

North-Holland

---

# Computational and Combinatorial Methods in Systems Theory



edited by  
C.I. Byrnes  
A. Lindquist

---

TP13  
B3

8761954

# COMPUTATIONAL AND COMBINATORIAL METHODS IN SYSTEMS THEORY

*edited by*

Christopher I. BYRNES

*Arizona State University  
Tempe, AZ 85281, U.S.A.*

*and*

Anders LINDQUIST

*Royal Institute of Technology  
100 44 Stockholm, Sweden*



E8761954



1986

NORTH-HOLLAND  
AMSTERDAM · NEW YORK · OXFORD · TOKYO

© ELSEVIER SCIENCE PUBLISHERS B.V., 1986

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without the prior permission of the copyright owner.

ISBN: 0 444 70031 5

*Publishers:*

ELSEVIER SCIENCE PUBLISHERS B.V.  
P.O. Box 1991  
1000 BZ Amsterdam  
The Netherlands

*Sole distributors for the U.S.A. and Canada:*

ELSEVIER SCIENCE PUBLISHING COMPANY, INC.  
52 Vanderbilt Avenue  
New York, N.Y. 10017  
U.S.A.

**Library of Congress Cataloging-in-Publication Data**

Computational and combinatorial methods in systems  
theory.

Selected papers originally presented at the 7th International Symposium on the Mathematical Theory of Networks and Systems, held at the Royal Institute of Technology in Stockholm, June 10-14, 1985.

Includes index.

1. Control theory--Congresses. 2. Numerical analysis--Congresses. 3. Combinatorial analysis--Congresses.

I. Byrnes, Christopher I., 1949- . II. Lindquist, Anders. III. MTNS International Symposium (7th : 1985 : Kungl. Tekniska högskolan)

QA402.3.C575 1986 629.8'312 86-9041  
ISBN 0-444-70031-5 (U.S.)

PRINTED IN THE NETHERLANDS

**COMPUTATIONAL AND COMBINATORIAL METHODS  
IN SYSTEMS THEORY**

## PREFACE



In classical mathematics the problems of computation and approximation occupied a central position in the application and the development of powerful new theories and there can be no doubt that the importance of these twin themes will endure for the foreseeable future. Systems theory, depending as it does on a variety of mathematical disciplines for its basic tools, provides a natural source of interesting problems for those classical and modern applications of mathematics.

In this volume, we present a collection of papers on both the discrete and the continuous aspects of these themes, ranging from fundamental systems theoretic applications of interpolation theory, numerical linear algebra and computational complex analysis to the development of programming languages and strategies for flexible manufacturing systems and VLSI design. The papers of this volume, which attest to the vitality and importance of this cross-fertilization between pure and applied, were selected from the invited and contributed papers presented at the 7th International Symposium on the Mathematical Theory of Networks and Systems held at the Royal Institute of Technology in Stockholm on June 10-14, 1985.

We would like to take this opportunity to thank the following research agencies for their generous support of MTNS-85: the National Swedish Board for Technical Development (STU), the Office of Naval Research Branch Office, London (ONRL), the Research Institute of National Defence (FOA), the Swedish Institute for Applied Mathematics (ITM), the Swedish Natural Science Research Council (NFR) and the USAF European Office of Aerospace Research and Development (EOARD).

Christopher I. Byrnes and Anders Lindquist  
Stockholm, January 29, 1986.

## CONTENTS

### 1. Algorithms for Estimation and Control

Algebraic Problems Arising in Robust Stabilization and Compensation S.P. Bhattacharyya and J.W. Howze	3
Reliable Algorithms for Reduced Order Observer Design K.-W.E. Chu and N.K. Nichols	11
The Numerical Solution of the Kalman Filtering Problem Sven Hammarling	23
A Numerical Algorithm for Eigenvalue Assignment by Output Feedback R.V. Patel and P. Misra	37

### 2. Applications of Classical and Complex Analysis to Circuits and Systems

Basic Study of Elliptic Low-Pass Filters by Means of the Minkowski Model of Lorentz Space E. Folke Bolinder	53
Spectral Representation of the Laplace Transform and Related Operators in $L_2(\mathbb{R}_+)$ D.S. Gilliam, J.R. Schulenberger and J.R. Lund	69
Deformations of Connections, the Riemann Hilbert Problem and $\tau$ -Functions Gerard Helminck	75
On Some Problems of Best Approximation James Rovnyak	91
Residue Formulas for Meromorphic Matrices J.M. Schumacher	97
Cascade Realizations of Multi-Port Networks via Factorization of Rational Matrix Functions Leang S. Shieh, Yih T. Tsay and Robert E. Yates	113

