlin – University of Applied Sciences, Department of Mathematics, Physics and Chemistry, Luxemburger Str. 10, 13353 Berlin, Germany
brick@tfh–berlin.de

Dominik Brunner Institute of Applied and Experimental Mechanics, Pfaffenwaldring 9, 70550 Stuttgart, Germany
brunner@iam.uni–stuttgart.de

Zhensheng Chen Acoustics Research Institute, Austrian Academy of Sciences, Wohllebengasse 12–14, A–1040 Vienna, Austria
zhensheng.chen@oeaw.ac.at

Gary Cohen INRIA, Domaine de Voluceau, Rocquencourt – BP 105, 78153 Le Chesnay Cedex, France
gary.cohen@inria.fr

Daniel Dreyer Hanse Wohnbau GmbH, Birkenweg 15, D–22850 Norderstedt, Germany
daniel.dreyer@h–wb.de

Max Escobar Friedrich–Alexander–Universität Erlangen–Nürnberg, Lehrstuhl für Sensorik, Paul–Gordan–Straße 3/5, 91052 Erlangen, Germany
max.escobar@lse.eei.uni–erlangen.de

Otto von Estorff Institute of Modelling and Computation, Hamburg University of Technology, Denickestraße 17, D–21073 Hamburg, Germany
estorff@tu–harburg.de

Pablo Gamallo Departamento de Matemática Aplicada II, Universidade de Vigo, 36310 Vigo, Spain
pgamallo@uvigo.es

Lothar Gaul Institute of Applied and Experimental Mechanics, Pfaffenwaldring 9, 70550 Stuttgart, Germany
gaul@iam.uni–stuttgart.de

Dan Givoli Department of Aerospace Engineering, Technion, Haifa 32000, Israel
givolid@aerodyne.technion.ac.il

Isaac Harari Faculty of Engineering, Tel Aviv University, 69978 Ramat Aviv, Israel
harari@eng.tau.ac.il

Andreas Hauck Friedrich–Alexander–Universität Erlangen–Nürnberg, Lehrstuhl für Sensorik, Paul–Gordan–Str. 3/5, 91052 Erlangen, Germany
andreas.hauck@lse.eei.uni–erlangen.de

Luis Hervella–Nieto Departamento de Matemáticas, Universidade da Coruña, 15707 A Coruña, Spain
luisher@udc.es

Günter Hofstetter Institute for Basic Sciences in Civil Engineering, University of Innsbruck, Technikerstraße 13, A–6020 Innsbruck, Austria
Guenter.Hofstetter@uibk.ac.at

Jeong–Guon Ih Centre for Noise and Vibration Control (NoViC), Department of Mechanical Engineering, Korea Advanced Institute of Science and Technology (KAIST), Science Town, Daejeon 305–701, Korea
J.G.Ih@kaist.ac.kr

Michael Junge Institute of Applied and Experimental Mechanics, Pfaffenwaldring 9, 70550 Stuttgart, Germany
junge@iam.uni–stuttgart.de

Sabine Langer Institute of Applied Mechanics, Technical University Clausthal, Adolph–Roemer-Str. 2a, 38678 Clausthal–Zellerfeld, Germany
s.langer@tu–clausthal.de

Haw–Ling Liew Department of Mechanical Engineering, Faculty of Mechanics and Computation, Stanford University, Stanford, CA, 94305, USA
hlliew@stanford.edu

Manfred Kaltenbacher Friedrich–Alexander–Universität Erlangen–Nürnberg, Lehrstuhl für Sensorik, Paul–Gordan–Straße 3/5, 91052 Erlangen, Germany
manfred@lse.eei.uni–erlangen.de

Herbert Mang Institute for Mechanics of Materials and Structures, Vienna University of Technology, Karlsplatz 13, A–1040 Vienna, Austria
Herbert.Mang@tuwien.ac.at

Steffen Marburg Institut für Festkörpermechanik, Technische Universität Dresden, 01062 Dresden, Germany
marburg@ifkm.mw.tu–dresden.de

