INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

Proceedings of the Fourth Trieste Conference

QUANTUM FIELD THEORY AND CONDENSED MATTER PHYSICS

Editors

S. Randjbar-Daemi & Yu Lu

World Scientific

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

Proceedings of the Fourth Trieste Conference

QUANTUM FIELD THEORY AND CONDENSED MATTER PHYSICS

Trieste, Italy 13–16 May 1991

Editors

S. Randjbar-Daemi *(ICTP)*
Yu Lu *(ICTP & ITP, CAS)*

World Scientific
Singapore • New Jersey • London • Hong Kong

Published by

World Scientific Publishing Co. Pte. Ltd.
P O Box 128, Farrer Road, Singapore 9128
USA office: Suite 1B, 1060 Main Street, River Edge, NJ 07661
UK office: 73 Lynton Mead, Totteridge, London N20 8DH

QUANTUM FIELD THEORY AND CONDENSED MATTER PHYSICS

ISBN 981-02-1622-X

Printed in Singapore by JBW Printers & Binders Pte. Ltd.

QUANTUM FIELD THEORY
AND
CONDENSED MATTER PHYSICS

OPENING ADDRESS

First of all, I would like to welcome all of you to the Centre.

The Conference on "Quantum Field Theory and Condensed Matter Physics" is the fourth of a series of Trieste Conferences. Like last year's Conference on "Topological Methods in Quantum Field Theory", it has an interdisciplinary character. That Trieste Conference brought together a number of distinguished physicists and mathematicians, and created a lively atmosphere in which many topics of common interest were discussed. For this year's meeting we have invited experts in condensed matter physics and in superstrings and conformal field theories, with the hope of learning from them the most salient features of the osmosis which has been taking place in these fields in the last several years. Indeed ideas like the Chern–Simons systems, which for the first time emerged in the context of supergravity theories, seem to play a pivotal role in some recently proposed theories of High-T_c superconductivity. In this meeting we shall hear about these interesting proposals from the founder of the anyonic superconductivity, Professor Laughlin. Chern–Simons Field Theories play an equally crucial role in theories of quantum Hall effect. There will be many talks covering the recent developments in this fascinating field, almost all of which will be related to the ideas originally introduced by Laughlin.

Integrable field-theoretical and statistical models are going to form another major theme of our meeting. Indeed, advances in the understanding of the central role of Yang–Baxter equations and the emergence of quantum groups in the analysis of such models should be regarded as a significant achievement of the last two decades. These techniques provide another useful common ground which enables the superstring specialists to enter into a useful intellectual discourse with their colleagues in the neighbouring field of condensed matter and statistical physics. This dialogue has of course a rather long history and its recent past includes the application of Wilson and others of the renormalization group ideas to the understanding of the critical phenomena. Conformal field theories are defined in terms of the fixed points of the renormalization group transformations and, as you may know, these theories have been applied successfully in the investigation of the critical behaviour of several low-dimensional systems. In this meeting Professor Zamolodchikov, one of the principal architects of conformal physics, will lecture on some interesting extensions of his ideas in this area. We shall also hear about the application of this method to the Hubbard model and the Kondo problem.

The other major advance in the last few years has been in the area of two-dimensional gravity. The idea of quantizing the Liouville mode of the two-dimensional metric goes back to the original work of Polyakov on covariant quantization of string theories. In 1988, Knizhnik, Polyakov and Zamolodchikov published their seminal paper on the light cone gauge quantization of two-dimensional gravity. The random lattice approach, of Gross and Migdal, and independently, Brezin and

Kazakov, triggered the new wave of interest which is still thriving. This approach, although technically not very straightforward, is very powerful and yields non perturbative information about the string partition function. Unfortunately, the barrier at $c = 1$ has not yet been overcome. We shall hear from Professor Migdal about the new achievements in this area.

I would like to express my sincere gratitude to the directors of this Conference, Professor E. Brezin from Ecole Normale Superieure of Paris and Professors S. Randjbar-Daemi and Yu Lu from the ICTP, for their idea of organizing this meeting.