### 3. Discrete Event and Large Scale Systems

- Optimal Scheduling of a Flexible Manufacturing System: A Stochastic Control Problem for a System with Jump Markov Disturbances  
R. Akella and P.R. Kumar 131
- Overlapping Decomposition of Large Scale Systems into Weakly Coupled Subsystems  
I.M. Arabacioglu, M.E. Sezer and Ö.H. Oral 135
- Fault Diagnosis in Dynamical Systems: A Graph Theoretic Approach  
S.V. Nageswara Rao and N. Viswanadham 149
- On Control of Discrete Event Systems  
W.M. Wonham 159

### 4. Numerical Linear Algebra

- Inverse Eigenvalue Methods and Flag Manifolds  
Gregory S. Ammar 177
- Numerical Stability and Instability in Matrix Sign Function Based Algorithms  
Ralph Byers 185
- Theoretical and Computational Aspects of Some Linear Algebra Problems in Control Theory  
B.N. Datta and Karabi Datta 201
- Algebraic Aspects of Generalized Eigenvalue Problems for Solving Riccati Equations  
Alan J. Laub 213
- Hessenberg Forms in Linear Systems Theory  
Alan J. Laub and Arno Linnemann 229
- The Block Jacobi Method for Computing the Singular Value Decomposition  
Charles F. Van Loan 245
- Stability Criteria for Internal Matrices  
Xu Daoyi 257

### 5. Toeplitz Operators, Interpolation and Positive Real Functions

- The Implementation and Use of the Generalized Schur Algorithm  
Gregory S. Ammar and William B. Gragg 265

Convergence of Schur Parameters and Transmission Zeros of a Meromorphic Spectrum A. Bultheel	281
Z-Plane Algorithms and Polynomial Matrix Spectral Factorization F.M. Callier and J. Winkin	297
Canonical Factorization of Pseudo-Carathéodory Functions P. Delsarte, Y. Genin and Y. Kamp	299
A Maximum Entropy Principle for Contractive Interpolants Harry Dym	309
Hankel Norm Approximation of Power Spectra Martin H. Gutknecht	315
Continued Fractions Associated with Wiener's Linear Prediction Method William B. Jones, Olav Njåstad and W.J. Thron	327
On the Structure and Application of the State Space of J-Lossless Systems C.V.K. Prabhakara Rao and E. Deprettere	341
<b>6. VLSI and Communication Networks</b>	
Dynamic Routing Policies in Communication Networks: The Finite Buffer Case M. Aicardi, G. Casalino, F. Davoli, R. Minciardi and R. Zoppoli	357
A Hierarchical Design System for VLSI Implementation of Signal Processing Algorithms Jurgen Annevelink	371
Subspaces: Factorization and Communication Armin B. Cremers and Thomas N. Hibbard	383
An Efficient VLSI Solver for a Special Set of Near Toeplitz Systems Equations Ed F.A. Deprettere and Kishan Jainandunsing	397
An Optimal and Flexible Delay Management Technique for VLSI Gert Goossens, Rajeev Jain, Joos Vandewalle and Hugo de Man	409
SIGNAL: A Data Flow Oriented Language for Real Time Signal Processing P. Le Guernic, A. Benveniste, P. Bournai and T. Gautier	419



# **1. ALGORITHMS FOR ESTIMATION AND CONTROL**



## ALGEBRAIC PROBLEMS ARISING IN ROBUST STABILIZATION AND COMPENSATION\*

S.P. Bhattacharyya and J.W. Howze

Department of Electrical Engineering  
Texas A&M University  
College Station, Texas 77843  
U.S.A.

The problem of designing a low order robust stabilizing compensator for a linear time invariant system is described and formulated in algebraic terms as a static output feedback stabilization problem. A numerical approach to this problem based on a suitably formulated optimization problem is presented. This is an extension of the recently introduced algorithm for pole assignment via Sylvester's equation. Some examples are given.

### INTRODUCTION

The problem of feedback stabilization of a given system by a compensator is the central problem of control theory and design and almost every major theoretical tool developed in the control literature is directed towards this problem. Among the main distinct approaches to this problem are the LQG theory, the state feedback observer approach, and the Brasch-Pearson pole placement approach which are now standard and may be found in textbooks such as [1].

From a practical point of view none of the above approaches are adequate or even reasonably satisfactory. The reason for this is twofold:

- 1) the dynamic order of the controllers that result in any of the above methods is very high, and
- 2) the controllers are not robust, i.e., do not provide protection against plant parameter perturbations.