Bodo Nolte Forschungsanstalt für Wasserschall und Geophysik, 24148 Kiel, Germany
BodoNolte@BWB.ORG

Martin Ochmann TFH Berlin – University of Applied Sciences, Department of Mathematics, Physics and Chemistry, Luxemburger Str. 10, 13353 Berlin, Germany
ochmann@tfh–berlin.de

Andrew Peplow Marcus Wallenberg Laboratory for Sound and Vibration Research, Department of Aeronautics & Vehicle Engineering, Kungl Tekniska Högskolan, S–100 44, Stockholm, Sweden
atpeplow@kth.se

Steffen Petersen Department of Mechanical Engineering, Stanford University, 488 Escondido Mall, Mail code 3035, Stanford, CA 94305, USA
steffenp@stanford.edu

Peter M. Pinsky Department of Mechanical Engineering, Faculty of Mechanics and Computation, Stanford University, Stanford, CA, 94305, USA
pinsky@stanford.edu

Andrés Prieto Departamento de Matemática Aplicada, Universidade de Santiago de Compostela, 15782 Santiago de Compostela, Spain
maprieto@usc.es

Toru Otsuru Oita University, 700 Dannoharu, Oita 870–1192, Japan
otsuru@cc.oita–u.ac.jp

Rodolfo Rodríguez GI^{2}MA, Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160–C, Concepción, Chile
rodolfo@ing–mat.udec.cl

Tetsuya Sakuma Graduate School of Frontier Sciences, University of Tokyo, 5–1–5 Kashiwanoha, Kashiwa, Chiba 277–8563, Japan
sakuma@k.u–tokyo.ac.jp

Duarte Santamarina Departamento de Matemática Aplicada, Universidade de Santiago de Compostela, 15782 Santiago de Compostela, Spain
duartesr@usc.es

Martin Schanz Institute of Applied Mechanics, Graz University of Technology, Technikerstraße 4, 8010 Graz, Austria
m.schanz@tugraz.at

Stefan Schneider CNRS/LMA, 31 chemin Joseph–Aiguier, 13402 Marseille Cedex 20, France
schneider@lma.cnrs–mrs.fr

James P. Tuck–Lee Department of Mechanical Engineering, Faculty of Mechanics and Computation, Stanford University, Stanford, CA, 94305, USA
jtucklee@stanford.edu

Shuo Wang Science Applications International Corporation (SAIC), 2450 NASA Road One, Houston, TX 77058, USA
shuowang_usa@yahoo.com

Ting–Wen Wu Department of Mechanical Engineering, University of Kentucky, Lexington, KY 40506, USA
timwu@engr.uky.edu

Yosuke Yasuda Graduate School of Frontier Sciences, University of Tokyo, 5–1–5 Kashiwanoha, Kashiwa, Chiba 277–8563, Japan
yyasuda@k.u–tokyo.ac.jp

Contents

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A Unified Approach to Finite and Boundary Element Discretization in Linear Time–Harmonic Acoustics

Steffen Marburg[1] and Bodo Nolte[2]

[1] Institut für Festkörpermechanik, Technische Universität, 01062 Dresden, Germany
marburg@ifkm.mw.tu-dresden.de
[2] Forschungsanstalt für Wasserschall und Geophysik, 24148 Kiel, Germany
BodoNolte@BWB.ORG

Summary. This chapter introduces the reader too important physical and mathematical concepts in acoustics. It presents an approach to finite and boundary element techniques for linear time–harmonic acoustics starting from the fundamental axioms of continuum mechanics. Based on these axioms, the wave equation is derived. Using a time–harmonic approximation, the boundary value problem of linear time–harmonic acoustics is formulated in the classic and in the weak form. Subsequently, two types of the weak form are used as the basis for discretization resulting in a Galerkin finite element formulation, in a collocation boundary element formulation and in a Galerkin boundary element formulation. Then, different representations of sources and incident wave–fields in finite and boundary element methods are discussed. In the final part of this chapter, the authors categorize the subsequent twenty chapters of this book. The chapter will be completed by an outlook and some open problems in the development of finite and boundary element techniques from the authors' points of view.