I hope this Conference will be as fruitful as the previous ones and wish you an enjoyable stay in Trieste.

Abdus Salam

CONTENTS

FIELD THEORY APPROACH TO CRITICAL QUANTUM IMPURITY PROBLEMS AND APPLICATIONS TO THE MULTI-CHANNEL KONDO EFFECT

Andreas W. W. LUDWIG

Physics Department,
Simon Fraser University, Burnaby, BC, V5A 1S6, Canada
and
Physics Department,
University of British Columbia Vancouver, BC, V6T 1Z1, Canada

ABSTRACT: A new, general approach to finding exact solutions of Quantum Impurity Problems is reviewed, in which a local quantum mechanical degree of freedom is coupled to an extended variable, having gapless excitations. The principle ideas, using Conformal Field Theory techniques, are demonstrated in the example of the Multi-Channel Kondo effect, which exhibits unusual non-Fermi-liquid behavior at low temperatures and which appears to be relevant for Heavy Fermion Materials. In particular, static and dynamic Green's functions as well as transport and pairing properties are calculated exactly at low temperatures. Quantum Impurity Problems provide realizations of non-trivial, fully interacting Conformal Field Theories.

INTRODUCTION

Much progress has recently[1,2,3,4,5,6,7,8] been made in understanding the physics of *Quantum Impurity problems* (QIP). This lecture intents to give a pedagogical review of these recent developments.

Quantum Impurity problems occur in a variety of different contexts. A common feature of all these problems is the coupling of a local quantum mechanical degree of freedom to some extended variable, often a critical field theory. In *Condensed Matter Physics* such problems have caught the attention of physicists for several decades in the form of the *Kondo effect* and its generalizations; the local variable is an impurity spin while (gapless) low lying excitations of the conduction band electrons near the Fermi surface represent the critical field theory. Kondo physics plays also an important role in understanding Heavy Fermion materials (possibly also High T_c materials). In *quantum dissipation problems* a quantum mechanical degree of freedom, e.g. a particle in a double well potential, is coupled to an enviroment, which can be represented by extended degrees of freedom. The *catalysis of proton decay*, and of other *Baryon-number violating* processes, by a magnetic monopole is a QIP belonging to the areas of *Particle Physics* and *Cosmology*. In *String Theory*, the effect of dynamic degrees of freedom (representing e.g. quarks), placed at the end of the open string, is a QIP relevant for Quantum Chromodynamics. These are only a few examples.

In this lecture we review a new approach, capable of generating exact solutions for a large variety of QIP's. The basic idea is simple: Under renormalization the local degrees of freedom disappear from the description and turn into a boundary condition on the field theory which describes the extended variables. The main problem reduces therefore to finding these boundary conditions. In many cases, such as e.g. the (single-impurity) multi-channel or the two-impurity Kondo problem, the field theory as well as the boundary condition have in fact a huge, namely conformal symmetry. Then, the powerful apparatus of Conformal Field Theory (CFT) can be brought into play, providing a complete solution of the universal features of the problem, which are relevant for the low temperature physics.

The main emphasis in this lecture will be on the *complete* exact solution of the *multi-channel (single impurity) Kondo* problem, which was recently[2,3,4,5,6,8] obtained using CFT techniques. This problem provides the simplest illustration of the general methods. At the same time, the results obtained from the CFT approach for this problem appear to be relevant for Heavy Fermion materials.