The problem of high order is serious because typically one is interested in low order controllers that are capable of controlling high order plants rather than vice versa. The control system designer who is handed a high order controller as the solution to the stabilization problem faces a high dimensional parameter space in which he has to carry out adjustments to meet various conflicting design specifications. This poses formidable conceptual and computational problems which are best avoided if at all possible.

The problem of robustness is well known in the literature: the controller must preserve stability of the closed loop system when the plant undergoes perturbations from a specified class. However, despite the extensive literature on the subject, the only result that gives a constructive synthesis procedure for generating a robust controller is, to the best of our knowledge, the recent result of Kimura [2]. The result of [2] is restricted to single input single

---

\*This research was supported by the National Science Foundation under Grant No. ECS-8309792

output plants, considers additive perturbations of the transfer function, and in general will provide high order solutions. With regard to the LQG approach it is now well known that the good stability margins provided by the LQ solutions disappear when implemented via output feedback.

In the present paper we attempt to provide a reasonably realistic formulation of the problem of designing a low order robust stabilizing compensator. This algebraic formulation is motivated by the observation that every compensator corresponds to a static output feedback controller for an augmented system derived from the plant. Robust compensator design then corresponds to the design of a robust static output feedback control law. It is proposed that analytical approaches be developed by researchers for the resolution of this unsolved problem. A numerical approach for solution of this problem will be described along with examples.

#### PROBLEM FORMULATION

Let

$$\begin{aligned} S_p : \dot{x} &= Ax + Bu, \quad x \in \mathbb{R}^n \\ y &= Cx \end{aligned}$$

denote the plant and

$$\begin{aligned} S_p : \dot{z} &= A_c z + B_c u_c, \quad z \in \mathbb{R}^q \\ y_c &= C_c z + D_c u_c \end{aligned}$$

the controller with the feedback connection

$$u_c = y$$

$$y_c = u$$

so that the closed loop system equations are:

$$\begin{pmatrix} \dot{x} \\ \dot{z} \end{pmatrix} = \underbrace{\begin{pmatrix} A + BD_c C & BC_c \\ B_c C & A_c \end{pmatrix}}_{A_{cl}} \begin{pmatrix} x \\ z \end{pmatrix}$$

The problem of feedback stabilization is:

Given the plant  $(A, B, C)$  find a controller  $(A_c, B_c, C_c, D_c)$  so that the eigenvalues of  $A_{cl}$  lie in the left half of the complex plane, i.e.,  $\sigma(A_{cl}) \subset \mathbb{C}^-$ .

The necessary and sufficient conditions for the existence of some controller is given by stabilizability and detectability of  $(A, B, C)$  and under this assumption the problem can be reformulated by considering  $(A, B, C)$  to be controllable and observable, without loss of generality [1]. When the order  $q$  of the controller is fixed, however, no necessary and sufficient conditions for the existence of  $(A_c, B_c, C_c, D_c)$  are known.

Now since

$$A_{cl} = \underbrace{\begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}}_{A_q} + \underbrace{\begin{bmatrix} B & 0 \\ 0 & I_q \end{bmatrix}}_{B_q} \underbrace{\begin{bmatrix} D_c & C_c \\ B_c & A_c \end{bmatrix}}_{K_q} \underbrace{\begin{bmatrix} C & 0 \\ 0 & I_q \end{bmatrix}}_{C_q}$$

$$= A_q + B_q K_q C_q$$

it is clear that the problem of feedback stabilization by a fixed order controller is equivalent to stabilizing  $A_q + B_q K_q C_q$  by choice of  $K_q$ . The minimal order of a stabilizing controller is the smallest integer  $q^*$  such that there exists  $K_{q^*}$  for which  $A_{q^*} + B_{q^*} K_{q^*} C_{q^*}$  is stable.

The problem of finding  $q^*$  is one of the outstanding unsolved problems of control theory. The available bounds on  $q^*$  are

1) the pole-placement result of Brasch-Pearson [3] which states that

$$q^* \leq \text{Min} \{P_c, P_o\}$$

where  $P_c$  is the controllability index of  $(A, B)$  and  $P_o$  is the observability index of  $(C, A)$ , and

2) the stabilization result of Kimura [4] according to which

$$q^* \leq n + 1 - r - m$$

where  $r$  and  $m$  are the number of inputs and outputs respectively.

In the following section we present a numerical approach to the output feedback stabilization problem. This algorithm can be applied to the problem of finding a low order stabilizing compensator by sequentially applying it to  $(A_i, B_i, C_i)$  for  $i = 0, 1, 2 \dots$  until a  $q$  is found such that  $A_q + B_q K_q C_q$  is stable for some  $K_q$ . The algorithm also provides for some robustness in the sense that a  $K$  is found that attempts to orthogonalize the eigenvector set of  $A + BKC$ . It is well known [5] and [6] that such a choice of  $K$  causes the eigenvalues of  $A + BKC$  to be relatively insensitive to first order perturbations in the entries of the matrix.