0.1 Introduction

It must be stated in the beginning that due to the wide use of numerical methods, the range of papers about finite and boundary element methods in acoustics is quite difficult to survey. The development of these methods started almost half a century ago and there are a number of monographs and editions, but only a limited number which are solely dedicated to computational acoustics in general and, in particular, to FEM and BEM in acoustics. With respect to finite element methods in acoustics, the authors are aware of the books by Givoli [36] (including a chapter on BEM too) and by Ihlenburg [47]; with respect to boundary element methods, we can mention the editions by Ciskowski and Brebbia [18], von Estorff [29], Wu [110] and the monograph by Kirkup [55]. There are a couple of interesting review papers on FEM and BEM, see for example Harari et al. [42], Harari [41] and Thompson [99], also the cost comparison of traditional FEM and BEM by Harari and Hughes [43]. Often, FEM and BEM for acoustics are discussed in other contexts such as computational methods for unbounded domains, cf. Geers [33] and Givoli [38], together with structural and/or electromagnetic wave propagation [1,44,78] and structural–acoustic optimization and noise control [56, 57, 76], see also the review paper by Marburg [63]. Numerical methods have been described and reviewed in books on acoustics such as Crighton et al. [23] and Mechel [73]. Furthermore, there are a number of special issues of journals on FEM and BEM (some of them primarily dedicated to treatment of the exterior problem), e.g. [7,37,48,61,62,66,71,80–82,104] and conference proceedings which contain high quality papers of the field, among many others, see for instance [101, 108]. Finally, books on fluid–structure interaction problems are often closely related to numerical methods and present specific formulations in detail, see for example [2, 19, 46, 75, 83].

It is the purpose of this chapter to present basic formulations of linear time–harmonic acoustics and, in this context, to categorize the remaining twenty chapters of this book. We will start with a short derivation of the linear wave equation. This will be followed by presenting the boundary value problem of time–harmonic acoustics with its partial differential equation (Helmholtz equation) and boundary conditions on one hand, and, on the other hand, a weak form which is the basis for the discretization process. Approximation and discretization will be discussed in Section 0.4. In the following Section 0.5, we will discuss different representations of sources and incident wave fields. In Section 0.6, we categorize the remaining chapters of this book. The chapter will be completed by an outlook and identification of areas for future work in the development of finite and boundary element techniques from the authors' points of view.

0.2 Approach to the Wave Equation

Fundamentals of linear acoustics are based on the basic equations of continuum mechanics. It is assumed that the dimensions of the problem are large with respect to the nanoscale in which the number of molecules is countable. For derivation of the wave equation, we will use the Eulerian representation and, thus, the Eulerian or spatial coordinates.

We consider problems defined in a domain Ω. The complement is denoted by Ω_c. Γ represents the closed boundary of Ω and Ω_c. This configuration includes the direction of the outward normal, pointing into the domain Ω_c as shown in Figure 0.1.

0.2.1 Fundamental Axioms of Continuum Mechanics

For derivation of the wave equation, two fundamental laws of the theory of continuum mechanics are required. These are the principle of conservation of mass and the principle of balance of momentum.

Conservation of Mass

The principle of conservation of mass means that the total mass M of the considered domain Ω