It was known experimentally since the 1930s that the addition of a small amount of magnetic atoms changes the properties of simple metals dramatically. There is a minimum in the resistivity as a function of temperature. In 1964 J. Kondo gave an explanation[9] of this minimum, in terms of what would be called today '*asymptotic freedom*', using the simplest model in which a $s = 1/2$ impurity spin is coupled to a single band of conduction electrons. It was only much later that Nozières and Blandin studied[10] the coupling of a *realistic* impurity *atom*, where the spin-carrying electron has in general non-zero *orbital* angular momentum, to a (3D) electron gas. The general model for describing this situation is the more complicated *multi-channel Kondo* model, where k degenerate bands of spin 1/2 conduction electrons, corresponding in the simplest case to the $k = 2l + 1$ different orientations of (3D) band electrons in the l^{th} partial wave around the origin, couple to an effective impurity spin of general size s, located there. [For more details see Ref.'s (10), (11), (12)]. Based on physical arguments Nozières and Blandin showed in their remarkable paper[10] that, depending on whether $k \leq 2s$ ('underscreening') or $k > 2s$ ('overscreening'), the low temperature physics, governed by a strong coupling fixed point, is totally different: When $k = 2s$, the electron-impurity system can be described at very low temperatures ($T << T_K$), where it is strongly coupled, by a (local) Fermi-liquid. The Fermi-liquid parameters can be viewed, in a renormalization group description, as irrelevant interactions between the otherwise free Fermionic excitations. When $k < 2s$ the picture is essentially the same, except that now a decoupled residual impurity spin of size $s - k/2$ is left over. However for overscreening, i.e. when $k > 2s$, Nozières and Blandin were able to show that the low temperature physics is not described by Fermi-liquid theory; but, there has been very little understanding of the nature of this fixed point, governing the low temperature physics in this case. The complete description of this fixed point is the subject of this lecture.

To date one of the most prominent realizations of the overscreened ($k > 2s$), i.e. non-Fermi-liquid multi-channel Kondo effect, occurs, amongst others, in the form of the *Quadrupolar* Kondo effect, having $k = 2$ and $s = 1/2$. It has been proposed[12] as a model for many Uranium-based Heavy Fermion materials. (It is also likely[12] to be relevant for a lattice of such impurity atoms.) Schematically, the physics of the Quadrupolar Kondo effect is the following: The impurity atom (in this case Uranium) posesses a degenerate doublet ground state, when placed in the crystal field of the other lattice ions. The degenerate states of this doublet have the same spin quantum numbers but differ in other, orbital quantum numbers, and can be viewed as a multiplet of a *pseudo-spin* (orbital) $SU(2)$ symmetry. The conduction electrons have, apart form the ordinary spin up-down index, pseudo-spin quantum numbers as well. The Kondo coupling between conduction electrons and impurity is an antiferromagnetic pseudo-spin/ pseudo-spin coupling, just like in the ordinary (spin-) Kondo interaction, except that spin is replaced by pseudo-spin. The only difference is that this coupling is generically *anisotropic* in pseudo-spin* space: $\sum_{j=\text{up,down}}\{J_{xy}(\tau^x_{imp}\tau^x_{cond,j} + \tau^y_{imp}\tau^y_{cond,j}) + J_z\tau^z_{imp}\tau^z_{cond,j}\}$. The ordinary spin index of the conduction electrons serves now as an additional *band* index; the spin up and down bands are exactly degenerate in zero magnetic field, thus realizing a $k = 2$ band, $s = 1/2$ (doublet) impurity overscreened Kondo model.

Some of the predictions of the theory, to be explained later, are for the $k = 2$ channel and $s = 1/2$ spin case the following: The anisotropy mentioned above can in fact shown to be irrelevant[6] for the low temperature physics, which is therefore described by the *iso*tropic model. This is shown to have a specific heat increment due to the impurity of the form $T\ln T$, while the added residual entropy at $T = 0$, coming from the impurity, is predicted to be $(1/2)\ln 2$. Furthermore, the resistivity saturates with a non-trivial power law[5] in T as $T \to 0$ rather than T^2 as in the normal Fermi-liquid Kondo problem[13]. Recent experiments[14] done on the dilute Heavy Fermion compound $U_xY_{1-x}Pd_3$ (where $x = 0.2$ and 0.1), which appear to show 2-channel Kondo behavior of the low temperature specific heat, entropy and resistivity (even though there seems to be, to date,

---

* $\vec{\tau}$ are Pauli matrices in pseudo-spin space

a descrepancy in the precise power law), seem to support a description[12] of this Heavy Fermion system by the Quadrupolar Kondo effect.