#### NUMERICAL ALGORITHM

In this section we display an algorithm for finding  $K$ , if it exists, so that  $A + BKC$  is stable. We shall extend the state feedback pole assignment algorithm via Sylvester's equation, developed in [7], to handle this problem. The advantage of this algorithm is that it parametrizes the entire family of solutions to the state feedback problem so that an efficient approximation to the output feedback problem may be obtained.

The state feedback pole assignment algorithm of [7] is:

1) Solve for  $X$  in

$$AX - \tilde{X}\tilde{A} = -BG$$

for some  $\tilde{A}$  with  $\sigma(\tilde{A}) = \Lambda$  and for any  $G$  so that  $(G, \tilde{A})$  is observable. Then by the result of [8]  $X$  will almost always be nonsingular.

2) Solve for F in

$$FX = G$$

Then  $\sigma(A + BF) = \Lambda$ .

To adapt this to the output feedback case we also want that F be approximated by KC, i.e.,  $\|F - KC\|$  be small. Moreover since the closed loop eigenvalues are those of  $A + BKC$  it is important to ensure that  $\sigma(A + BKC)$  and  $\sigma(A + BF)$  be "close" if F and KC are "close." This can be taken into consideration by attempting to orthonormalize the eigenvector set of  $A + BF$ . Finally the assigned eigenvalues are required to lie in a region  $\Omega \subset \mathbb{C}^-$  which is appropriately chosen. Based on these consideration we formulate the optimization problem:

Find F and K so that

$$J = \alpha_1 \text{trace} [I - XX^T]^2 + \alpha_2 \text{trace} (F - KC)^T (F - KC) \text{ is minimized where } \sigma(A + BF) \subset \Omega \text{ and } X \text{ is the eigenvector matrix of } A + BF.$$

As mentioned before we may take  $\sigma(\tilde{A}) \subset \Omega$  with  $\tilde{A}$  diagonal so that in

$$\begin{aligned} AX - X\tilde{A} &= -BG \\ FX &= G \end{aligned}$$

X becomes the eigenvector matrix of  $A + BF$ . The gradient of J with respect to G may be calculated. The details of this calculation are omitted and may be found in [9]. The result derived in [9] is that

$$\frac{\Delta J}{\Delta G} = 2 \{ \alpha_1 F [I - 2RC + (RC)^T RC] (X^T)^{-1} - B^T U^T \}$$

where U satisfies

$$\begin{aligned} \tilde{A}U - UA &= \alpha_1 X^{-1} [I + 2(RC)^T + (RC)(RC)^T] F^T F \\ &\quad + 2\alpha_2 [I - X^T X] X^T \end{aligned}$$

and  $R = C^T (CC^T)^{-1}$ .

Based on this we may formulate the gradient based algorithm:

Step 0: Pick  $\sigma(\tilde{A}) \subset \Omega$  and  $G_0$

Step 1: Find F and K that minimizes

$$J = \alpha_1 \text{Trace}\{I - XX^T\}^2 + \alpha_2 \text{Trace}\{(F - KC)^T (F - KC)\}$$

with respect to G.

Step 2: IF  $\sigma(A + BKC) \subset \Omega$   
THEN stop (controller found)  
ELSE continue to Step 3.

Step 3: Compute  $\frac{\partial J}{\partial A}$  by numerical procedures

$$\text{IF } \left\| \frac{\partial J}{\partial A} \right\| > \epsilon$$

THEN continue to Step 4.  
ELSE stop

Step 4: Obtain a new  $\tilde{A}_{i+1}$  using a gradient based method.  
Set  $i = i + 1$  and continue to Step 1.

### EXAMPLES

All examples shown here indicate that lower order output feedbacks were achieved and the region  $\Omega$  was defined as the whole left half complex plane to achieve a stabilizing controller.

#### Example 1

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$C = (1 \quad 0.5 \quad 1.5)$$

$$\text{Initial guess of } G = \begin{bmatrix} 1 & 0.5 & 1.5 \\ -0.5 & -1 & 1 \end{bmatrix}$$

Eigenvalues of  $A = (16.1168, -1.1168, 0)$ .

Initial eigenvalues of  $\tilde{A} = (-1, -2, -3)$ .

Initial cost  $J_0 = 659.42478159$ .

Final cost  $J_{\text{opt}} = 6.3778387856$ .