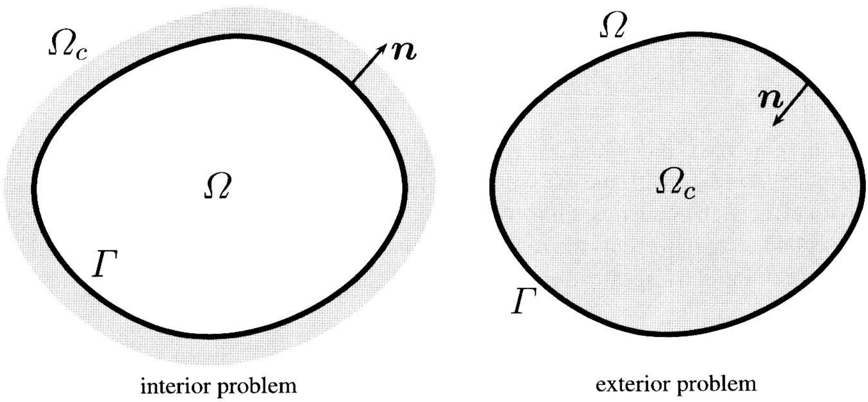

Fig. 0.1 Definition of regions Ω and Ω_c, boundary Γ and outward normal vector $\boldsymbol{n}$.

$$M(t) = \int_\Omega \varrho(\boldsymbol{x}, t) d\Omega \tag{0.1}$$

remains constant during the motion, where $\boldsymbol{x}$ and t denote position vector and time. Often, these dependencies will not be shown in this section. The principle of conservation of mass implies that the material derivative (or total time derivative) vanishes, i.e.

$$\dot{M} = \frac{dM}{dt} = \int_\Omega \left(\frac{\partial \varrho}{\partial t} + \varrho \boldsymbol{\nabla} \cdot \boldsymbol{v} \right) d\Omega = 0 . \tag{0.2}$$

The material derivative introduces the flow velocity vector $\boldsymbol{v}$ which results from $\partial \boldsymbol{x} / \partial t$. In addition to the global validity of the conservation of mass, we require that it is also valid for an arbitrarily small neighborhood of each material point which implies the local conservation of mass as

$$\frac{\partial \varrho}{\partial t} + \varrho \boldsymbol{\nabla} \cdot \boldsymbol{v} = 0 . \tag{0.3}$$

Balance of Momentum

The principle of balance of momentum means that the time rate of change of momentum is equal to the resultant force $\boldsymbol{F}_R$ acting on the body. With momentum vector $\boldsymbol{P}$, also known as the linear momentum vector, this is written as

$$\dot{\boldsymbol{P}} = \frac{d\boldsymbol{P}}{dt} = \boldsymbol{F}_R . \tag{0.4}$$

Herein, the momentum vector is given by

$$\boldsymbol{P} = \int_\Omega \varrho \boldsymbol{v} \, d\Omega \tag{0.5}$$

whereas the resultant force combines volume forces and external forces as

$$\boldsymbol{F}_R = \int_\Omega \boldsymbol{b}\varrho \, d\Omega - \int_\Gamma p\,\boldsymbol{n}\, d\Gamma \,. \tag{0.6}$$

In Equation (0.6), the first term on the right hand side is known as the resultant external body force with the external body force $\boldsymbol{b}$. Using this term, we may consider gravity effects. In acoustics, this term is usually not relevant and, consequently, zero. The second term represents the resultant contact force which can be transformed into a domain integral by application of the Gauss' theorem

$$\int_\Gamma p\boldsymbol{n}d\Gamma = \int_\Omega \nabla p \, d\Omega \,. \tag{0.7}$$

The material derivative of the momentum is given as

$$\frac{d\boldsymbol{P}}{dt} = \frac{d}{dt}\left(\int_\Omega \varrho\,\boldsymbol{v}\, d\Omega\right) = \int_\Omega \frac{d\,(\varrho\,\boldsymbol{v})}{dt}\, d\Omega =$$

$$= \int_\Omega \left[\frac{\partial\varrho}{\partial t}\boldsymbol{v} + \varrho\,(\nabla\cdot\boldsymbol{v})\,\boldsymbol{v} + \varrho\frac{\partial\boldsymbol{v}}{\partial t} + \varrho\,(\boldsymbol{v}\cdot\nabla)\,\boldsymbol{v}\right] d\Omega \,. \tag{0.8}$$

The first two terms of the integrand vanish with respect to the conservation of mass in Equation (0.2) and (0.3), respectively. This yields

$$\frac{d\boldsymbol{P}}{dt} = \int_\Omega \left[\varrho\frac{\partial\boldsymbol{v}}{\partial t} + \varrho\,(\boldsymbol{v}\cdot\nabla)\,\boldsymbol{v}\right] d\Omega \,. \tag{0.9}$$