The CFT approach provides[5,7] also all exact (3D) space- and time- dependent Green's functions for the impurity problems. In particular, the local pair field response function has been shown to diverge logarithmically as $T \to 0$ in the Quadrupolar case. This means, that the Quadrupolar Kondo impurity strongly affects pairing and could therefore provide a new mechanism for unusual superconductivity. (Arguments have also been given[12] that the multi-channel Kondo effect may provide a mechanism for High-T_c superconductivity of the Cuprates).

Amongst the *new exact results* which have, to date, been obtained explicitly for the multi channel Kondo effect using CFT, for general values of k and s, are the following: The complete analytic asymptotic (Wilsonian) Finite Size spectrum(Section 3), all critical exponents at low T(Section 4), the complete analysis of the stability properties of the low T fixed point[6], universality of the Wilson ratio in the overscreened cases ($k > 2s$) and its exact value (Section 5), universality of the $T = 0$ entropy of the impurity (and its relationship with the modular S-matrix of CFT, Section 5), *all* two- and multi- point (3D) space and time dependent Green's functions (Section 6), transport properties such as the low T resistivity (as obtained from the Kubo formula) and single-particle life-time (Section 6), etc..

The appearance of Conformal Field theory in the multi-channel (single-impurity) Kondo effect can be rather easily understood. Since only the s-wave conduction electrons interact with the impurity spin, the fundamental geometry of the problem is, adopting a space-time picture, the upper complex plane: The positive imaginary axis labels the radial distance from the impurity, while the real axis labels time. Moreover, only the s-wave conduction electrons near the Fermi surface interact at low T with the impurity; since the dispersion law of these low lying excitations is linear they are described by a conformal field theory. According to the central hypothesis, by now very convincingly verified, which is being made in the approach reviewed here, the impurity spin disappears form the description at the low temperature fixed point and turns into a boundary condition, which must be scale invariant at the fixed point. It turns out that it is even invariant under conformal transformations, corresponding to scale transformations with a space and time dependent scale factor. We will show later how to find and describe this boundary condition explicitly.

I would like to mention in passing that a similar approach has lead recently[8], may be in a slightly less obvious way, to an exact solution also of the (non-trivial) multicritical fixed point which is known to occur[15] in the problem of *two impurity spins* of size 1/2 in a sea of conduction electrons. Apart from its intrinsic conceptual interest, the two-impurity Kondo effect is relevant for Heavy Fermion physics. A discussion of the two-impurity problem is however not included in this review.

There has been an explosion of activity in two-dimensional Conformal Field theory (CFT) within the past 7 years, starting with the pioneering work of Belavin, Polyakov and Zamolodchikov[16]. Exact solutions for an immense variety of 2D CFT's have been found (see e.g. Ref.(17, 18, 19)), all of which describe some kind of critical behavior**. The subject of CFT has in fact been, intellectually, one of the most successful developments in theoretical physics, providing one of the most prolific examples of interdisciplinary cross-fertilization between Statistical Mechanics, Quantum Field Theory, String Theory and diverse branches of Mathematics, revealing deep connections among these subjects.

As with any area of theoretical physics, the ultimate goal is to contribute an understanding of nature. Even though CFT plays an important rôle in String Theory, applications of the impressive body of exact information coming from CFT to experimentally accessible systems has been, unfortunately, rather limited so far: Systems of absorbed monolayers of atoms on crystal surfaces, described by 2D Ising and 3-state Potts statistical mechanics models, realize a very limited number of CFT's. One-dimensional quantum spin chains are also realizations[20,21] of CFT's; however, the stable spin chains occuring in nature, provide only realizations of trivial, i.e. non-interacting CFT's.

**These exact solutions are in fact so detailed that essentially any universal quantity related to this critical behavior could be calculated.

Furthermore, the excitations occuring at the edge of 2D Fractional Quantum Hall Effect samples are realizations[22] of, again, only non-interacting, Gaussian CFT's.