Obtained  $K = (-0.7538303194 \quad -9.560368516)$

Eigenvalues of  $A + BKC = -2.402794 \pm j4.94279, -1.19972433$ .

#### Example 2

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 & 0 & 1 \\ -1 & 1 & 0.5 & 0 \end{bmatrix}$$

$$\text{Initial guess of } G = \begin{bmatrix} -5 & 2 & 2 & 1 \\ -1 & -1 & 2 & 1 \end{bmatrix}$$

Eigenvalues of  $A = (1, 0, 0, 0)$ .

Initial eigenvalues of  $\tilde{A} = (-1 \pm j2, -2 \pm j1)$ .

Initial cost of  $J_0 = 159.758483$ .

Final cost  $J_{opt} = 3.492771$ .

$$\text{Obtained } K = \begin{bmatrix} -1.559706 & -1.225891 \\ 2.355928 & -10.53672 \end{bmatrix}$$

Eigenvalues of  $A + BKC = (-1.183542 \pm j1.392888, -3.230272, -1.790421)$ .

### Example 3

$$A = \begin{bmatrix} 1 & 0.5 & 0.2 & 0 & 0 \\ 1 & -1 & 0 & -1.5 & 0 \\ 0 & 0 & -1.5 & 0.5 & 1 \\ 0 & 0 & 1.5 & 2 & 1.5 \\ 0 & 0 & 0 & -1.5 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & -1 \\ 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0.5 & -1 & 0 & 1 \\ -1 & 0 & -0.5 & 1.5 & 0 \end{bmatrix}$$

$$\text{Initial guess of } G = \begin{bmatrix} 0 & 1 & -1 & 0 & 1 \\ 1 & 1 & -1 & 0 & 1 \end{bmatrix}$$

Eigenvalues of  $A = (-1.22474, 1.22474, -1.8336, 1.66682 \pm j1.52788)$ .

Initial eigenvalues of  $\tilde{A} = (-2.5 \pm j2.5, -3 \pm j3, -3.5)$ .

Initial cost  $J_0 = 440.983766$ .

Final cost  $J_{opt} = 6.650208$ .

$$\text{Obtained } K = \begin{bmatrix} 253.10249 & -50.30768 \\ 25.604106 & -16.62546 \end{bmatrix}$$

Eigenvalues of  $A + BKC = (-1.7441 \pm j9.2792, -5.5774 \pm j2.3377, -0.4822)$ .

### CONCLUDING REMARKS

The problem of finding a low order if not the smallest order controller that stabilizes a system has been shown to be equivalent to finding the smallest  $q$  such that  $A + B K C$  is stable for some  $K$ . This is an unsolved problem and deserves to be solved in an efficient manner. The determination of  $K$  so that  $A + B K C$  remains stable for a class of perturbations in  $(A, B, C)$  is the problem of robust stabilization and deserves the attention of researchers. This paper has indicated a numerical approach to address some of these problems but obviously the whole problem is open for solution.



- [1] W.M. Wonham, "Linear Multivariable Control: A Geometric Approach." Springer Verlag, 1984.
- [2] H. Kimura, "Pole assignment by gain output feedback." IEEE T-AC, Vol. AC-20, No. 4, pp. 509-516, August 1975.
- [3] F.M. Brasch and J.B. Pearson, "Pole placement using dynamic compensators." IEEE T-AC, Vol. AC-15 (1), pp. 34-43, February 1970.
- [4] H. Kimura, "Robust Stabilizability of a class of transfer functions." IEEE T-AC, Vol. AC-29, No. 9, pp. 788-793, September 1984.
- [5] J.H. Wilkinson, "The Algebraic Eigenvalue Problem." Clarendon Press, 1964.
- [6] R.K. Cavin III and S.P. Bhattacharyya, "Robust and well conditioned eigenstructure assignment via Sylvester's equation," Optimal Control Applications and Methods, Vol. 4, pp. 205-212, 1983.
- [7] S.P. Bhattacharyya and E. de Souza, "Pole assignment via Sylvester's Equation," System and Control Letters, Vol. 1, No. 4, pp. 261-163, January 1982.
- [8] E. de Souza and S.P. Bhattacharyya, "Controllability, Observability and the Solution of  $AX - XB = C$ ," Lin. Alg. and its Applic., Vol. 39, pp. 167-188, 1981.
- [9] L.H. Keel and S.P. Bhattacharyya, "Compensator Design for Robust Eigenstruct Assignment via Sylvester's Equation," Proc. of the 1985 American Control Conference, June 1985, Boston.

Acknowledgement: The authors are grateful to L.H. Keel for generating the numerical examples.