Summarizing these manipulations, we incorporate Equations (0.6), (0.7) and (0.9) into Equation (0.4) to obtain the so–called Euler equation

$$\int_\Omega \left[\varrho\frac{\partial\boldsymbol{v}}{\partial t} + \varrho\,(\boldsymbol{v}\cdot\nabla)\,\boldsymbol{v} + \nabla p\right] d\Omega = 0 \tag{0.10}$$

or, in local form,

$$\varrho\frac{\partial\boldsymbol{v}}{\partial t} + \varrho\,(\boldsymbol{v}\cdot\nabla)\,\boldsymbol{v} + \nabla p = 0 \,. \tag{0.11}$$

In continuum mechanics, Euler's equations of motion comprise the balance of momentum and the balance of momentum of momentum, also known as the balance of angular momentum. The latter axiom can be neglected since shear effects are not considered herein. Euler's equation (0.11) can be considered as a special local form of Newton's equation of motion $\boldsymbol{F}_R = \partial(m\boldsymbol{v})/\partial t$.

Linearization and Simplification

Commonly, problems of linear acoustics refer to small perturbations of ambient quantities. These ambient quantities are referred to by using the subscript 0. The small fluctuating parts of pressure, density and flow velocity vector are represented as $\tilde{p}$, $\tilde{\varrho}$ and $\tilde{\boldsymbol{v}}$. With this notation, we can substitute for the quantities pressure, density and flow velocity as

$$\begin{aligned} p &= p_0 + \tilde{p}\,, \\ \varrho &= \varrho_0 + \tilde{\varrho}\,, \\ \boldsymbol{v} &= \boldsymbol{v}_0 + \tilde{\boldsymbol{v}}\,. \end{aligned} \tag{0.12}$$

For simplicity for the wave equation approach, we assume that there is no ambient flow, i.e. $\boldsymbol{v}_0 = \mathbf{0}$.

Substituting for the major quantities in Equation (0.3) and considering only first order terms, we write

$$\frac{\partial \tilde{\varrho}}{\partial t} + \varrho_0 \, \boldsymbol{\nabla} \cdot \tilde{\boldsymbol{v}} = 0\,. \tag{0.13}$$

Similarly, Euler's equation (0.11) is linearized and simplified as

$$\varrho_0 \frac{\partial \tilde{\boldsymbol{v}}}{\partial t} + \boldsymbol{\nabla} \tilde{p} = 0\,, \tag{0.14}$$

where it is assumed that ϱ_0 and p_0 are independent of time and spatial coordinates.

0.2.2 Constitutive Equation

In fluids, sound propagates through pressure waves only. The velocity of the sound pressure wave – better known as the speed of sound – depends on the propagation material. For wave propagation in linear fluid acoustics, the speed of sound is one of two relevant material parameters. It can be understood as the result of mathematical relations of other material parameters which are not solely relevant for our considerations.

The constitutive relations are usually referred to as the equations of state. With respect to thermodynamics, the pressure fluctuation and, thus, sound propagation occurs with negligible heat flow because the changes of the state occur so rapidly that there is no time for the temperature to equalize with the surrounding medium. This is the property of an adiabatic process. If fluctuation amplitudes and frequency remain small enough, the process can be considered as reversible and isentropic.

Derivation of the speed of sound is different for gases, liquids and solids. Since we limit our considerations to fluids herein, we will only discuss derivation of the speed of sound for gases and liquids in what follows.

The speed of sound c may be introduced as a constant to relate the fluctuating parts of pressure and density to each other as

$$\tilde{p} = c^2 \, \tilde{\varrho}\,. \tag{0.15}$$

This is equivalent to

$$c = \sqrt{\frac{\partial p}{\partial \varrho}}\,. \tag{0.16}$$

For gases, we will present finding the relation (0.15) whereas for liquids, we will derive the speed of sound based on Equation (0.16).