From a theoretical point of view it is possibly one of the most important aspects of the work reviewed in this lecture that *Quantum Impurity Problems do provide*, in general, *realizations of non-trivial, fully interacting*[§] *CFT*'s. It may come as a surprise that some of the most prominent realization of CFT in the laboratory appear to be found in the physics of Heavy Fermion type materials (and possibly High-T_c superconductivity). It is also interesting to notice that it is the *boundary* versions of non-trivial CFT's, first proposed[23] in their abstract form by J. Cardy, which are more easily realized in the laboratory, and not their bulk versions, which are somewhat more popular in the literature.

An interesting feature of the realizations of CFT addressed in this review is that some of the deep, purely mathematical structures[24,25,26,27], which are considered at the heart of CFT, such as e.g. the *Fusion Rules* as well as the *modular S-matrix* which represents a so-called modular transformation on the characters of irreducible representations of the infinite-dimensional conformal algebra, appear directly in experimentally observable quantities. For example, the change in the residual entropy at zero temperature, caused by the impurity (which has e.g. been measured in the dilute Heavy Fermion compound $U_xY_{1-x}Pd_3$, proposed as a realization of the $k = 2$ channel, $s = 1/2$ model), is universal and simply equal to the logarithm of a ratio of such S-matrix elements (Section 5.3). Another ratio of such S-matrix elements appears in the residual resistivity (Section 6.1), induced by dilute impurities at low temperature (also a measurable quantity). This same ratio also enters the *Friedel sum rule*, when generalized to the non-Fermi-liquid situation. Moreover, the *Verlinde Formula*[25], which relates the modular S-matrix to the the Fusion Rules, is a crucial ingredient in deriving all these results. Furthermore, the *Monodromy matrices*[28,27] of the conformal blocks of the (conformal) Wess-Zumino-Witten model, related to non-trivial representations of the *Braid group* (see e.g. Ref.(26, 29)), describe precisely[5,7] the physical process of moving operators (such as e.g. the spin or pair-field operator) from infinity, where they are equivalent to free Fermion

§ typically Wess-Zumino-Witten models at level $k > 1$

fields since all the effects of the impurity have died out, to a position close to the impurity, where they turn into non-trivial, interacting fields. In physical language, *spin, charge* or *flavor* degrees of freedom, which in the bulk can only occur bound together in certain allowed combinations, may become liberated near the impurity in this process.

The outline of this review is as follows: In Section 1 some basic background about the Kondo effect is given. Section 2 reviews the mapping of the Kondo problem into a Field Theory problem and discusses the physical picture of Nozières and Blandin. In Section 3 the exact solution of the strong coupling fixed point using Conformal Field Theory is reviewed. The so-obtained solution is put into the more general context of boundary critical phenomena in Section 4. As an application of the ideas developped so far, the low temperature thermodynamics of the Kondo problem is reviewed. Section 6 discusses the exact computation of space and time dependent Green's functions and transport properties. In Appendix A the rôle of modular transformations and of the modular S-matrix in boundary problems, as well as their relationship with the Fusion Rules and the Verlinde Formula, are reviewed briefly.

1: Some Physics of the Kondo Effect

The Kondo problem is concerned with the effect of magnetic impurities* at low concentration x on the properties of metals. Physical observables such as $A = \chi$ (susceptibility), C(specific heat), M(magnetization), S (entropy), ρ(resistivity),.. have in general a bulk piece which measures the (for our purposes uninteresting) properties of the pure metal as well as a piece linear in the concentration x, which describes the interaction of the conduction electrons with a *single* impurity

$$A_x(T, ...) = A_{bulk}(T, ...) + xA_{imp}(T, ...) + O(x^2) \tag{1.1}$$

*For the sake of clarity, I will always adopt the language of a magnetic impurity spin, coupled to (in general several bands of) conduction electrons; other situations, as e.g. the Quadrupolar Kondo effect, can always be cast in this form.

Higher powers in x, which we are not concerned with in this review, are related to interactions amongst impurities, as they occur e.g. for the first time in the two-impurity Kondo effect[15,8].

At high enough temperatures, i.e. when the coupling energy of electrons and impurity is much smaller than $k_B T$, the impurity is only weakly coupled and the linear term in Eq.(1.1) reflects simply the properties of an isolated quantum mechanical spin. The susceptibility, for example, has Curie-Weiss form

$$\chi_0(T) = s(s+1)/3T \tag{1.2}$$

The essential physics of the completely screened ($k = 2s$) Kondo effect, a special case of which is the simplest version with one band ($k = 1$) and spin 1/2, is that the impurity spin gets *screened* by the conduction electrons at low T. The crossover from Curie-Weiss to this low T behavior occurs at a characteristic scale T_K, the Kondo temperature. This screening manifests itself in a susceptibility which goes to a constant at low T, instead of diverging; hence, it is as if s was reduced to zero in Eq.(1.2). In the underscreened case ($k < 2s$), only a part of the impurity spin is screened, and the impurity susceptibility still diverges as in Eq.(1.2), where however s is replaced by $s' = s - k/2$, the size of the unscreened spin. The physical picture by Nozières and Blandin[10], to be reviewed below, gives a transparent explanation of screening in these cases. In the overscreened cases ($k > 2s$), $\chi_{imp}(T)$ still diverges at low T, however with a non-trivial power, smaller than 1 (or as $\ln T$ for $k = 2, s = 1/2$). There is no simple physical picture available for the screening mechanism in these, overscreened cases. A complete description of the system at low temperatures in these cases is given, for the first time,in the work reviewed in this lecture.

As already pointed out, the *in*crease of the resistivity $\rho_{imp}(T)$, as T is lowered, counteracting the ordinary *de*crease caused by interaction with phonons, manifests the dramatic effect of magnetic impurities on the electronic properties of the host metal. $\rho_{imp}(T)$ does however not increase to infinity but saturates to a *finite* value, the *residual resistivity* $\rho_{imp}(0)$, as $T \to 0$, which has been computed exactly[5] using the CFT approach. This value is approached, in the overscreened cases ($k > 2s$), with a non-trivial power law, as was recently shown[5]. In the remaining simpler cases, Nozières's Fermi-liquid theory predicted[13], in contrast to that, a T^2 saturation.

2: Field Theory Mapping of the Multi-Channel Kondo Problem

The general Hamiltonian for the multi-channel Kondo model is

$$\hat{H} = \hat{H}_0 + \hat{H}_I \tag{2.1}$$

Here

$$\hat{H}_0 = \int d^3\vec{k} \cdot \epsilon_{\vec{k}} \cdot c^{\dagger\alpha,i}(\vec{k}) c_{\alpha,i}(\vec{k}) \tag{2.2}$$

represents the propagation of free band electrons ($\epsilon_{\vec{k}}$ is the electron dispersion relation) and $\alpha = 1, 2$ labels up- and down- spin, while $j = 1, 2, ..., k$ is a channel index, labelling the different bands**. The (non-relativistic) Fermions

$$c_{\alpha,i}(\vec{r}) = \int \frac{d^3\vec{k}}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{r}} c_{\alpha,i}(\vec{k}) \tag{2.3}$$

satisfy canonical anticommutation relations

$$\{c^{\dagger\alpha,i}(\vec{k}), c_{\beta,j}(\vec{k}')\} = \delta_{\alpha,\beta}\delta_{i,j}\delta^{(3)}(\vec{k} - \vec{k}')$$

$$\{c^\dagger, c^\dagger\} = \{c, c\} = 0 \tag{2.4}$$

The interaction hamiltonian represents simply an antiferromagnetic coupling of the impurity spin to the conduction electron spin density

$$\vec{\mathcal{S}}(\vec{r}) = \frac{1}{2} c^{\dagger,\alpha,i}(\vec{r}) (\vec{\sigma})_\alpha^\beta c_{\beta,i}(\vec{r}) \tag{2.5}$$

at the origin, where the impurity spin $\vec{S}$ is located:

$$\hat{H}_I = \tilde{\lambda}_K \vec{\mathcal{S}}(\vec{r} = 0) \cdot \vec{S}, \quad (\text{where} \quad \tilde{\lambda}_K \geq 0) \tag{2.6}$$

** summation over repeated indices is understood throughout